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Passive *Q*-switching of an erbium-doped glass laser by using a Co^{2+} : ZnSe crystal

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***Abstract*. The operation of a passively *Q*-switched laser emitting the TEM_{mn} mode is described. The excited-state absorption losses and nonlinear absorption in the shaded ends of the active element are taken into account. We measured the laser pulse energy for the TEM_{00} mode, the ratios of the output energy in transverse TEM_{mn} modes with $m = 0 - 4$, $n = 0$ and $m = 0 - 3$, $n = 1$ to the TEM_{00} mode energy in an erbium-doped glass laser and determined its temporal parameters in the passive *Q*-switching regime achieved with the help of a Co^{2+} : ZnSe crystal. It is shown that on passing from one type of the transverse mode to another for invariable parameters of the resonator, the laser pulse energy changes discretely. A change in the radius of the transverse distribution of the TEM_{00} mode at the output mirror of the passively *Q*-switched laser during the development of lasing is estimated. It is found that the relative change in the laser beam cross section on the output mirror under our conditions is less than 2 %. The experimental data obtained in the study are in good agreement with our calculations.**

Keywords: *lasers, passive Q-switching, transverse resonator modes.*

1. Introduction

The passive *Q*-switching regime has been studied in many papers [1 – 10], in particular, quite recently [2]. In most cases it was assumed theoretically that the transverse distribution of the radiation intensity is rectangular. The exclusion in this respect is paper [3], where it was assumed that the transverse intensity distribution corresponds to the TEM_{00} mode of an empty resonator and remains invariable during lasing. The same assumption was made later in papers [2, 9, 10], where it was also assumed that the radiation profile is Gaussian.

One of the aims of our paper is to develop the method for calculating parameters of a passively *Q*-switched Er^{3+}-doped glass laser for the transverse intensity distribution corresponding to an arbitrary TEM_{mn} mode of a passive resonator. The method proposed in the paper is the development of the energy balance method described in [3]. A part of the paper is devoted to the experimental substantiation of the assumption that the transverse intensity distribution remains invariable during the development of lasing under our conditions. We also studied the influence of the excited-state absorption in Er^{3+} at a wavelength of 1.54 μm on the laser parameters.

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2. Theory

We will consider the energy balance for a transverse mode with arbitrary indices during a short period of time Δt. Such an approach allows us to avoid a quite intricate calculation of the interaction between spatially inhomogeneous radiation and a medium with a nonuniform gain. The validity of the adopted assumptions can be verified by comparing conclusions obtained with the framework of our model with experimental results.

Let us make the following assumptions. First, we assume that lasing occurs on the transverse TEM_{mn} mode with the field distribution radius equal to the radius of the field distribution of a mode of a passive resonator, i.e. a resonator containing all optical elements, including an unpumped active element (AE). This assumption is quite strong because the transverse density distributions of the population of the upper and lower laser levels change during lasing, which should affect the transverse intensity distribution of radiation propagating through the AE. The distortion of the transverse intensity distribution in a saturated amplifying medium was studied in a number of papers. Thus, it was shown in [11] and some other papers that in the case of high gains (7.4 – 20), the intensity profile changes during the development of lasing and differs from the gain calculated in [12]. On the other hand, it was shown in [13, 14] that these distortions at low gains were small. In our case, the gain was lower than 1.3, which is much smaller than that in [11], and in addition, in [11] a resonator with plane mirrors was considered, which is very sensitive to perturbations.

Note also that a resonator contains, as a rule, an aperture to filter the spatial intensity distribution, which partially reconstructs the lasing profile after each transit of radiation in the resonator. As shown in [15], the TEM_{00} mode of a resonator with a Gaussian aperture is stable to perturbations, i.e. initial deviations from the stationary parameters (radius of curvature and diameter) of a Gaussian beam propagating in the resonator tend to zero with increasing the number of transits of radiation in the resonator. If an aperture is absent in the resonator, deviations do not decrease with increasing the number of transits, as for example, during the propagation of radiation in an optical delay line [16]. For a three-level laser medium in the presence of the shaded regions of the AE, i.e. the regions that are not pumped, the bleaching of these regions caused by laser radiation will produce a soft aperture, which will also maintain a certain mode of the resonator.

Second, we assume that the lasing intensity, which is a sum of intensities of the forward and backward waves in a linear resonator, is constant at any instant of time along the AE. Third, we neglect a change in the population of levels caused by spontaneous decay because the laser pulse duration (~ 100 ns) is considerably shorter than the lifetime of the upper level of Er^{3+} (5–10 ms). Fourth, we neglect the energy spent for bleaching a Q switch. This assumption is justified by the fact that under usual conditions the lasing energy emitted by the AE considerably exceeds the energy required for bleaching a passive Q switch. Fifth, we include into the balance also the radiation energy accumulated in the resonator. In addition, we assume that the width of the laser emission spectrum is large enough, so that the spatial distribution of the population inversion along the AE is assumed homogeneous. We also assume that the initial distribution of the gain in the AE volume is homogeneous.

A part of radiation emitted by an AE during lasing for the time Δt is lost due to absorption and diffraction and another part emerges from the resonator through a semi-transparent mirror. In addition, a part of energy is accumulated in the form of radiation in the resonator. We will take into account the excited-state absorption (ESA). Within the framework of our consideration, this absorption cannot be distinguished, generally speaking, from usual absorption, for example, by foreign impurities. However, the consideration in the presence of ESA is somewhat more complicated, therefore we will perform it for the case when ESA is present.

Let us assume that the laser radiation intensity in the AE is

$$I(x,y,t) = I_0(t)G_{mn}(x,y),$$

where x and y are coordinates in the transverse plane and $G_{mn}(x,y)$ are functions describing the transverse intensity distribution for modes with indices m and n. In the rectangular coordinates [17, 18],

$$G_{mn}(x,y) = H_m\left(\frac{\sqrt{2}x}{w}\right)^2 H_n\left(\frac{\sqrt{2}y}{w}\right)^2 \times \exp\left(-2\,\frac{x^2+y^2}{w^2}\right),$$

where $H_n(s)$ is the Hermitean polynomial of the nth degree and w is the radius of the Gaussian distribution. For the circular symmetry, the function G_{mn} is expressed in terms of the Laguerre polynomials.

We will proceed from the known (see, for example, [19]) kinetic equation for the concentration n_2 of ions occupying upper laser level 2:

$$\frac{\mathrm{d}n_2}{\mathrm{d}t} = [-(\sigma_{12}+\sigma_{21})n_2 + \sigma_{12}n_0]I(x,y,t),$$

where n_0 is the total concentration of impurity ions; σ_{12} and σ_{21} are the cross sections for stimulated upward and downward transitions, respectively. The solution of this equation has the form

$$n_2(x,y,t) = \xi n_0 + [n_2(t=-\infty) - \xi n_0] \times \exp\left[-(\sigma_{12}+\sigma_{21})\int_{-\infty}^{t} I(x,y,t')\mathrm{d}t'\right],$$

where $\xi = \sigma_{12}/(\sigma_{12}+\sigma_{21})$. The initial concentration $n_2(t = -\infty)$ for the irradiated region of the AE is equal to its threshold value, while for the shaded regions of the AE we have $n_2(t=-\infty) = 0$.

It can be shown that the energy extracted from the AE for the time Δt during lasing is

$$E_{\mathrm{extr}} = \hbar\omega\,\frac{\mathrm{d}x(t)}{\mathrm{d}t}\,\Delta t\,\frac{\chi_0 + 2l_{\mathrm{ends}}\alpha_0 + 2\eta l_{\mathrm{ill}}\alpha_0}{2(\sigma_{12}+\sigma_{21})(1-\eta)}\,F_{mn}(x(t)), \tag{1a}$$

where

$$F_{mn}(x(t)) = \int_S G_{mn}\exp[-x(t)G_{mn}]\mathrm{d}s;$$

$\hbar\omega$ is the laser photon energy;

$$x(t) = (\sigma_{12}+\sigma_{21})\int_{-\infty}^{t} I_0(t)\mathrm{d}t$$

(note that hereafter $x(t)$ is not a coordinate); $\eta = \sigma_{23}/(\sigma_{12}+\sigma_{21})$; σ_{23} is the excited-state absorption cross section; $\chi_0 = \ln(1/R) + 2\ln(1/T_0)$; R is the reflectance of the output mirror; T_0 is the initial transmission of a passive Q switch; l_{ill} is the length of the illuminated AE region; l_{ends} is the length of the shaded AE region near the AE ends (we assume that pumping is performed by a flashlamp and a part of the AE is shaded by the AE holders); $L_{\mathrm{a}} = l_{\mathrm{ill}} + l_{\mathrm{ends}}$ is the AE length; and $\alpha_0 = \sigma_{12}n_0$ is the unsaturated absorption coefficient at the laser wavelength. Integration is performed over the area S in the transverse plane ($\mathrm{d}s$ is the area element). Expression (1a) was derived taking into account that the radius w of the transverse intensity distribution in the AE is independent of the longitudinal coordinate z and time.

The energy absorbed from the excited state during lasing in the illuminated AE region is

$$E_{\mathrm{ill}}^{\mathrm{esa}} = \eta\hbar\omega\,\frac{\mathrm{d}x(t)}{\mathrm{d}t}\,\Delta t\left[\frac{l_{\mathrm{ill}}\alpha_0}{\sigma_{12}+\sigma_{21}}\,S_{mn} + \frac{\chi_0 + 2l_{\mathrm{ends}}\alpha_0 + 2\eta l_{\mathrm{ill}}\alpha_0}{2(\sigma_{12}+\sigma_{21})(1-\eta)}\,F_{mn}(x(t))\right], \tag{1b}$$

where $S_{mn} = \int_S G_{mn}\mathrm{d}s$. The energy absorbed from the ground state in the shaded AE region is

$$E_{\mathrm{ends}}^{\mathrm{s}} = \hbar\omega\,\frac{\mathrm{d}x(t)}{\mathrm{d}t}\,\Delta t l_{\mathrm{ends}}\alpha_0\,\frac{1}{\sigma_{12}+\sigma_{21}}\,F_{mn}(x(t)). \tag{1c}$$

The energy absorbed from the excited state during lasing in the shaded AE region is

$$E_{\rm ends}^{\rm esa} = \hbar\omega\,\frac{{\rm d}x(t)}{{\rm d}t}\,\Delta t\eta l_{\rm ends}\alpha_0\,\frac{1}{\sigma_{12}+\sigma_{21}}\,[S_{mn} - F_{mn}(x(t))]. \quad (1d)$$

The energy lost in the resonator due to escape of radiation through the semitransparent output mirror and due to absorption in the passive Q switch (we assume that the Q switch has the residual absorption) is [19, 20]

$$E_{\rm res}^{\rm loss} = \frac{1}{2}\,\hbar\omega\,\frac{1}{\sigma_{12}+\sigma_{21}}\,\frac{{\rm d}x(t)}{{\rm d}t}\,\Delta t\chi_1 S_{mn}, \quad (1e)$$

where $\chi_1 = \ln(1/R) + 2\ln(1/T_1)$ is the intracavity loss during lasing and T_1 is the transmission of the open Q switch. The energy accumulated in the resonator in the form of radiation changes for the time Δt by

$$E_{\rm res}^{\rm r} = \frac{1}{2}\,T\hbar\omega\Delta t\,\frac{{\rm d}I_0}{{\rm d}t}\,S_{mn}, \quad (1f)$$

where T is the round-trip transit time of a photon in the resonator. The energy extracted from the AE will be scattered in all processes considered above.

The energy balance is described by the expression

$$E_{\rm extr} = E_{\rm ill}^{\rm esa} + E_{\rm ends}^{\rm s} + E_{\rm ends}^{\rm esa} + E_{\rm res}^{\rm loss} + E_{\rm res}^{\rm r}. \quad (2)$$

By substituting expressions (1) into (2), we obtain the equation

$$T\,\frac{{\rm d}I_0}{{\rm d}t} = \frac{\chi_1 + 2L_{\rm a}\eta\alpha_0}{\sigma_{12}+\sigma_{21}}\,\frac{{\rm d}x(t)}{{\rm d}t}\left[(1+v)\,\frac{F_{mn}(x(t))}{S_{mn}} - 1\right], \quad (3)$$

where $1 + v = (\chi_0 + 2L_{\rm a}\eta\alpha_0)/(\chi_1 + 2L_{\rm a}\eta\alpha_0)$. From (3) and the condition ${\rm d}I_0/{\rm d}t = 0$, we can obtain the equation for determining $x_{\max}$ at which the radiation intensity during lasing achieves the maximum:

$$(1+v)\,\frac{F_{mn}(x_{\max})}{S_{mn}} - 1 = 0. \quad (4)$$

By integrating the right- and left-hand sides of Eqn (3) with respect to time from $-\infty$ to t, passing to a new integration variable $u = x(t)$ in the right-hand side, and taking into account that $x(t = -\infty) = 0$, we obtain the equation for $x(t)$:

$$T\,\frac{{\rm d}x(t)}{{\rm d}t} = (\chi_1 + 2L_{\rm a}\eta\alpha_0)$$
$$\times\left\{(1+v)\,\frac{1}{S_{mn}}\int_S [1 - \exp[-x(t)G_{mn}]]{\rm d}s - x(t)\right\}. \quad (5)$$

From (5), taking into account that ${\rm d}x(t)/{\rm d}t \to 0$ for $t \to \infty$ (the radiation intensity after the pulse end is zero), we obtain the equation, whose solution gives the value $x_\infty^{(mn)} = x(t \to \infty)$ determining the laser pulse energy:

$$(1+v)\,\frac{1}{S_{mn}}\int_S \left\{1 - \exp\left[-x_\infty^{(mn)}G_{mn}\right]\right\}{\rm d}s - x_\infty^{(mn)} = 0. \quad (6)$$

The output energy $W^{(mn)}$ can be found from the expression [19, 20]

$$W^{(mn)} = \frac{1}{2}\,S_{mn}x_\infty^{(mn)}E_{\rm s}\ln\frac{1}{R}, \quad (7)$$

where $E_{\rm s} = \hbar\omega/(\sigma_{12} + \sigma_{21})$ is the saturation energy density. From (5) and (4), we obtain the intensity in the laser pulse maximum

$$I_{\max} = \frac{\chi_1 + 2L_{\rm a}\eta\alpha_0}{T(\sigma_{12}+\sigma_{21})}$$
$$\times\left\{(1+v)\,\frac{1}{S_{mn}}\int_S [1 - \exp[-x_{\max}G_{mn}]]{\rm d}s - x_{\max}\right\}. \quad (8)$$

Note that Eqns (3)–(6) and (8) are independent of the radius w. By defining the pulse duration as the ratio of the energy density to the maximum intensity, we obtain the expressions for the duration of the leading (τ_1) and trailing (τ_2) edges of the pulse and its total duration τ:

$$\tau_1 = \frac{x_{\max}}{I_{\max}(\sigma_{12}+\sigma_{21})},$$
$$\tau_2 = \frac{x_\infty - x_{\max}}{I_{\max}(\sigma_{12}+\sigma_{21})}, \quad (9)$$
$$\tau = \frac{x_\infty}{I_{\max}(\sigma_{12}+\sigma_{21})}.$$

For the TEM_{00} mode, Eqn (6) has the form [3]

$$(1+v)\int_0^{x_\infty}\frac{1 - \exp(-y)}{y}\,{\rm d}y - x_\infty = 0,$$

in this case, the lasing cross section is $S_{00} = \pi w^2/2$. If we set formally $G_{mn} = 1$ for $r \leqslant r_0$ and $G_{mn} = 0$ for $r > r_0$ (where r is the radius in the polar coordinate system and r_0 is the radius of the lasing region), then Eqn (6) transforms to the equation [1, 6]

$$(1+v)[1 - \exp(-x_\infty)] - x_\infty = 0,$$

and $S_{mn} = \pi r_0^2$.

It follows from (7) that the output energy $W^{(mn)}$ corresponds to each transverse mode TEM_{mn}. It is clear from the above expressions that the output energy of the passively Q-switched laser should change discretely during the resonator tuning, when one type of oscillations passes to another. It can be shown that the ratio $\gamma_{mn}(v) = W^{(mn)}/W^{(00)}$ is determined by the indices m and n and is independent of the radius w for the chosen geometry of the resonator. A comparison of the experimental values of this ratio with calculated values can be used as a criterion for validity of assumptions made in the derivation of expressions (2)–(9).

One can see from (6) that the ratio γ_{nm} is a function of the parameter v. It can be shown that

$$\gamma_{mn}^{(0)} = \gamma_{mn}(v \to 0) = \frac{\left(\int_S G_{mn}{\rm d}s\right)^2}{\int_S G_{mn}^2{\rm d}s}\,\frac{\int_S G_{00}^2{\rm d}s}{\left(\int_S G_{00}{\rm d}s\right)^2}$$
$$= \frac{1}{\pi}\,\frac{\left(\int_S G_{mn}{\rm d}s\right)^2}{\int_S G_{mn}^2{\rm d}s}. \quad (10)$$

As v is increased from 0 to 3, which exhausts most of the practically important cases, the ratio γ_{mn} ($m, n \leqslant 4$) changes insignificantly, decreasing by $\sim 2.5\,\%-4\,\%$ of $\gamma_{mn}^{(0)}$; therefore, the values $\gamma_{mn}^{(0)}$ can be used for estimates.

When ESA is absent, but there exist losses caused, for example, by foreign impurities, all final Eqns (3)–(9) retain their form if we assume that $\eta\alpha_0$ is the absorption coefficient of these impurities and $(1-\eta)\alpha_0$ is the ground-state absorption coefficient of Er^{3+} ions.

An important advantage of this description is the use of the parameter $x(t)$ as the main characteristic of radiation. This parameter for the TEM_{00} mode is the energy density in the maximum of its transverse distribution from the onset of lasing up to the moment t, normalised to the saturation energy density. For glass doped with Er^{3+} ions, the saturation energy density is high (~ 10 J cm^{-2}), which requires a careful choice of the reflectance of the output mirror and the initial transmission of a passive Q switch to avoid the optical breakdown of the resonator elements by laser radiation.

It also follows from (6) that the shaded AE regions do not affect the output energy. This can be explained by the fact that the initial absorption in these regions requires an increase in the energy accumulated in the AE for the gain to exceed losses caused by absorption in these regions. This additional energy is completely spent in turn to bleach the shaded AE region, i.e. the presence of shaded AE regions leads to the increase in the threshold pump energy and does not affect the output energy. For the modes of higher orders than TEM_{00}, absorption regions remain near the field zeroes, in shaded AE regions, and in the passive Q switch volume in which diffraction of radiation occurs, resulting in the increase of loses during lasing and a change in radiation parameters. The consideration of this circumstance is beyond the scope of our study.

Note that the equations presented above are also valid for the four-level laser energy diagram if we set formally $\sigma_{12} = 0$.

3. Experiment

The passive Q-switching regime was studied for two erbium-doped glass AEs: the AE of size $\varnothing 5 \times 105$ mm with the illuminated region of length 80 mm and $\alpha_0 = 0.1$ cm^{-1} at the laser wavelength and the AE of size $\varnothing 4 \times 80$ mm with the illuminated region of length 75 mm and $\alpha_0 = 0.052$ cm^{-1}. The AE end-faces had AR coatings. The reflectances for all the end-faces were measured to be less than 0.1 %. The lifetime of the upper laser level measured by the luminescence decay was 6–6.5 ms. The saturation energy density and residual losses measured for these AEs were $E_s = 10.5 \pm 1.5$ J cm^{-2} and $\eta < 0.06$.

The laser resonator was formed by the output mirror with the reflectance 75 % and the radius of curvature 1 m and a plane mirror with the reflectance $\sim 100\,\%$. The resonator length was 66 cm, the AE middle was at a distance of 22 cm from the output mirror. No polarisers were used in the resonator.

Passive Q-switching was obtained by using a Co^{2+}-doped ZnSe crystal Q switch. Doping was performed by the diffusion method under the thermodynamic equilibrium conditions [21]. Two passive Q switches were used, which we denote by A and B. The transmission of the Q switches measured in weak (T_0) and strong (T_1) fields were $T_0 = 91.4\,\% \pm 0.3\%$, $T_1 = 96\,\% \pm 0.2\,\%$ for Q switch A and $T_0 = 91.9\,\% \pm 1.8\,\%$, $T_1 = 95.6\,\% \pm 0.3\,\%$ for Q switch B. Passive Q switches were mounted in front of the output mirror.

Pumping was performed by a flashlamp with a pump pulse repetition rate of 0.1 Hz. The absolute measurements of the laser pulse energies were performed with an IMO-2H calorimeter, and the relative measurements were performed with a PM-4 pyroelectric detector with a removed entrance germanium window.

To compare the experimental results with calculations, it is necessary to know the area of the transverse distribution of the energy density in the AE. We measured this distribution in the output mirror plane for the TEM_{00} mode. The measurements were performed in the following way. The output mirror plane was imaged with the help of a concave gold mirror with the radius of curvature 1 m, and the transverse distribution of the energy density was measured in the image plane by using a slit that was moved by a step motor. The accuracy of the slit displacement was 1 μm. The magnification factor was determined by using a test scale placed near the output mirror plane, all the elements of the optical path remaining in their places. The image size was measured with a microscope by illuminating its scale by an incandescent lamp. Figure 1 shows the experimental transverse distribution of the radiation energy density in the output mirror plane (circles) and the distribution corresponding to the TEM_{00} mode of a passive resonator (solid curve). One can see that these distributions coincide.

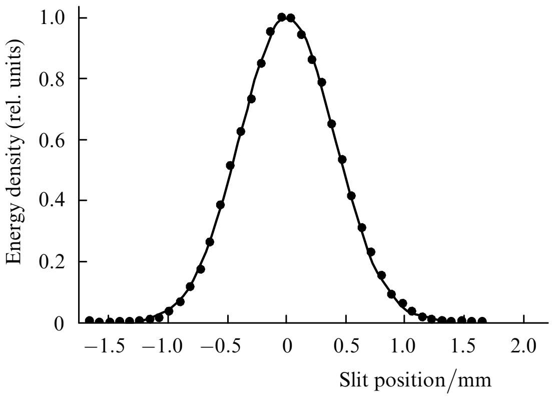

Figure 1. Transverse distribution of the radiation energy density in the output mirror plane (AE of size $\varnothing 4 \times 80$ mm, passive Q switch B). Circles are experiment, the solid curves is $\exp[-2(x/w)^2]$ for $w = 803$ μm.

Table 1 presents the measured and calculated values of the radius w in the output mirror plane. Calculations were performed by using the $ABCD$ matrix formalism [18] for the resonator geometry described above. One can see that the experimental and calculated values w are close. The beam radius value calculated in the AE middle was taken as the beam radius. Table 1 also gives pulse energies for the TEM_{00} mode obtained for the two AEs with passive Q switches available. Note that the two AEs were mounted into the same illuminator and had different lengths (5 and 25 mm) l_{ends} of shaded regions. However, the output pulse energy in these two cases was the same when the same Q switch was used. This confirms the conclusion made above that the pulse energy is independent of the length of the AE shaded region. Table 1 also presents the calculated values of

Table 1.

AE size/mm	Q switch	Beam radius on the mirror/µm		Laser pulse energy/mJ	
		Experiment	Calculation	Experiment	Calculation
$\varnothing 4 \times 80$	A	823	803	10.2	10.4
	B	805	803	7.6	8.1
$\varnothing 4 \times 105$	A	827	796	10.3	10.3
	B	811	796	8.1	8.0

the output energy for the parameters of passive Q switches indicated above. It was assumed in calculations that $E_s = 10.5$ J cm^{-2} and the excited-state absorption is absent ($\eta = 0$). One can see that the measured and calculated energies are close. The polarisation of radiation was almost linear, which was estimated by the ratio of the minimal radiation energy transmitted through a Glan prism to the maximal energy (depending on the prism rotation angle), which was $\sim 1:300$.

We studied a change in the transverse distribution of the radiation intensity during the development of passively Q-switched lasing. For this purpose, instead of a slit in the scheme for measuring the transverse distribution of the energy density at its maximum, an aperture of diameter 0.6 mm with transmission $\sim 18\,\%$ was mounted. Radiation incident on the aperture and transmitted through it was detected with two LFD 2a photodiodes (with the time resolution ~ 1 ns) and a DPO 7254 oscilloscope. The two photodiodes were used to increase the measurement accuracy. The ratio r_d of the output signals of photodiodes and the ratio r_i of signals in the absence of the aperture were measured as functions of time. The ratio $T_d = r_d/r_i$ gave the time dependence of transmission.

Figire 2a presents transmission T_d normalised to the average value and averaged over 15 realisations. Also, a laser pulse is presented to show for which parts of the pulse the values of T_d were measured. One can see that the values of T_d are close to unity, but the presence of noise complicates the measurement of their deviations. The high-frequency noise component appears because the pulse is not smooth and also due to the noise of a detector. The noise was eliminated by filtering the high-frequency radiation components with frequencies exceeding the frequency equal to the inverse round-trip transit time of radiation in the resonator. Figure 2b shows the values of T_d processed by using two different filtration frequencies. One can see that the deviation of T_d from unity during the pulse development is less than 2 %. This means that the relative change in the area of the transverse intensity distribution during the development of lasing is also less than 2 %, and, therefore, the assumption that this distribution is constant during the development of lasing is valid under our conditions.

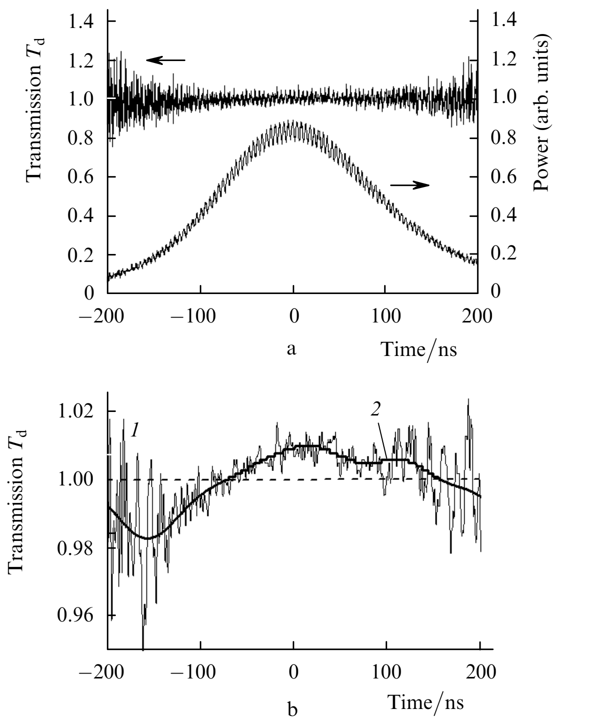

Figure 2. Transmission T_d of the aperture normalised to the average value and averaged over 15 realisations, and a laser pulse (a) and T_d values processed at different noise filtration frequencies (b); the filtration frequency for curve (*2*) is lower than that for curve (*1*).

Figure 3a shows the shape of a laser pulse obtained in an AE of size $\varnothing 4 \times 80$ mm with passive Q switch B (signal is normalised to the maximum), and Fig. 3b presents the pulse shape calculated with parameters corresponding to passive

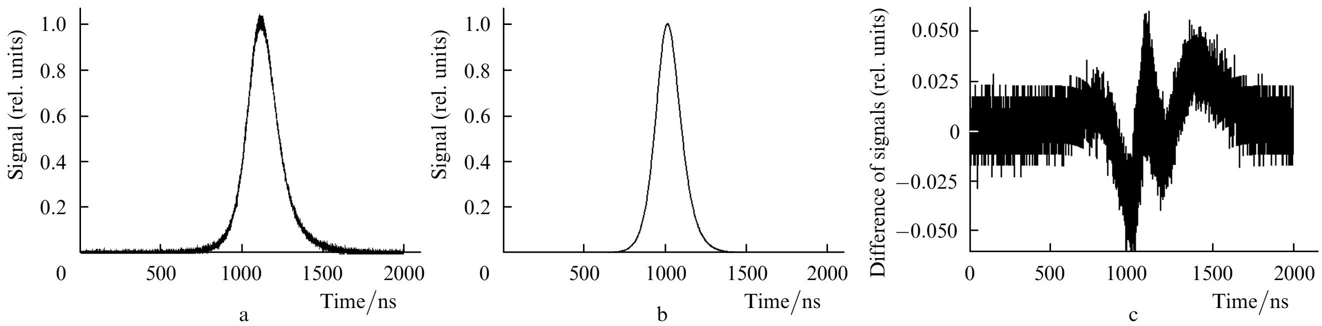

Figure 3. Experimental time dependence of the output pulse power of a passively Q-switched laser (Q switch B) (a), the calculated pulse shape (b), and (c) the difference of pulses in Figs 3a and b.

Table 2.

Q switch	τ_1/ns		τ_2/ns		τ/ns	
	Experiment	Calculation	Experiment	Calculation	Experiment	Calculation
A	95	95	95	106	190	201
B	111	121	111	131	222	251

Q switch B (signal is normalised to the maximum). The calculation was performed by using Eqn (5) for $\eta = 0$. It follows from Fig. 3c, which shows the difference of these signals, that the deviation of the experimental pulse shape from the calculated shape is less than 0.05. This deviation is observed at the trailing edge of the pulse and can be caused by the specific features of the time response of a photodiode used in experiments.

The durations of the leading and trailing edges of the pulse were found from oscillograms in Figs 3a, b by using expressions (9). For this purpose, we found from a set of signal values U_i corresponding to instants t_i $(i = 0, 1, 2, \ldots)$ in the oscillogram in Fig. 3a the position $i_{\max}$ of the maximum and the maximum signal value $U_{\max}$, as a well as the area under the curve up to the maximum $S_{\max} = \sum_{i=0}^{i_{\max}} U_i \Delta$ and the total area under the curve $S_\infty = \sum_{i=0}^{N} U_i \Delta$, where Δ is the data digitisation step in time. Then, the leading edge duration is $\tau_{1\,\exp} = S_{\max}/U_{\max}$, the trailing edge duration is $\tau_{2\,\exp} = (S_\infty - S_{\max})/U_{\max}$, and the total pulse duration is $\tau_{\exp} = S_\infty/U_{\max}$. Table 2 presents these durations obtained in experiments and in calculations. One can see that the experimental and calculated durations coincide within 10 %.

The values of γ_{mn} were found by measuring pulse energies generated at modes with high transverse indices. The transverse distribution of the radiation energy density was determined by the choice of the lasing region in the AE and an intracavity aperture. To obtain lasing at high-order transverse modes, a filament of diameter 40 μm was placed into the cavity. The output energy averaged over ten pulses was measured for each type of this transverse distribution. Figure 4 presents the values of γ_{mn} measured for $m = 0 - 4$ and $n = 0$, and also for $m = 0 - 3$ and $n = 1$. The calculated values of $\gamma_{mn}^{(0)}$ are also given. The polarisation of radiation in all cases was close to linear.

The measurements of γ_{mn} involved a difficulty because, to obtain the experimental values of γ_{mn} that can be compared with the calculated values, the intracavity loses should be the same for all the modes being compared. Another experimental difficulty was that the transverse distribution of the gain in the AE was inhomogeneous, which manifested in the change in the lasing threshold on passing from one AE lasing region to another. Therefore, we performed a few series of measurements, the resonator being differently aligned in each of the series. The average values of γ_{mn} were taken. The errors shown in Fig. 4 characterise the scatter of the obtained values of γ_{mn}. One can see that the calculated and experimental values are in satisfactory agreement and their difference is within the experimental error.

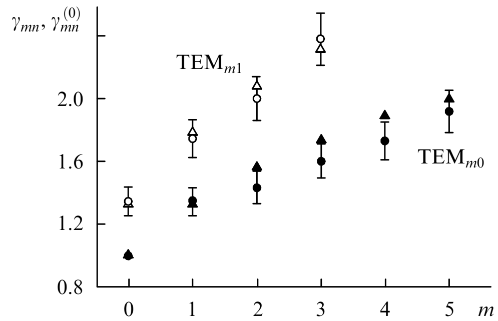

Figure 4. Dependences of γ_{mn} (circles) and $\gamma_{mn}^{(0)}$ (triangles) on the TEM$_{mn}$ mode index m for $n = 0$ and 1. Circles are experiment, triangles are calculations by (10) for $\nu = 0$.

4. Conclusions

(i) The expressions have been found that describe the operation of a passively Q-switched laser generating an arbitrary transverse TEM$_{mn}$ mode. Excited-state absorption losses and nonlinear absorption in the shaded AE ends have been taken into account. It is shown that shaded AE regions have no effect on the laser pulse energy while the lasing threshold is achievable. Within the framework of the model used, it is impossible to distinguish the excited-state absorption from absorption by foreign impurities.

(ii) The transverse distribution of the radiation energy density has been measured in the plane of the output mirror of the passively Q-switched laser emitting the TEM$_{00}$ mode. It has been shown that the experimental Gaussian-beam radius coincides with the calculated radius within the experimental error; therefore, the Gaussian-beam radius inside the AE can be set equal to the radius of the TEM$_{00}$ mode of a passive resonator (when the AE is not pumped) in calculations. This assumption allows one to calculate the output energy of a Q-switched laser by using the parameters of a passive resonator.

(iii) The TEM$_{00}$ pulse energy and temporal characteristics of a Q-switched erbium glass laser measured in experiments have been compared with calculations. Q-switching was performed by using a Co^{2+} : ZnSe crystal Q switch. It is shown that the experimental data are in good agreement with calculations performed by using the expressions derived in the paper.

(iv) The change in the radius of the transverse distribution of the TEM$_{00}$ mode intensity on the output mirror of the Q-switched laser during the development of lasing has been estimated. It is found experimentally that the relative change in the laser-beam cross section in the output mirror plane is less than 2 %. Therefore, our assumption that the transverse radiation intensity distribution is constant during lasing is correct under our conditions.

(v) The output energies have been measured for modes with different transverse indices. It is shown that the laser pulse energy changes discretely on passing from one type of oscillations to another. The values of γ_{mn} form a family of points in the (γ_{mn} – index m) plane combined over the index n. The experimental values of γ_{mn} for $m = 0 - 4$, $n = 0$, and $m = 0 - 3$, $n = 1$ are in good agreement with the calculated values of $\gamma_{mn}^{(0)}$.

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References

1. Basiev T.T., Denker B.I., Il'ichev N.N., Malyutin A.A., Mirov S.B., Osiko V.V., Pashinin P.P. *Kvantovaya Elektron.*, **9**, 1536 (1982) [*Sov. J. Quantum Electron.*, **12**, 984 (1982)].
2. Bhardwaj A., Agrawal L., Pal S., Kumar A. *Appl. Phys. B*, **86**, 293 (2007).
3. Il'ichev N.N., Gulyamova E.S., Pashinin P.P. *Kvantovaya Elektron.*, **24**, 1001 (1997) [*Quantum Electron.*, **27**, 972 (1997)].
4. Kalashnikov V.L., Shcherbitsky V.G., Kuleshov N.V., Girard S., Moncorge R. *Appl. Phys. B*, **75**, 35 (2002).
5. Zhang X., Zhao S., Wang Q., Liu Y., Wang J. *IEEE J. Quantum Electron.*, **30**, 905 (1994).
6. Degnan J.J. *IEEE J. Quantum Electron.*, **31**, 1890 (1995).
7. Xiao G., Bass M. *IEEE J. Quantum Electron.*, **33**, 41 (1997).
8. Zhang X., Zhao S., Wang Q., Zhang Q., Sun L., Zhang S. *IEEE J. Quantum Electron.*, **33**, 2286 (1997).
9. Chen Y.F., Lan Y.P., Chang H.L. *IEEE J. Quantum Electron.*, **37**, 462 (2001).
10. Zhang X., Zhao S., Wang Q., Ozygus B., Weber H. *J. Opt. Soc. Am. B*, **17**, 1166 (2000).
11. McAllister G.L., Mann M.M., DeShazer L.G. *IEEE J. Quantum Electron.*, **6**, 44 (1970).
12. Fox A.G., Li T. *Bell Syst. Tech. J.*, **40**, 453 (1961).
13. Statz H., Tang C.L. *J. Appl. Phys.*, **36**, 1816 (1965).
14. Fox A.G., Li T. *IEEE J. Quantum Electron.*, **2**, 774 (1966).
15. Yariv A., Yeh P. *Opt. Commun.*, **13**, 370 (1975).
16. Kogelnik H. *Bell Syst. Tech. J.*, **44**, 455 (1965).
17. Kogelnik H., Li T. *Appl. Opt.*, **5**, 1550 (1966).
18. Prokhorov A.M. (Ed.) *Spravochnik po lazeram* (Handbook on Lasers) (Moscow: Sov. Radio, 1967) Vol. 2.
19. Mikaelyan A.L., Ter-Mikaelyan M.L., Turkov Yu.G. *Opticheskie generatory na tverdom tele* (Solid-State Lasers) (Moscow: Sov. Radio, 1967).
20. Mak A.A., Anan'ev Yu.A., Ermakov B.A. *Usp. Fiz. Nauk*, **92**, 373 (1967).
21. Il'ichev N.N., Shapkin P.V., Mosaleva S.E., Nasibov A.S. *Kvantovaya Elektron.*, **34**, 1169 (2004) [*Quantum Electron.*, **34**, 1169 (2004)].