WAVEGUIDES AND RESONATORS

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Formation of a non-Gaussian intensity profile in a waveguide quasi-optical resonator with an aspherical reflector

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Abstract. **A method of phase conjugation for formation of non-Gaussian light beams with the specified transverse intensity distribution in waveguide quasi-optical resonators is described. The method is based on the correction of the phase of the fundamental resonator mode by changing the shape of one of the reflectors. The possibility of producing beams with the super-Gaussian and doughnut-like intensity profiles in a waveguide resonator with an aspherical mirror is shown theoretically. The region of geometrical parameters of the resonator is found for which the difference between the phase functions of aspherical and convex spherical resonators is small. The existence of beams with the super-Gaussian intensity profile at the output of a waveguide submillimetre ($\lambda = 0.4326$ mm) resonator with a convex spherical mirror is confirmed experimentally.**

Keywords: *waveguide resonator, beam formation, aspherical reflector, single-mode regime.*

1. Introduction

Due to diffraction, the output distribution of the radiation field outside a laser resonator in the general case represents a complex superposition of transverse modes and has an intricate shape. Wide applications of laser radiation in modern fields of physics, technological processes, diagnostic systems, and medicine require the formation of the specified profile of a laser beam and its control, as well as the increase in the operation efficiency of laser systems. To achieve the optimal result in each practical application, it is necessary to use a certain distribution of the laser radiation intensity. Such distributions of the beam intensity in resonators are usually called custom modes [1] because their spatial profile is optimised for the specific application. In practice, it is often necessary to use the nearly uniform super-Gaussian distribution [2] or the doughnut-like transverse field distribution on the output mirror of a resonator or in a specified plane outside the resonator. For example, in laser applications for material surface machining, annealing of defects in semiconductors, location, in systems for optoelectronic data processing, nonlinear conversion, and ophthalmosurgery, it is desirable to use laser beams with the uniform transverse intensity distribution, which sharply decreases at the aperture edges [3]. It is shown that a laser beam with the doughnut-like intensity profile experiences the smallest nonlinear distortions compared to the beams of other shapes propagating in a nonlinear and turbulent medium [4]. Such beams attract great interest for applications in laser pincers for capturing and moving micron and submicron particles in experiments in the fields of quantum electrodynamics, microbiology, biomechanics, micro- and nanotechnologies [5].

At present beams with a specified profile in open laser resonators are produced by using aspherical graded-phase mirrors [6]. The problem of the development of an open resonator for producing the specified output beam was first formulated in [7]. An analytic method was proposed for determining the required mirror shape from the specified field distribution in it. In [8 – 10], the energy losses and transverse distributions of the fundamental-field mode in stable and unstable resonators with aspheric mirrors of a finite aperture were investigated numerically. It was shown that in stable resonators with specially selected asphericity parameters of mirrors, a highly uniform distribution of the laser radiation field amplitude can be achieved by selecting proper parameters; however, mode losses increase significantly in this case. These theoretical studies were summarised in paper [11], where the general approach was formulated according to which the resonator parameters (complex reflection coefficients of resonator mirrors) are found from the given amplitude and (or) phase distribution of the field at the output aperture and closed integral equations for their determination are obtained.

The modern practical approach to the application of aspherical mirrors in laser resonators is presented and analysed in papers [1, 6, 12] from the point of view of producing the required (non-Gaussian) fundamental mode with the given field distribution at the output mirror of Nd:YAG, CO_2 and He – Ne lasers. Resonators with deformable mirrors attract increasing attention with the development of the methods and devices of adaptive optics [13]. Diffraction optical elements also can efficiently transform one radiation wavefront to another. Such elements are used in Nd:YAG laser resonators to form the output beams with the required radiation profile [14]; however, their manufacturing is more complicated than that of aspherical mirrors.

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Waveguide quasi-optical resonators (WQORs) represent a new class of resonators which provided the optimal energy and spectral characteristics of gas-discharge and solid active media in lasers [15]. Such resonators allow the formation of non-Gaussian radiation beams not only by using mirrors but also due to the coherent summing of a set of the transverse modes of an oversized waveguide placed between mirrors. It was proposed recently to use such resonators for generating quasi-uniform output beams of gas and solid-state lasers [16, 17]. However, their application requires the introduction of additional focusing elements into a resonator, which complicates the laser design. Therefore, the search for simpler and more efficient methods for shaping the output beam of WQORs is of current interest.

This paper is devoted to the theoretical and experimental study of one of the schemes of a submillimetre waveguide resonator, in which the output beam with a required intensity distribution is generated by using an aspherical mirror of the specified shape as one of the reflectors.

2. Theoretical relations and calculation results

Our theoretical treatment is based on the methods of phase conjugation and eigenmodes [13, 18]. The phase conjugation method involves the determination of the phase correction function of a reflector which would change the field incident on the reflector in such a way that the output radiation returned back to the initial plane would have the specified transverse intensity and (or) phase distribution. We will use this method to calculate the mode parameters of a WQOR with a circular cross section of a hollow dielectric channel closed by reflectors at its ends, one of which has a plane surface and the other – the required aspherical surface (Fig. 1).

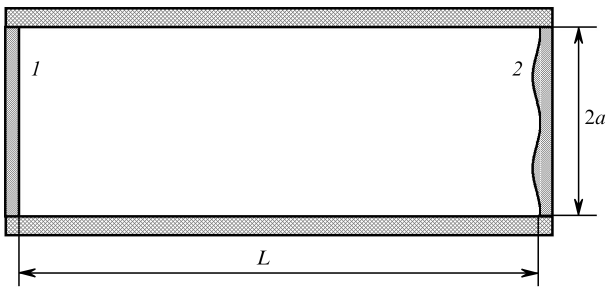

Figure 1. Scheme of a WQOR with an aspherical mirror: (*1*) plane reflector; (*2*) aspherical reflector.

Our calculation is based on the interpretation of the resonator modes as the result of the interference of counter-propagating wave beams in the waveguide reflected by mirrors, as accepted for quasi-optical systems [18]. Let us represent the required field distribution functions, describing the types of resonator modes, in the form of the expansion in slowly leaking waveguide modes of the channel-in-dielectric (LCD) type [19]. We will describe the interaction of radiation with the aspherical surface of one of the mirrors by using the phase correction function [20]. Let us assume that the diameters of the waveguide and resonator mirrors are $2a$ and the waveguide length (equal to the separation between mirrors) is L. We also assume in the calculation of this resonator that quasi-optical approximation conditions are fulfilled [18]: the resonator dimensions exceed the wavelength λ ($L/\lambda \gg 1$ and $a/\lambda \gg 1$), and the resonator length is considerably greater than its width ($L/a \gg 1$). The time dependence proportional to $\exp(-\mathrm{i}\omega t)$ (where $\omega = ck$, c is the speed of light in vacuum, and $k = 2\pi/\lambda$) was neglected.

We will seek the required profile of an aspherical mirror taking into account that the problem is axially symmetric, i.e. the radial field distribution in the resonator should be considered, and denote the radial orthonormalised functions for the LCD modes by $V_m(\rho)$, where $\rho = r/a$ is the dimensionless radial coordinate in the waveguide. Let us write the expression for the required complex amplitude of the wave beam field incident on plane reflector (*1*) in the form

$$U^{(1)}(\rho) = \sum_m C_m V_m(\rho), \tag{1}$$

where C_m are excitation coefficients for the waveguide modes. Then, the expression for the field amplitude on aspherical mirror (*2*) located at a distance of L from the plane reflector is written in the form

$$U^{(2)}(\rho) = \sum_m C_m V_m(\rho) \exp(\mathrm{i}\gamma_m L), \tag{2}$$

where γ_m are propagation constants for the LCD modes [19].

To recover the initial shape of the field at the resonator output after reflection from mirror (*2*), it is necessary to use the phase correction function $F(\rho)$ to compensate for the phase shift acquired by the radiation beam after the round trip in the resonator. Thus, we will seek $F(\rho)$ in the form

$$F(\rho) = \exp[\mathrm{i}\Psi(\rho)], \tag{3}$$

where $\Psi(\rho)$ is the phase function of mirror (*2*). According to the principle of the initial-beam phase compensation, this function is described by the expression

$$\Psi(\rho) = 2\mathrm{Arg}[U^{*(2)}(\rho)], \tag{4}$$

where the sign * means complex conjugation.

We will find the equation describing the resonator eigenmodes by the matrix method [21]. According to expressions (1) and (2), we have for the beam reflected from the aspherical mirror:

$$U^{(3)}(\rho) = \sum_m C_m F(\rho) V_m(\rho) \exp(\mathrm{i}\gamma_m L). \tag{5}$$

Let us rewrite (5), by representing $F(\rho)V_m(\rho)$ as a sum similarly to (1):

$$U^{(3)}(\rho) = \sum_m C_m \exp(\mathrm{i}\gamma_m L) \sum_l D_{ml} V_l(\rho), \tag{6}$$

where

$$D_{ml} = \int_0^1 F(\rho) V_m(\rho) V_l(\rho) \rho \mathrm{d}\rho.$$

The expression for the complex amplitude of the waveguide beam incident on plane reflector (*1*) can be written in the form

$$U^{(4)}(\rho) = \sum_m C_m \exp(\mathrm{i}\gamma_m L) \sum_l D_{ml} V_l(\rho) \exp(\mathrm{i}\gamma_l L). \quad (7)$$

Note that the amplitude $U^{(4)}(\rho)$ is obtained as a result of the round trip of the wave described by function $U^{(1)}(\rho)$ (1) in the resonator. For the established oscillations in the resonator, the field component in the wave beam before and after the round trip changes according to the expression $U^{(4)}(\rho) = \exp(\mathrm{i}2kL)\Lambda U^{(1)}(\rho)$ [18]. After transformations, we obtain from (1) and (7) a system of equations for determining the main characteristics of the resonator modes:

$$\Lambda C_l = \sum_m C_m D_{ml} \exp[\mathrm{i}(\gamma_m + \gamma_l - 2k)L]. \quad (8)$$

The solution of a system of equations (8) gives the eigenvalues Λ and eigenvectors $\boldsymbol{C}$ with components representing the coefficients of expansion of the resonator modes in the waveguide modes. The relative energy loss δ and the phase shift φ after the round trip in the resonator, which is additional to the geometrical-optic phase shift, are described by the expressions

$$\delta = 1 - |\Lambda|^2, \quad \varphi = \arg \Lambda. \quad (9)$$

We will restrict our numerical study to a practically important case of linearly polarised radiation. Among the LCD modes, there exists a set of EH_{1m} modes, whose fields for $m \leqslant (a/\lambda)^{1/2}$ [22] are linearly polarised and have complex amplitudes described by the orthonormalised functions $V_m(\rho) = \sqrt{2} J_0(U_m\rho)/J_1(U_m)$ forming a complete system, where J_0 and J_1 are the Bessel functions of the first kind and U_m are the roots of the equation $J_0(U_m) = 0$ [19]. The propagation constants of these modes are

$$\gamma_m \approx k\left[1 - \frac{1}{2}\left(\frac{U_m\lambda}{2\pi a}\right)^2\left(1 - \frac{\mathrm{i}n_1\lambda}{\pi a}\right)\right], \quad (10)$$

where $n_1 = 0.5(n^2+1)/(n^2-1)^{1/2}$ for the EH_{1m} modes and n is the refractive index of the waveguide walls.

The phase correction function for the reflector (*2*) of the waveguide resonator required for obtaining the uniform output field was calculated by the phase conjugation method. The complex matrix in (8) was calculated with a computer by using the Mathcad software package. We used the matrix method in [23] to calculate the parameters of the modes of the waveguide dielectric resonator in a 0.4326-mm HCOOH laser with mirror (*2*) representing a convex spherical reflector. The phase correction function of this reflector is described by the expression [20]

$$F(\rho) = \exp[-\mathrm{i}\Phi(\rho)], \quad (11)$$

where $\Phi(\rho) = 2\pi N v \rho^2$ is the phase function of the reflector; $N = a^2/(\lambda L)$ is the Fresnel number of the resonator; $v = L/R < 0$ is the confocal parameter of the mirror; and R is radius of curvature of the mirror. In this paper, we studied the root-mean-square difference Δ between the phase functions of the aspherical reflector [$\Psi(\rho)$] and the convex spherical reflector [$\Phi(\rho)$] calculated from the expression [24]

$$\Delta = \left[\int_0^1 |\Psi(\rho) - \Phi(\rho)| \mathrm{d}\rho\right]^{1/2}.$$

The Fresnel number N of the resonator was varied in the range from 0.1 to 1, which is typical for waveguide laser resonators, the modulus of the confocal parameter $|v|$ of the convex mirror was varied from 0.1 to 10. The calculations were performed for the waveguide radius $a = 9$ mm ($a/\lambda \approx 21$) by varying the resonator length from 187 to 1870 mm. It was assumed that the waveguide was made of a Pyrex glass with the refractive index $n \approx 2.58 + \mathrm{i}\,0.08$ at the given wavelength [25]. Figure 2 shows the region of geometrical parameters of the resonator ($N \approx 0.259 - 0.275$, $|v| \approx 1.17 - 1.96$), which is characterised by a small root-mean-square difference Δ not exceeding 10 %. One can see from Fig. 3 that the difference between the phase function of the aspherical reflector, calculated by the phase conjugation method, and the phase function of the convex spherical reflector in this region is small. Therefore, we studied theoretically and experimentally the conditions of application of the phase conjugation method in WQORs for formation of the uniform radiation field by analysing the

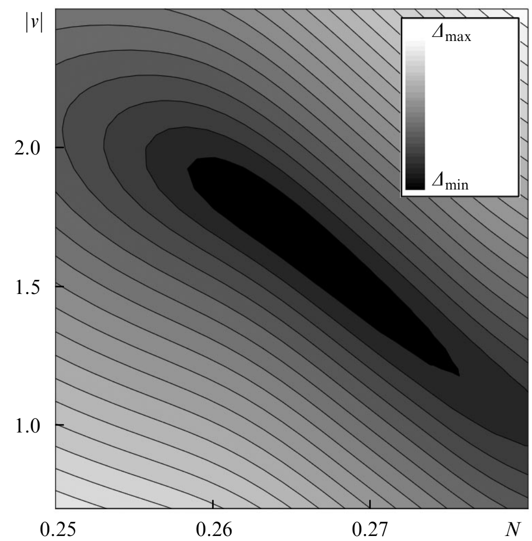

Figure 2. Dependence of the root-mean-square difference Δ between the phase functions of aspherical and convex spherical WQOR reflectors on the Fresnel number N and the confocal parameter v of a mirror.

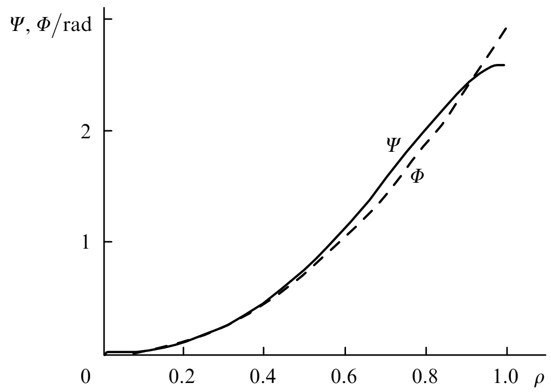

Figure 3. Phase functions of the aspherical (Ψ) and convex spherical (Φ) reflectors of a WQOR for $N \approx 0.26$ and $|v| \approx 1.8$.

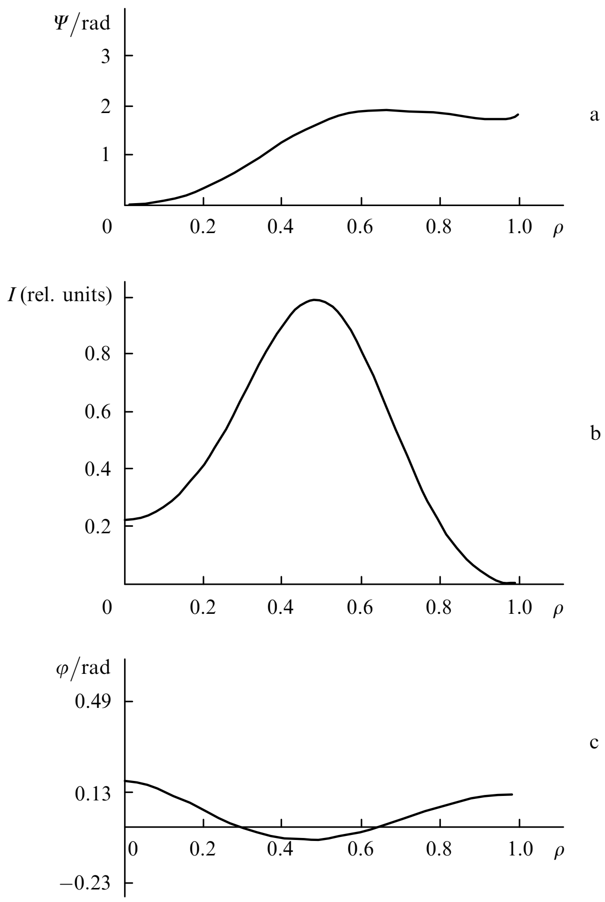

Figure 4. Phase function of the aspherical reflector (a) and the radial intensity (b) and phase (c) distributions on the output WQOR mirror.

mode parameters of a resonator with a convex spherical reflector.

The phase conjugation method was also used to study numerically the possibility of obtaining the doughnut-like profile of the output WQOR beam. Figure 4a shows the calculated phase function of resonator mirror (*2*) (Fig. 1) ($N = 0.26$) required for this case. Figures 4b, c present the transverse field intensity and phase distributions produced on the output mirror in this case.

3. Experimental setup. Comparison of experimental and numerical results

Figure 5 presents the scheme of the experimental setup for studying the mode spectrum of WQORs and their output radiation intensity distribution. To obtain symmetric resonance curves and to study the output radiation intensity of the resonator, the resonator was investigated in the 'passage' regime [26]. The resonator was formed by hollow glass waveguide (*9*) of diameter 18 mm and length 720 mm. Reflector (*8*) represents a two-dimensional nickel grid with a period 100 μm made of strips of width 25 μm and thickness 17 μm. The transmission coefficient of the grid at a wavelength of 0.4326 mm, at which measurements were performed, was 6 %. To obtain the plane or spherical profile of mirror (*8*), Teflon plane–convex lens (*7*) with the radius of curvature of the convex surface equal to 400 mm was used. By pressing tightly one of the lens surfaces to the nickel grid, the plane of spherical profile of

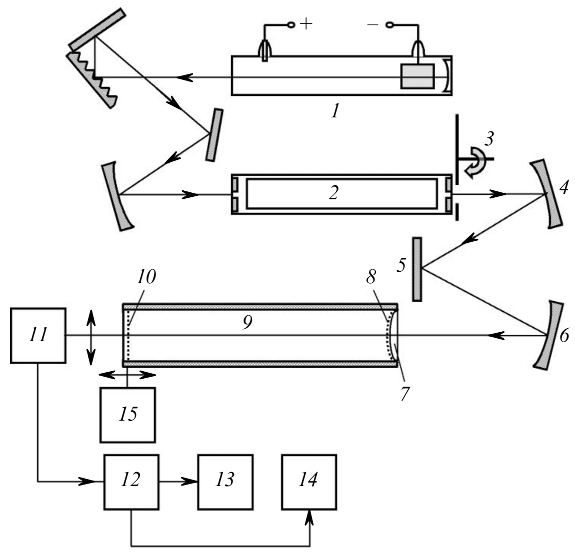

Figure 5. Scheme of the experimental setup: (*1*) CO_2 laser; (*2*) submillimetre cell; (*3*) chopper; (*4–6*) mirrors; (*7*) Teflon lens; (*8*, *10*) reflectors; (*9*) glass waveguide; (*11*) pyroelectric detector; (*12*) amplifier; (*13*) oscilloscope; (*14*) recorder; (*15*) electric motor.

mirror (*8*) could be obtained. In this case, the transparency of the reflector was not changed within the measurement error. Reflector (*10*) is made of a plane–parallel silica plate with a capacitive aluminium grid with reflecting squares of size 86×86 μm and period 14 μm applied on the inner surface of the plate. The measured transparency of the capacitive grid was 18 %.

All the elements of the resonator were mounted on an IZA-2 measuring line which provided the precision displacement (with a misalignment of no more than $1''$) of output reflector (*10*) along the resonator optical axis with the help of electric drive (*15*). The resonator was excited through semitransparent reflector (*8*) by an optically pumped submillimetre laser consisting of CO_2 pump laser (*1*) and submillimetre cell (*2*). The submillimetre laser operating on formic acid molecules (HCOOH) emitted at 0.4326 mm. The laser radiation was modulated with chopper (*3*) and was matched with the resonator by using mirrors (*4–6*). The radius of curvature $R = 5$ m of mirror (*4*) was chosen taking into account the divergence of the output beam of the laser; mirror (*5*) was plane. The reflector of the resonator under study was placed at a distance of 115 cm from mirror (*6*) with the radius of curvature 2 m. In this plane, a Gaussian radiation beam of diameter 18 mm at the 1/10 maximum level with a plane phase front was formed. The radiation passed through the resonator was detected with devices (*11–14*).

The measurement method was similar to that described in [26]. The spectrum of resonator eigenmodes was recorded by varying the resonator length with the help of electric drive (*15*). The total energy losses δ per round trip in the resonator were determined from the measured width of the resonance curve. The transverse modes were identified by intermode intervals, which were calculated from their phase shifts after a round trip in the resonator, and from the theoretical transverse intensity distributions [19]. The transverse intensity distributions near the output reflector of the resonator were measured by scanning pyroelectric detector

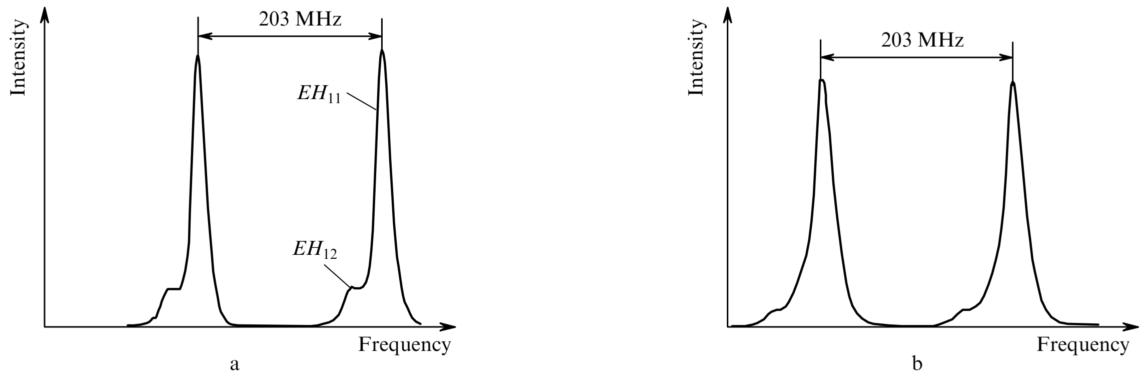

Figure 6. Mode spectra of waveguide resonators with two plane (a) and a plane and convex spherical reflectors (b).

(*11*) with a spatial resolution of 1 mm in a plane perpendicular to the laser beam.

The size of the experimental resonator model described above were chosen to match the Fresnel number of the resonator $N \approx 0.26$ and the modulus of the confocal parameter of the convex reflector $|v| \approx 1.8$, i.e. in the region (Fig. 2) where the field distributions on the output mirror differ insignificantly when aspherical or convex spherical reflectors are used (Fig. 3). Figure 6 presents the mode spectra for resonators with two plane reflectors and with a plane and spherical reflector. In both cases, two modes are observed during resonator tuning. In the case of the plane–parallel geometry, the spectrum contains the EH_{11} and EH_{12} modes. The experimental intermode distances correspond to calculations and are equal to 37 and 53 MHz for plane and convex spherical reflector (*8*), respectively. The measured total losses after a round trip in the resonator were 43 % and 53 % in the case of plane reflectors and the spherical reflector, respectively. Because the coupling and thermal losses in mirrors are the same for different resonator geometries, the difference of total losses is explained by the increase in waveguide losses in the resonator with the convex reflector and is 10 %. The waveguide losses for the resonator with plane mirrors calculated from (10) are equal to 16 %, while in the case of a convex reflector, the losses amount to 25 %, i.e. the calculated increase in losses in the latter case is 9 %. Taking into account that the experimental error of measuring losses was ± 5 %, the experimental data and calculations are in good agreement.

Figure 7 presents the experimental and calculated intensity distributions on the output mirrors of resonators for modes having the minimal losses. The transverse intensity distribution for a resonator with plane reflectors corresponds to the distribution for the fundamental waveguide EH_{11} mode. The experimental intensity distribution in the case of a convex spherical reflector coincides qualitatively with the calculated distribution and has the nearly uniform super-Gaussian profile sharply decreasing at the aperture edges. Some difference between the calculated and experimental field distributions in both cases is related to the inaccurate alignment of resonator mirrors and deviations of the dimensions of the glass waveguide for the ideal values (due to ellipticity, conicity, and surface roughness), which were neglected in calculations.

Thus, we have confirmed experimentally the possibility to obtain the uniform output intensity distribution by using a spherical convex reflector instead of a more complex aspherical reflector.

4. Conclusions

We have described the phase conjugation method for producing non-Gaussian light beams with the specified intensity distribution in a WQOR, which is based on the correction of the phase of the fundamental resonator mode by varying the shape of one of the reflectors. The possibility of generating beams with super-Gaussian and doughnut-like intensity profiles has been shown theoretically. The region of geometrical parameters of the resonator has been found where the difference between the phase functions of

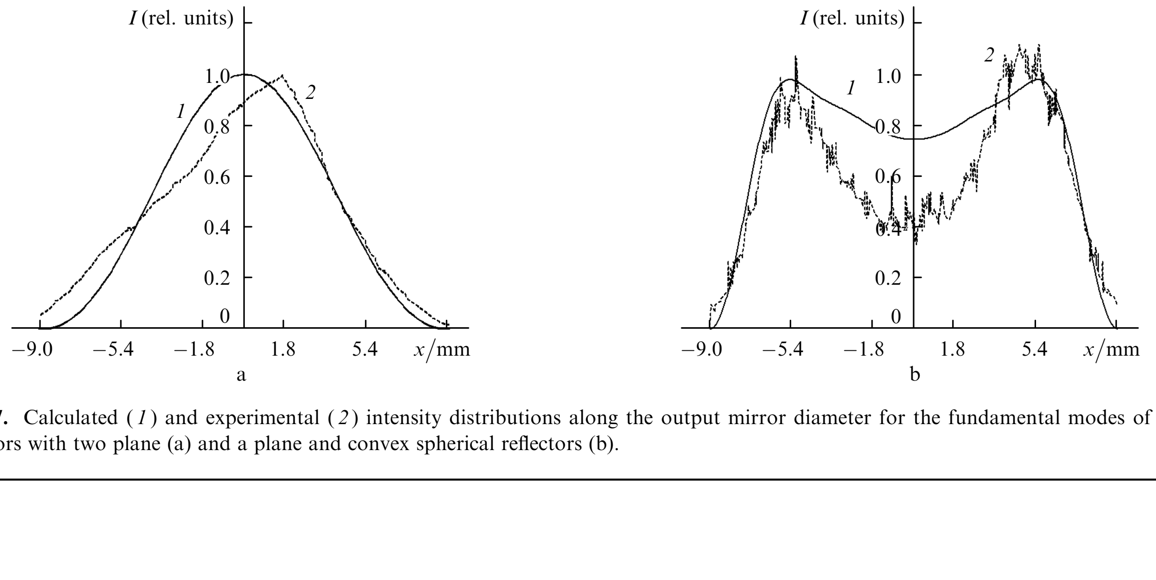

Figure 7. Calculated (*1*) and experimental (*2*) intensity distributions along the output mirror diameter for the fundamental modes of waveguide resonators with two plane (a) and a plane and convex spherical reflectors (b).

aspherical and convex spherical reflectors is small, and the existence of a transverse mode with the super-Gaussian intensity profile at the output of a waveguide submillimetre resonator with a convex mirror has been confirmed experimentally. In this case, the good selection of the transverse resonator modes over energy losses is observed, i.e. single-mode lasing can be provided at this mode.

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