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Relativistic motion and radiation of an electron in the field of an intense laser pulse

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***Abstract*. The motion of an electron in the fields of relativistic-intensity linearly and circularly polarised laser radiation is analysed. The analysis is based on the numerical solution of the Newton equation with the Lorentz force. The electromagnetic radiation of an electron interacting with a laser pulse is studied. It is shown that this radiation is emitted in the form of extremely short attosecond pulses. It is found that an initially immobile electron does not move along figure-eight trajectories in the field of a linearly polarised laser pulse.**

Keywords: *relativistic electron motion, generation of attosecond electromagnetic pulses.*

1. Introduction

The type of motion of a charged particle (for example, an electron) in the field of a relativistic laser pulse is determined by the spatiotemporal intensity distribution and polarisation of radiation. The parameters of the electron motion can be found by solving the Newton equation with the Lorentz force. In most papers, focused laser beams with the Gaussian transverse intensity distribution have been considered. Such beams have a waist at the focus. The phase front in the focal plane called the beam caustic can be considered plane.

The dynamics of an electron in an electromagnetic field has been investigated by using the Newton equation with the Lorentz force in a number of papers (see, for example, [1 – 6]). It has been shown in [3, 4] that the electron is 'captured' during some time by a laser pulse and moves together with it. The analysis of the electron motion in the case of Gaussian beams [7] has shown that an electron initially at rest on the laser beam axis is accelerated by laser radiation to a great velocity at the laser pulse front and then is decelerated at the pulse tail. An electron located outside the laser beam axis is ejected at an angle to the laser beam axis. The kinetic energy of such an electron can achieve great values. Sometimes, this process is interpreted as electron scattering. Note that in the case of very intense laser radiation, the amplitude of electron oscillations becomes comparable with the size of the optical-field caustic waist.

The description of the electron motion in a high-frequency field involves considerable difficulties due to a great number of field oscillations. Therefore, the electron motion is often analysed by using the Lorentz force averaged over high-frequency oscillations instead of the usual Lorentz force [8 – 11]. This force is called the pondermotive force in the literature. It is obvious that the concept of the pondermotive force for very short, few-cycle pulses becomes meaningless, and the electron motion should be analysed by using the exact Lorentz force.

Previous papers have not paid a proper attention to comparing electron motions in the fields of linearly and circularly polarised radiation, which is one of the problems studied in our paper. Also, the ejection of electrons with a high kinetic energy from the interaction region should be additionally analysed. In addition, it is not clear whether the figure-eight trajectory can be realised in the case of a linearly polarised laser pulse.

It is also interesting to study in detail the generation of electromagnetic radiation by an electron interacting with an intense electromagnetic field. The questions concerning the radiation of a charged particle moving in a plane field have been considered, for example, in paper [12].

2. Equations of motion

We investigate in this paper the dynamics of an electron in an intense electromagnetic field. The electron motion in the field of a short laser pulse of the relativistic intensity with different polarisations (linear and circular) is analysed by solving the Newton equation with the Lorentz force.

Consider a focused laser beam with the Gaussian transverse intensity distribution. The phase front of the beam in the caustic vicinity can be assumed plane. We will use the approximation in which the interaction of laser radiation with the electron is localised in the given vicinity. Under certain conditions due to interaction with the field, the electron can leave this region with the nonzero kinetic energy. We will estimate the longitudinal size of a caustic by the double Rayleigh length $2L_{\rm R}$, where $L_{\rm R} = \pi\rho_0^2/\lambda$ and ρ_0 and λ are the beam radius at the caustic centre and its wavelength, respectively.

The electron is subjected to the high-frequency Lorentz force, and the equation of electron motion has the form

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$$\frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t} = -e\boldsymbol{E} - \frac{e}{c}[\boldsymbol{vH}], \quad (1)$$

where $\boldsymbol{p}$ is the electron momentum; $\boldsymbol{E}$ and $\boldsymbol{H}$ are the electric and magnetic laser field strengths; and $e > 0$ is the absolute value of the electron charge. Equation (1) is supplemented with the initial conditions

$$\boldsymbol{v}(0) = \boldsymbol{v}_0, \quad \boldsymbol{r}(0) = \boldsymbol{r}_0 \quad (2)$$

for the electron velocity and position.

Let us choose the coordinate system in which the laser pulse propagates along the z axis. The phase front of the laser pulse is plane and the constant-phase surface is perpendicular to the z axis.

Equation (1) written for individual components has the form

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{mv_x}{(1-v^2/c^2)^{1/2}} = -e\Big(E_x - \frac{v_z}{c}H_y\Big), \quad (3)$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{mv_y}{(1-v^2/c^2)^{1/2}} = e\Big(E_y + \frac{v_z}{c}H_x\Big), \quad (4)$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{mv_z}{(1-v^2/c^2)^{1/2}} = -e\Big(\frac{v_x}{c}H_y - \frac{v_y}{c}H_x\Big), \quad (5)$$

where m is the electron rest mass. In the case of linear polarisation, we assume that the electric field is directed along the x axis. In this case, the magnetic field is directed along the y axis: $E_x = H_y = E_0(x, y, \xi)\cos\omega\xi$, where $\xi = t - z/c$ and $\omega = 2\pi c/\lambda$. Similarly, in the case of circular polarisation, $E_x = H_y = (1/\sqrt{2})E_0(x, y, \xi)\cos\omega\xi$ and $E_y = -H_x = (1/\sqrt{2})E_0(x, y, \xi)\sin\omega\xi$. The field amplitude $E_0(x, y, \xi)$ is related to the intensity I by the expression

$$I(x, y, \xi) = \frac{c}{8\pi}E_0^2(x, y, \xi). \quad (6)$$

Note that, for the same laser radiation intensity, the amplitudes of fields with different polarisations differ by a factor of $\sqrt{2}$.

We will describe the time and coordinate dependence of the amplitude $E_0(x, y, \xi)$ by the expression

$$E_0(x, y, \xi) = E_{\mathrm{m}}\exp\left\{-\left(\frac{\xi - z_{\mathrm{d}}/c}{\tau}\right)^s - \left[\frac{(x^2 + y^2)^{1/2}}{\rho_0}\right]^q\right\}. \quad (7)$$

Here, E_{m} is the maximum field strength; z_{d} is the initial time shift of the laser pulse with respect to the electron providing the smooth switching on of the field in the numerical solution; and τ is the pulse duration. The parameter s determines the temporal pulse shape, and the parameter q determines the transverse field strength distribution. In this paper, we studied mainly laser beams with Gaussian temporal and transverse field distributions ($s = q = 2$). For $\rho \to \infty$, the field represents an infinite plane wave.

If the longitudinal displacements of a charged particle interacting with an intense light pulse exceed the caustic size, distribution (7) cannot be used. The phase and group velocities of the pulse in vacuum are equal to c.

The calculations were performed by assuming, as a rule, that for $t = 0$ a charged particle is located at the point $z_0 = 0$. Many calculations were performed for the case when a charged particle is located on the beam axis ($x_0 = y_0 = 0$) for $t = 0$, but the influence of the initial radial displacement on the motion type was also considered. If according to initial conditions (2), we have $v_{0y} = 0$, the electron motion is localised in the xz plane.

3. Electron motion in a linearly polarised radiation field

We performed calculations by using expression (7) for different values of s and q, but the electron dynamics was analysed in most detail for $s = 2$ and 4 and $q = 2$.

Equations (3)–(5) can be written in the dimensionless variables x/λ, z/λ, ct/λ, and v/c. In the case of the numerical solution, of interest are the dimensionless coordinates, the components of the velocity and acceleration ($\lambda v'_x/c^2$ and $\lambda v'_z/c^2$), and the total kinetic energy $W/(mc^2)$ of the particle. Hereafter, the prime means the time derivative.

The dimensionless field amplitude is expressed in terms of the dimensionless intensity I/I_{r}, where I_{r} is the relativistic intensity. In the literature, several expressions for I_{r} are used which differ in the numerical factor. In our opinion, the most correct criterion for defining I_{r} can be based on a comparison of the maximal total energy of an electron oscillating in the field of a short laser pulse with the rest energy mc^2. Then,

$$I_{\mathrm{r}} = \frac{m^2c^3\omega^2}{8\pi e^2} = 1.37 \times 10^{18}\lambda^{-2}.$$

Here, I_{r} is expressed in W cm^{-2} and λ in micrometres.

Figure 1 shows the time profiles for x/λ, v_x/c, z/λ, v_z/c, $\lambda v'_z/c^2$, and $W/(mc^2)$. These profiles were obtained for distribution (7) in the case of a short pulse with parameters $I/I_{\mathrm{r}} = 25$ (relativistic case), $s = 2$ (Gaussian pulse), $c\tau/\lambda = 4$, $q = 2$, and $\rho_0/\lambda = 5$ for the zero initial velocity and zero initial transverse displacement of the electron.

The case of linear polarisation under study has the following specific features:

(i) The transverse coordinate x and velocity v_x oscillate with a variable frequency, the oscillation frequency at the beginning and end of a pulse (at low intensities) being coincident with the frequency of the initial electromagnetic radiation; for high intensities, the frequency decreases considerably and the shape of oscillations very strongly differs from the sinusoidal shape;

(ii) the longitudinal velocity v_z oscillates at the double oscillation frequency of the transverse coordinate and velocity, the velocity magnitude being always nonnegative, the shape of oscillations approaching the rectangular shape and the longitudinal velocity being zero after completion of the pulse;

(iii) the average longitudinal velocity can achieve values close to c, resulting in the peculiar 'capture' of the particle by the laser pulse field and a considerable increase in the time of interaction between the particle and field;

(iv) the longitudinal acceleration v'_z oscillates around zero synchronously with oscillations of the longitudinal velocity, the positive and negative acceleration values being approximately equal, and oscillations themselves representing narrow peaks at the instants of time corresponding to the zero longitudinal velocity;

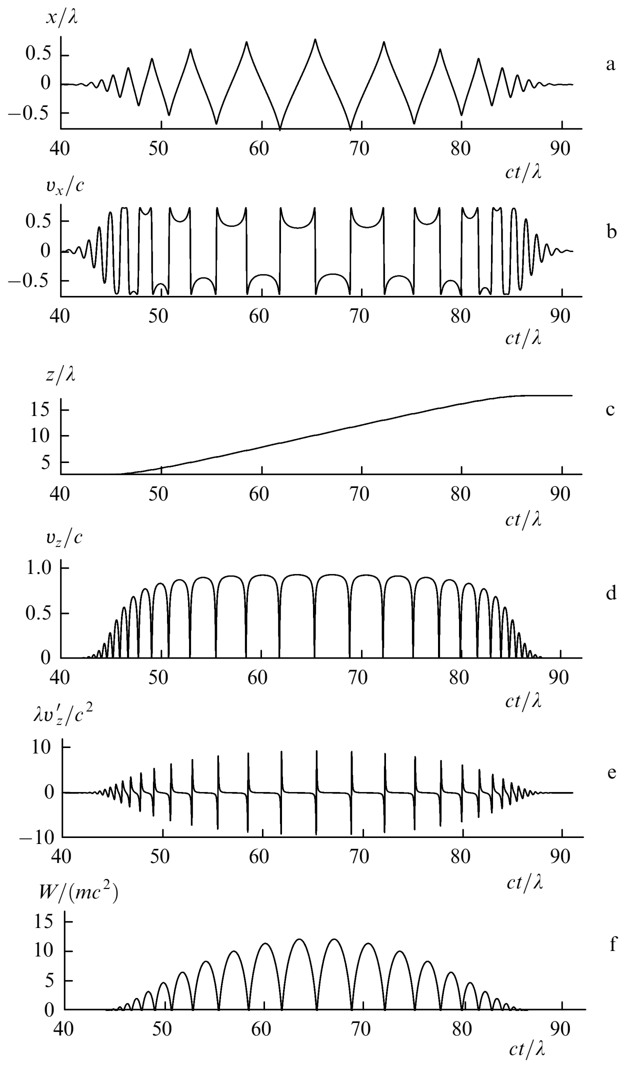

Figure 1. Time profiles of the parameters x/λ (a), υ_x/c (b), z/λ (c), υ_z/c (d), $\lambda\upsilon_z'/c^2$ (e), and $W/(mc^2)$ (f) of the relativistic motion ($I/I_r = 25$) of an electron for linearly polarised radiation. The profiles are obtained for a short pulse with the field distribution (7) for $s = 2$ (Gaussian shape), $c\tau/\lambda = 4$, $q = 2$, $\rho_0/\lambda = 5$, the zero initial velocity and zero initial transverse displacement of the electron.

(v) the longitudinal coordinate υ_z increases with time almost linearly, and the longitudinal displacement during the pulse is finite, its value being quite large (a few tens of wavelengths in the case under study).

The effect of electron 'capture' by an intense light field and the anharmonicity of its oscillations were reported in papers [3, 4, 7, 13]. In [14], the analytic expression describing the longitudinal displacement of an electron was proposed.

Note that a charged particle has no figure-eight trajectories. These trajectories were predicted in [15] for a plane monochromatic linearly polarised wave for the reference system in which a particle is at rest on average. During the interaction of an initially immobile particle with a laser pulse, the particle is immobile only outside the laser pulse, while the average (over high-frequency oscillations) longitudinal velocity of the particle inside the pulse is always nonzero, and the maximum value υ_m of this velocity is determined by the intensity and is achieved at the pulse maximum. Therefore, the figure-eight trajectory can be realised only on a flat top of the laser pulse, and to do this the particle should propagate initially with the velocity υ_m in the opposite direction to the laser pulse propagation.

Figure 2 presents an example of the figure-eight trajectory calculated for an electron moving at the velocity υ_m in the opposite direction to the laser pulse propagation. In this case, the calculation was performed for a longer pulse having a flat top ($s = 4$, $c\tau/\lambda = 16$, $q = 2$, $\rho_0/\lambda = 5$, and $I/I_r = 25$). For such parameters, $\upsilon_m/c = -0.862$. One can see (Fig. 2a) that there is no longitudinal drift displacement of the electron along the z axis in the flat part of the pulse, and only oscillations with respect to an average value are observed. Figure 2b shows the region of the electron trajectory in coordinates x, z at the centre of the flat part of the pulse. This region corresponds to motion along the figure-eight trajectory during five oscillations.

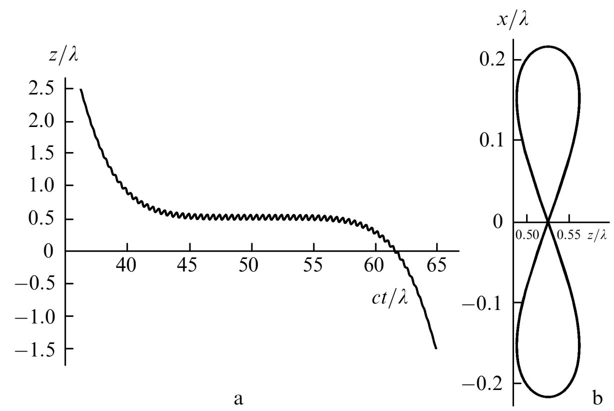

Figure 2. Trajectories of an electron moving at the velocity υ_m in the opposite direction to a laser pulse, calculated in coordinates z, t (a) and x, z (b).

The next series of calculations was devoted to the study of the motion of a charged particle that was initially displaced with respect to the laser beam axis in the direction x. The main result is that the particle is ejected from a light pulse, both in the radial and longitudinal directions. The particle continues to move after the propagation of the light pulse. A very important property of such a movement is that the nonzero radial and longitudinal velocity components of the particle are preserved. The movement of this type was investigated earlier, for example, in [5].

Figure 3a presents the trajectories for electrons with different initial displacements with respect to the pulse axis. The parameters of the laser pulse are as in Fig. 1. Figure 3b shows the kinetic energy of the electron after its interaction with the laser pulse as a function of the initial displacement.

The electron escape angle strongly depends on the initial displacement. Electrons accelerated by the pulse can be conditionally divided into three groups. The first group includes electrons with the initial displacement $x_0/\lambda = 0.05 - 1$. They leave the interaction region near the laser pulse maximum and have the maximal kinetic energy. For this group of electrons, the longitudinal component of the force dominates, and they escape predominantly forward. The second group contains electrons with the initial displacement $x_0/\lambda > 1$. They leave the interaction region at the very beginning of the pulse and have the minimal kinetic energy. For this electron group, the transverse component of the force dominates and their escape angle tends to the right angle with respect to the propagation

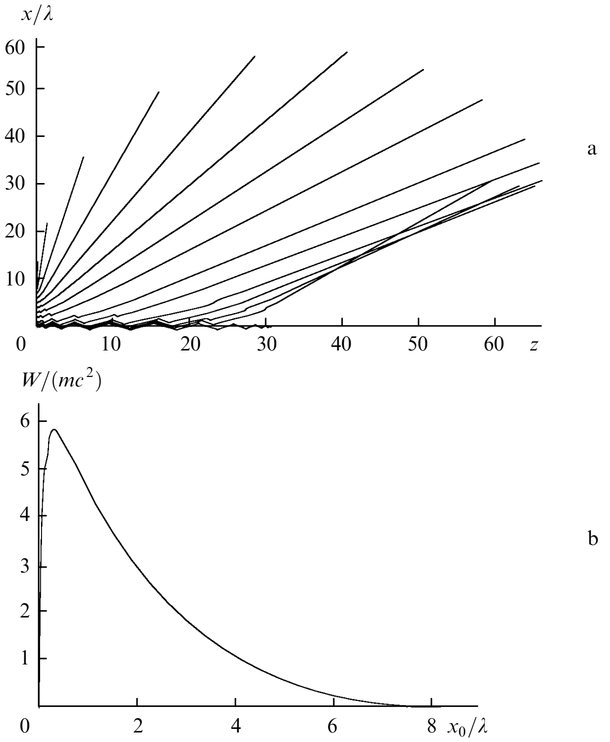

Figure 3. Trajectories of electrons for different initial displacements with respect to the pulse axis (a) and the kinetic electron energy after interaction with the laser pulse as a function of the initial displacement (b). The parameters of the laser pulse are the same as in Fig. 1.

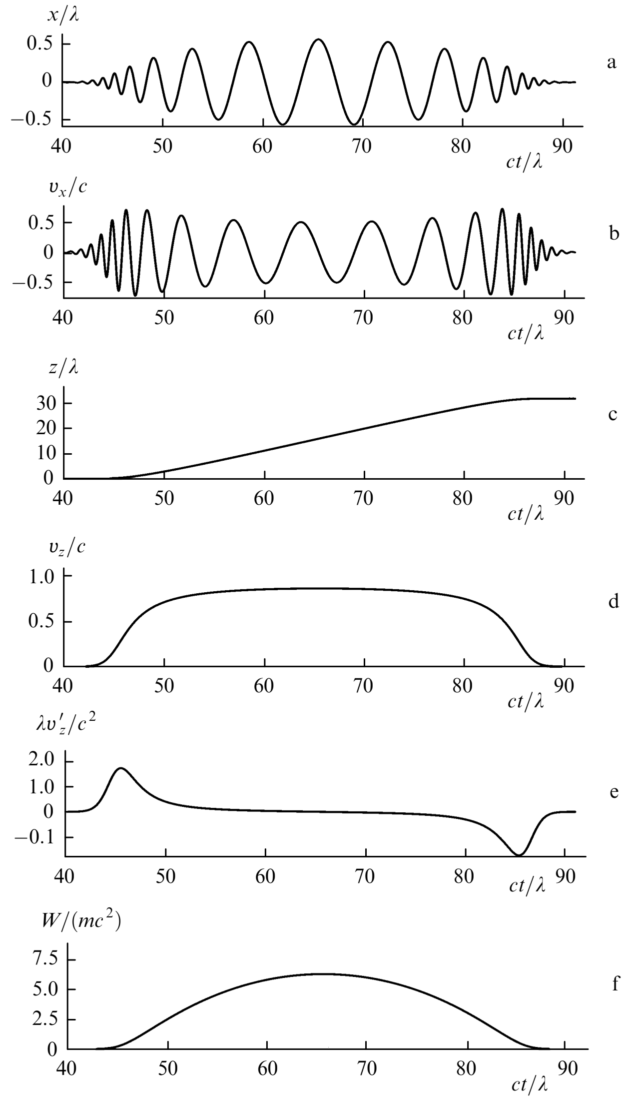

Figure 4. Time profiles of the parameters x/λ (a), v_x/c (b), z/λ (c), v_z/c (d), $\lambda v'_z/c^2$ (e), and $W/(mc^2)$ (f) of the relativistic motion ($I/I_r = 25$) of an electron for circularly polarised radiation. The laser pulse parameters and initial data for the electron are as in Fig. 1.

direction of the pulse. The third group contains electrons with the initial displacement $x_0/\lambda < 0.05$. They oscillate for a long time inside the pulse and escape from the interaction region at the laser pulse 'tail'. For this electron group, the transverse component of the force also dominates, and their escape angle tends to the right angle with respect to the propagation direction of the pulse.

For the case of linear polarisation and the initial displacement along the y axis under study, electrons are not ejected from the beam.

4. Electron motion in a circularly polarised radiation field

Figure 4 shows the time profiles for x/λ, v_x/c, z/λ, v_z/c, $\lambda v'_z/c^2$, and $W/(mc^2)$. These profiles are obtained for distribution (7) in the case of a short pulse with parameters $I/I_r = 25$ (relativistic case), $s = 2$ (Gaussian time profile), $c\tau/\lambda = 4$, $q = 2$, and $\rho_0/\lambda = 5$ for the zero initial velocity and the zero initial transverse displacement of an electron. The case of circular polarisation under study has the following specific features:

(i) Oscillations along coordinates x and y are sinusoidal in contrast to the case of linear polarisation;

(ii) the transverse coordinate x and velocity v_x oscillate at a variable frequency, the oscillation frequency at the beginning and end of the pulse (for low intensities) coinciding with the frequency of the initial electromagnetic radiation and considerably decreasing for high intensities;

(iii) the longitudinal velocity v_z does not oscillate, and the time dependence of the longitudinal coordinate z has no steps;

(iv) the longitudinal acceleration v'_z has a 'smooth' type: an electron is accelerated by the leading edge of the pulse and is decelerated at the pulse trailing edge.

The longitudinal displacements for linear and circular polarisations are the same in the case of the same shape and maximal intensity of pulses. The 'smooth' longitudinal velocity in the case of circular polarisation is the exact average of the oscillating longitudinal velocity in the case of linear polarisation.

5. Electromagnetic radiation of an electron moving in the field of an intense light pulse

Consider the electromagnetic radiation of an electron moving under the action of the Lorentz force produced by the field of an intense laser pulse. A point at which radiation is studied is specified by the radius vector $\boldsymbol{R}_0$ drawn to it from the origin of the coordinate system used to analyse the electron motion.

We will use the characteristics of the electron motion in the laser-pulse field obtained above to study the radiative parameters of the electron. The expression for the electric field of radiation obtained by using the Lienard – Wiechert potential has the form (for the electron)

$$E = -e\frac{1 - v^2/c^2}{(R - \boldsymbol{R}\boldsymbol{v}/c)^3}\left(\boldsymbol{R} - \frac{v}{c}R\right) - \frac{e}{c^2(r - \boldsymbol{R}\boldsymbol{v}/c)^3}$$

$$\times\left[\boldsymbol{R}\left[\left(\boldsymbol{R} - \frac{\boldsymbol{v}}{c}R\right)\boldsymbol{v}'\right]\right], \qquad (8)$$

where $v = \mathrm{d}\boldsymbol{r}/\mathrm{d}t$ and $\boldsymbol{v}' = \mathrm{d}^2\boldsymbol{r}/\mathrm{d}t^2$. The vector $\boldsymbol{R}$ connects the electron with the observation point. The radius vector of the electron at the same coordinate system is denoted by $\boldsymbol{r}$. These vectors are related by the expression $\boldsymbol{r}(t) + \boldsymbol{R}(t) = \boldsymbol{R}_0$. All the quantities in (8) are taken at the instant of time t. They can be recalculated to the moment t_{reg} of radiation arrival to the observation point. Taking into account the delay, t and t_{reg} are related by the expression $t + R(t)/c = t_{\mathrm{reg}}$.

Below, the electric field of radiation of a moving electron is determined at the observation point. This point is located in the xz plane at a distance of $R_0/\lambda = 7500$. The angle θ between $\boldsymbol{R}_0$ and the z axis was varied in a broad range.

The amplitude of the electron radiation field in the propagation direction of the laser pulse ($\theta = 0$) completely reproduces the initial field amplitude. Figure 5 presents the time dependences of the electric field at observation points calculated for angles $\theta = 15°$, $20°$, $25°$, and $30°$. One can see that radiation represents a train of very short pulses. The number of pulses and their mutual location correspond to the motion trajectories presented in Fig. 1.

Figure 6 presents at the enlarged scale the time dependences of the central-pulse field (Fig. 5) calculated for the same angles θ. One can see that the parameters of the radiation pulse depend substantially on the observation angle. There exists a critical angle at which the pulse shape and structure drastically change. As follows from Fig. 1, the trajectory of the relativistic electron consists of the parts close to linear; the critical angle coincides with the angle of inclination of the trajectory part from which radiation occurs. For parameters presented above, the critical angle is 22°. The pulse for the observation angle smaller than the critical angle ($\theta = 15°$) is presented in Fig. 6a. The pulse field takes negative and positive values and the total pulse duration is approximately $8 \times 10^{-2}\lambda/c$. For the observation angle close to the critical one ($\theta = 20°$), the pulse begins to change its polarity and acquires a more complicated shape (Fig. 6b). For the observation angle exceeding the critical one ($\theta = 25°$), the pulse becomes considerably shorter and completely changes its polarity. The total pulse duration is approximately $10^{-2}\lambda/c$ (Fig. 6c). For a wavelength of $\lambda = 8 \times 10^{-5}$ cm (Ti : sapphire laser), this corresponds to the pulse duration of 26 as. As the value of θ is further increased, radiation splits into two separated pulses with positive and negative polarities (Fig. 6d). The duration of each of them is approximately $2 \times 10^{-3}\lambda/c$, corresponding to 5.2 as. The maximum field strength is achieved for the observation angle equal to the critical one. Note that the critical angle and parameters of ultrashort pulses depend on the value of I/I_{r}.

The pulse shortening observed in the relativistic case is explained by the fact the electron propagates along linear trajectory parts at the velocity close to the speed of light. As

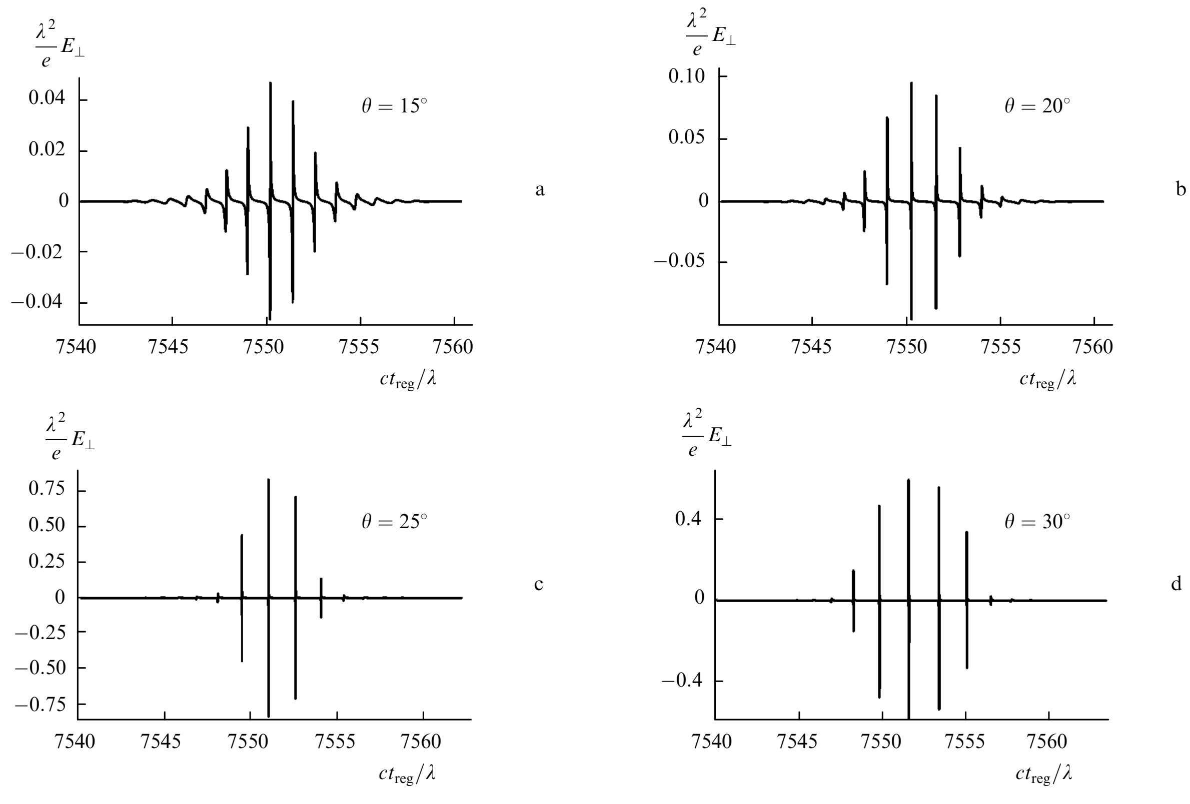

Figure 5. Time dependences of the perpendicular component $E_{\perp}$ of the electric field of electromagnetic radiation of a moving electron (with the same parameters as in Fig. 1) at the observation points at a distance of $R_0/\lambda = 7500$ for angles $\theta = 15°$, $20°$, $25°$, and $30°$.

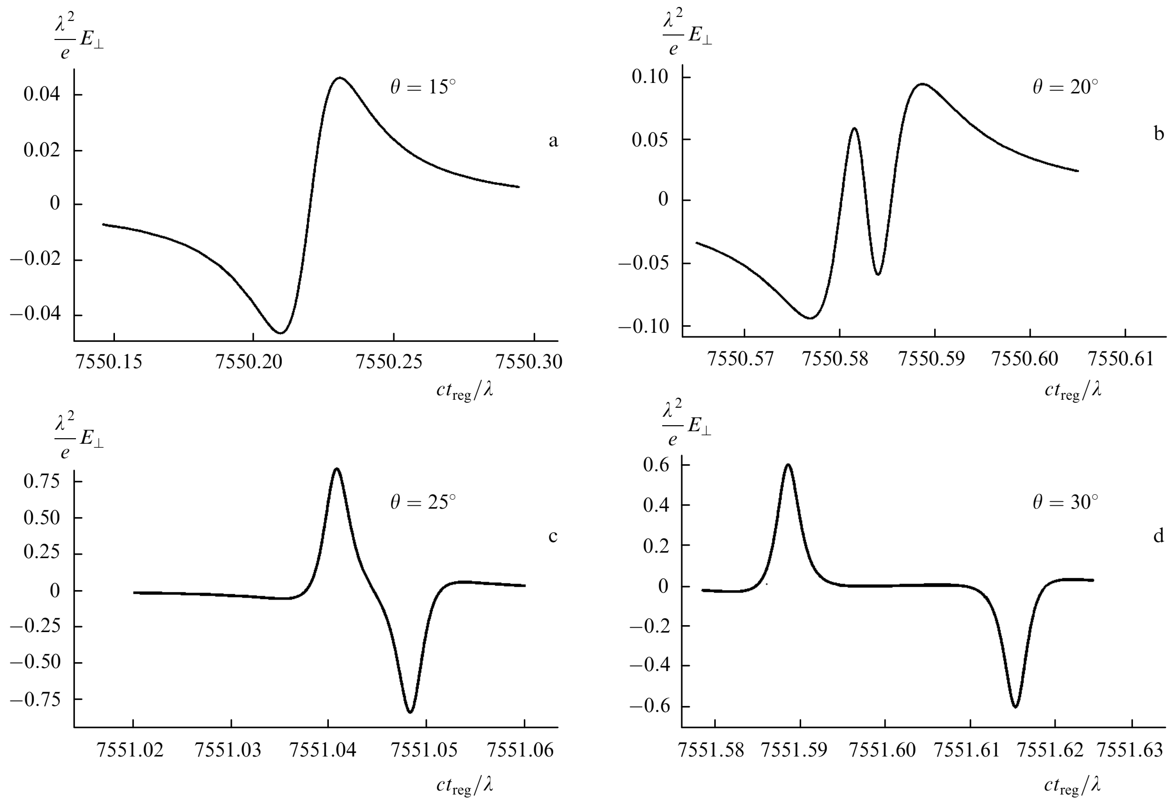

Figure 6. Time dependences of the perpendicular component $E_\perp$ of the electric field of the central pulse of the train (Fig. 5) for angles $\theta = 15°$, 20°, 25°, and 30°.

a result, the contributions of the field generated by the electron at different moments of its motion arrive at the observation point virtually simultaneously.

6. Parametric representation of the electron motion in the Lorentz force field

The motion of a charged particle in a high-frequency linearly polarised field in the form of a wave with a plane wavefront propagating along the z axis is characterised by the motion invariant [16]

$$(p^2 + m^2c^2)^{1/2} - p_z = \text{const}. \tag{9}$$

Below, this variant is used to obtain the parametric representation of the motion of a charged particle that was initially located on the beam axis.

To obtain such a representation, v_z is expressed in terms of v_x by using invariant (9) (assuming that $v_y = 0$) and is substituted into Eqn (3). Taking into account the accepted approximation $E(0,0,\xi) = \Psi(\xi)$, Eqn (3) is transformed to a differential equation for $x(\xi)$, depending only on one variable ξ. This equation is solved by the multiscale asymptotic method, which is similar to that used in [14]. Its solutions for x and p_x (for the case $v_{z0} = 0$) can be represented in the form of the converging series

$$x = \frac{e}{m\omega^2}\sum_{n=0}^{\infty}(n+1)\omega^{-n}\Psi^{(n)}(\xi)\cos\left(\omega\xi + \frac{n\pi}{2}\right), \tag{10}$$

$$p_x = -\frac{e}{\omega}\sum_{n=0}^{\infty}\omega^{-n}\Psi^{(n)}(\xi)\sin\left(\omega\xi + \frac{n\pi}{2}\right), \tag{11}$$

where $\Psi^{(n)}$ is the amplitude of the corresponding expansion term of $\Psi(\xi)$ in powers of $1/\omega$.

Solutions (10) and (11) were obtained by assuming that the intensity at the maximum is such that the amplitude of transverse oscillations is small compared to the characteristic transverse scale ρ_0 of intensity variations. In some sense, these solutions are the generalisation of the solution [15] for the electron motion in the stationary electromagnetic field of a plane wave to the nonstationary case. They can be used to refine the expression for the ponderomotive force in relativistic fields.

Expressions (10) and (11) were used for numerical calculations of x, z, v_x, and v_z at the pulse axis. These solutions were compared with the above results of numerical calculations with the use of the Lorentz force, which were performed for a charged particle located initially at the axis. The use of only the two first terms of series (10) and (11) in calculations provided good accuracy.

7. Conclusions

(i) The electromagnetic radiation of an electron moving in the field of a linearly polarised relativistic laser pulse in a rather broad range of observation angles (the front hemisphere except for axial values) represents a train of short pulses. The duration of a pulse in the train is much shorter than the light oscillation period. The parameters of these ultrashort pulses depend on the value of I/I_r (I_r is the relativistic intensity). The amplitude of the radiation field of an electron in the propagation direction of the laser pulse completely reproduces the amplitude of the initial field.

(ii) An electron in a relativistic field is partially 'captured' by this field, resulting in the dependence of its oscillation period on the local intensity. The oscillations of an electron in a linearly polarised optical field are substantially anharmonic, while in a circularly polarised field they are sinusoidal.

(iii) An electron initially displaced from the laser beam axis is ejected from it in the longitudinal and transverse directions with the kinetic energy comparable to the oscillation energy.

(iv) In the case of linear polarisation, a particle does not move along figure-eight trajectories. Such a trajectory can be realised only on the flat top of a laser pulse when a particle moves at a strictly specified velocity in the opposite direction to the laser pulse.

(v) For an electron that is initially at rest at the laser pulse axis, the longitudinal displacement is the same for both polarisations for the same intensity of the pulse. The longitudinal velocity in the case of circular polarisation is the exact average of the oscillating longitudinal velocity in the case of linear polarisation.

(vi) The parametric representation of the electron motion in the field of a linearly polarised intense electromagnetic pulse has been proposed.

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