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Influence of the difference in polarisations of counterpropagating waves on the dynamics of solid-state ring lasers

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***Abstract*. The influence of polarisations of counterpropagating waves on the dynamics of self-modulation oscillations of the first kind in solid-sate ring lasers is studied theoretically and experimentally. The characteristic features of amplitude and frequency characteristics of radiation and the spectra of relaxation oscillations appearing at different polarisations of counterpropagating waves are analysed. The obtained experimental results are well described by the vector model of a solid-state ring laser, which takes into account different polarisations of the ring resonator for counterpropagating waves.**

Keywords: *solid-state ring lasers, nonlinear dynamics, polarisation of counterpropagating waves, self-modulation oscillations, relaxation frequencies.*

1. Introduction

The dynamics of solid-state ring lasers is often studied by using one of the simplest mathematical models, the so-called standard model (see, for example, review [1]). This model completely ignores possible differences in polarisations of counterpropagating waves, although these polarisations can be considerably different in some studies of nonplanar cavity ring lasers (see, for example, [2]). The standard model was earlier improved [2, 3] to take into account the difference in polarisations of counterpropagating waves. In [2], the vector model of a solid-state ring laser was developed in which the interaction of counterpropagating waves with different elliptic polarisations was considered. The interaction of polarised radiation with active ions in this model leads to the azimuthal inhomogeneous burning of the inversion population, which substantially complicates the model, increasing the number of initial equations. A simpler vector model, which takes into account different polarisations of counterpropagating waves, was proposed in [3].

Vector models can predict and analyse a number of new features in the nonlinear dynamics of solid-state ring lasers. It was shown in [2 – 4] that the difference of polarisations of counterpropagating waves affects the amplitude and frequency characteristics of radiation in the self-modulation lasing regime of the first kind.

In this paper, we performed more detailed theoretical studies, which revealed some new features of the dynamics of self-modulation oscillations appearing in the interaction of counterpropagating waves with different polarisations. It is shown that the polarisation of radiation affects the frequencies of relaxation oscillations in the self-modulation lasing regime of the first kind and also upon stationary lasing with counterpropagating waves with substantially different intensities. Some features of the dynamics of self-modulation oscillations of the first kind related to the difference in polarisations of counterpropagating waves are studied experimentally in the paper.

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2. Basic equations of the vector model

In this paper, we use the vector model proposed in [3]. This model considers the interaction of counterpropagating waves with polarisations characterised by arbitrary unit vectors $\boldsymbol{e}_{1,2}$. As in the standard model, the amplification line is assumed homogeneously broadened and the linear coupling of counterpropagating waves is determined by phenomenologically introduced complex coupling coefficients written in the form

$$\tilde{m}_1 = m_1 \exp(\mathrm{i}\vartheta_1), \quad \tilde{m}_2 = m_2 \exp(-\mathrm{i}\vartheta_2), \tag{1}$$

where $m_{1,2}$ are the moduli of coupling coefficients and $\vartheta_{1,2}$ are their phases.

The difference between polarisations of counterpropagating waves leads first of all to the weakening of the nonlinear coupling of counterpropagating waves appearing due to back reflections from inverse population gratings. The weakening of the nonlinear coupling is determined by the factor

$$\beta = (\boldsymbol{e}_1\boldsymbol{e}_2)^2 = \cos^2\gamma, \tag{2}$$

where γ is the angle between the unit vectors $\boldsymbol{e}_{1,2}$. Note that the polarisation of counterpropagating waves in the cavity and the angle γ depend on the coordinate of a point considered in the cavity. Expression (2) contains the values $\boldsymbol{e}_{1,2}$ averaged over the resonator length. The basic set of equations of the vector model has the form

$$\frac{\mathrm{d}\tilde{E}_{1,2}}{\mathrm{d}t} = -\frac{\omega}{2Q_{1,2}}\tilde{E}_{1,2} \pm \mathrm{i}\,\frac{\Omega}{2}\,\tilde{E}_{1,2} + \frac{\mathrm{i}}{2}\,\tilde{m}_{1,2}\tilde{E}_{2,1} + \frac{\sigma l}{2T}\big(N_0\tilde{E}_{1,2} + N_{\pm}\tilde{E}_{2,1}\big),$$

$$T_1\,\frac{\mathrm{d}N_0}{\mathrm{d}T} = N_{\mathrm{th}}(1+\eta) - N_0 - N_0 a\big(|E_1|^2 + |E_2|^2\big) - N_+ aE_1E_2^* - N_- aE_1^*E_2, \quad (3)$$

$$T_1\,\frac{\mathrm{d}N_+}{\mathrm{d}T} = -N_+ - N_+\,a\big(|E_1|^2 + |E_2|^2\big) - \beta N_0 aE_1^*E_2.$$

Here, $\tilde{E}_{1,2}(t) = E_{1,2}\exp(\mathrm{i}\varphi_{1,2})$ are the complex amplitudes of counterpropagating waves; $Q_{1,2}$ are the Q factors of the resonator for counterpropagating waves; Ω is the frequency nonreciprocity; σ is the laser transition cross section at the amplification line centre; l is the active medium length; T, T_1 are the round-trip transit and relaxation times, respectively;

$$N_0 = \frac{1}{L}\int_0^L N\mathrm{d}z, \quad N_{\pm} = \frac{1}{L}\int_0^L \boldsymbol{e}_1^*\boldsymbol{e}_2 N\exp(\pm\,\mathrm{i}2kz)\mathrm{d}z \quad (4)$$

are the spatial harmonics of the inverse population N; N_{th} is the threshold population; $\eta = P/P_{\mathrm{th}} - 1$ is the excess of the pump power over the threshold power; a is the nonlinearity parameter; and L is the resonator length. The set of equations (3) differs from the equations of the standard model [3] only by the presence of the polarisation factor β in the last equation. Note that Eqns (3) are written for lasing at the amplification line centre.

3. Amplitude and frequency characteristics of self-modulation oscillations

In a broad range of parameters of solid-state ring lasers there exists the self-modulation lasing regime of the first kind which is characterised by the out-of-phase sinusoidal modulation of the intensities of counterpropagating waves [1]. The amplitude and frequency characteristics of radiation in this lasing regime were investigated within the framework of the vector model in [3, 4]. In this paper, we will consider for simplicity the case of symmetrically coupled counterpropagating waves, by assuming that the moduli of coupling coefficients are the same:

$$m_1 = m_2 = m. \quad (5)$$

The phase difference of the coupling coefficients is

$$\vartheta = \vartheta_1 - \vartheta_2. \quad (6)$$

The intensities $I_{1,2}$ of counterpropagating waves in the self-modulation regime of the first kind can be written in the form

$$I_{1,2} = I_{1,2}^0 \pm I_{1,2}^{\mathrm{m}}\cos(\omega_{\mathrm{m}}t + \varphi_{1,2}), \quad (7)$$

where $I_{1,2}^0$ are constant components (average values); $I_{1,2}^{\mathrm{m}}$ are the modulation amplitudes of the intensity of counterpropagating waves; and ω_{m} is the self-modulation oscillation frequency. In the absence of the amplitude nonreciprocity of a ring resonator in the self-modulation regime of the first kind, a strictly out-of-phase modulation of the intensities of counterpropagating waves takes place ($\varphi_1 = \varphi_2$).

The square of the frequency of self-modulation oscillations in the vector model [3, 4] depends on the parameters of a ring laser as

$$(\omega_{\mathrm{m}}^0)^2 = m^2\cos\vartheta + \frac{m^4\sin^2\vartheta - [\beta\eta\omega/Q(1+\eta)]^2\Delta^2}{M} + \Omega^2\left(1 - \frac{4\Delta^2}{M}\right) - \Delta^2, \quad (8)$$

where $M = 2m^2(1+\cos\vartheta)$; $\omega/Q = \frac{1}{2}(\omega/Q_1 + \omega/Q_2)$, and $\Delta = \frac{1}{2}(\omega/Q_2 - \omega/Q_1)$. The superscript of ω_{m}^0 means that the self-modulation frequency was calculated by neglecting the correction related to the modulation of the population inversion [5].

If the difference Δ of the Q factors of the resonator for counterpropagating waves is nonzero, the self-modulation frequency depends on the pump excess η over the threshold. The dependence of the self-modulation frequency on η also appears in the correction to ω_{m}^0 related to the modulation of the population inversion. Taking this correction into account, the self-modulation frequency is determined by the expression

$$\omega_{\mathrm{m}} = \omega_{\mathrm{m}}^0\left[1 + \frac{\omega\eta}{4QT_1(\omega_{\mathrm{m}}^0)^2}\right]. \quad (9)$$

The dependence of the self-modulation frequency on the excess over the threshold, calculated from (8) and (9), is shown in Fig. 1 for several values of the polarisation factor β. One can see that the difference between polarisations of counterpropagating waves can considerably weaken the dependence of the self-modulation frequency on the pump.

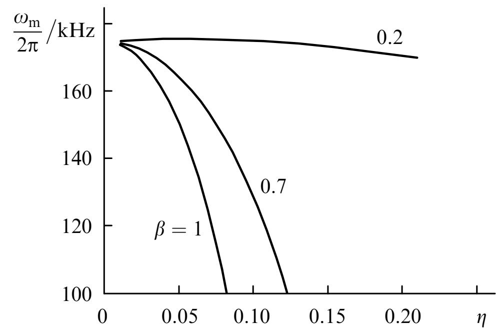

Figure 1. Dependences of the self-modulation frequency on the pump excess over the threshold calculated by expressions (8) and (9) for $m/2\pi = 333$ kHz, $\vartheta = 0.648\pi$, $\Delta = 10^5$ s^{-1} and different values of the polarisation factor β.

By generalising the results obtained in [5] to the case of arbitrarily polarised counterpropagating waves, we can obtain the expressions for parameters I_2^0 and I_2^{m} determining the self-modulation of the intensity of one of the waves:

$$I_2^0 = \frac{\eta}{2} - \frac{\beta\eta^2\Delta\omega}{MQ(1+\eta)}, \quad (10)$$

$$I_2^{\mathrm{m}} = \left\{\left(I_2^0\right)^2 - \left[\frac{m^2\sin\vartheta(1+\eta)Q}{2\beta\omega_{\mathrm{m}}\omega}\right]^2\right\}^{1/2}. \quad (11)$$

For simplicity, expressions (1) and (11) are presented for the particular case $\Omega = 0$. The amplitude characteristics of self-modulation oscillations were considered earlier in [4], however, the expression for our particular case following from the relation for $I_2^{\rm m}$ presented in [4] differs from (11) due to a misprint committed in [4].

The amplitude characteristics of self-modulation oscillations can be used to find the phase difference ϑ of coupling coefficients. By using expressions (10) and (11), we can easily find the depth of the intensity modulation of counterpropagating waves. In the case of equal Q factors of the resonator for counterpropagating waves ($\Delta = 0$), the relations $I_1^0 = I_2^0 = I_0$, $I_1^{\rm m} = I_2^{\rm m} = I$ are fulfilled, and from (10) and (11) a simple expression

$$\cos\vartheta = \frac{\omega_{\rm m}^2 - \alpha}{\omega_{\rm m}^2 + \alpha} \qquad (12)$$

for the phase difference of coupling coefficients can be obtained, where

$$\alpha = \frac{\beta^2 T_1^2 \omega_{\rm r}^4}{4(1+\eta)^2}\left[1 - \left(\frac{I}{I_0}\right)^2\right]$$

and $\omega_{\rm r}$ is the relaxation frequency.

Note that all the parameters entering into (12) (except ϑ) can be measured experimentally and then we can find the phase difference ϑ for coupling coefficients from (12).

Figure 2 shows the calculated dependences of the depth of the intensity modulation $h = I/I_0$ on the ring laser parameters. It follows from the results presented above that the consideration of different polarisations of counterpropagating waves is very important to determine correctly the phase difference of the coupling coefficients of counterpropagating waves from the experimentally measured characteristics of self-modulation oscillations. A decrease in the polarisation factor β leads to a decrease in the self-modulation depth and narrowing of the pump region in which the self-modulation regime of the first kind appears.

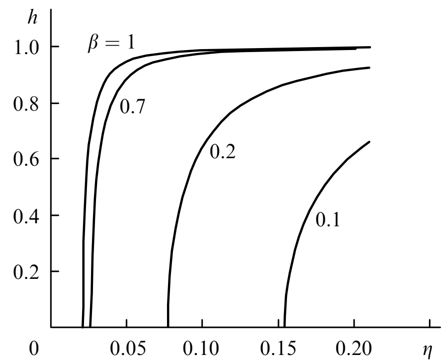

Figure 2. Dependences of the intensity self-modulation depth on the pump excess over the threshold for $m/2\pi = 333$ kHz, $\vartheta = 0.648\pi$, $\Delta = 0$ and different values of the polarisation factor β.

4. Relaxation oscillations in the self-modulation lasing regime

The difference between polarisations of counterpropagating waves also considerably affects relaxation oscillations in the self-modulation regime, which are characterised by the fundamental and additional frequencies $\omega_{\rm r}$ and $\omega_{\rm r1}$, respectively. By generalising the theoretical analysis of relaxation oscillations performed in [6, 7], we can show that the fundamental frequency is independent of the polarisation factor and is determined by the expression

$$\omega_{\rm r} = \left(\frac{\omega\eta}{QT_1}\right)^{1/2}, \qquad (13)$$

and the additional frequency is determined by the expression

$$\omega_{\rm r1} = \frac{1}{\sqrt{2}}\left[\beta\omega_{\rm r}^2 + \omega_{\rm m}^2 - \left(\omega_{\rm m}^4 + 2\beta\Omega^2\omega_{\rm r}^2\right)^{1/2}\right]^{1/2}. \qquad (14)$$

It follows from this expression that the additional relaxation frequency in the limiting case of orthogonal polarisations ($\beta = 0$) proves to be zero. Figure 3 shows the dependences of $\omega_{\rm r1}$ on the frequency nonreciprocity of the ring resonator for different values of the polarisation factor β. Note that the expressions for relaxation frequencies presented above are valid under the condition that the self-modulation oscillation frequency considerably exceeds the fundamental relaxation frequency. This condition is violated, in particular, if the phase difference of coupling coefficients is close to π.

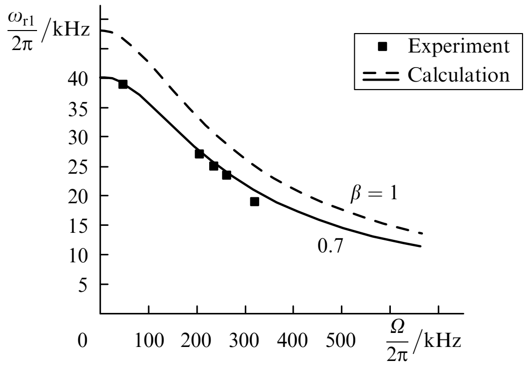

Figure 3. Dependences of the additional relaxation frequency on the frequency nonreciprocity of a ring resonator; the fundamental relaxation frequency is $\omega_{\rm r}/2\pi = 68$ kHz.

5. Relaxation oscillations in the stationary lasing regime with substantially different intensities of counterpropagating waves

The difference in polarisations of counterpropagating waves also results in a change of some relaxation frequencies in stationary lasing regimes. In the stationary regime with substantially different intensities of counterpropagating waves, three relaxation frequencies exist. The relaxation frequencies in this regime were found in [8] in the case of counterpropagating waves of the same polarisation. By generalising the results obtained in [8] to the case of arbitrary polarisations, we can obtain the expressions

$$\omega_{\rm r} = \left(\frac{\omega\eta}{QT_1}\right)^{1/2},$$

$$\omega_{\rm r1} = \left(\frac{\beta\omega\eta}{2QT_1} + \frac{\Omega^2}{4}\right)^{1/2} - \frac{\Omega}{2}, \qquad (15)$$

$$\omega_{r2} = \left(\frac{\beta\omega\eta}{2QT_1} + \frac{\Omega^2}{4}\right)^{1/2} + \frac{\Omega}{2}$$

for relaxation frequencies.

It follows from these expressions that the fundamental relaxation frequency is independent of polarisation of counterpropagating waves, whereas the two other relaxation frequencies decrease with increasing the angle between the polarisation vectors of counterpropagating waves. In the absence of the frequency nonreciprocity, the relaxation frequencies ω_{r1} and ω_{r2} prove to be degenerate.

6. Experimental results

We studied experimentally some of the features of the dynamics of a solid-state ring laser in the self-modulation regime of the first kind. The investigations were performed with a nonplanar cavity monolithic ring Nd : YAG laser. We described a similar laser in [9]. We analysed the polarisations of counterpropagating waves emerging from the cavity through a spherical mirror with a selective coating. For this purpose, we provided the same conditions for the propagation of detected waves with the help of identical beamsplitters. The study was performed for the excess η over the threshold in the region from 0.04 to 0.2. For the higher values of η, the self-modulation lasing regime became unstable and complicated quasi-periodic self-modulation appeared.

We measured the dependence of the additional relaxation frequency ω_{r1} on the frequency nonreciprocity of the ring resonator in the self-modulation regime of the first kind (Fig. 3). The frequency nonreciprocity changed when a magnetic field was applied to the active element. By comparing the experimental results presented in Fig. 3 with theoretical expression (14), we estimated the polarisation factor as $\beta = 0.7$.

The study of polarisations of counterpropagating waves emerging from a spherical mirror with a selective coating has shown that the azimuthal angle between the axes of polarisation ellipses of counterpropagating waves is 35°. This well agrees with the estimate of the average azimuthal angle determined from the results of investigations of the relaxation frequency ω_{r1}.

We also studied experimentally the dependence of the intensity modulation depth on the pump excess η over the threshold (Fig. 4). The difference Δ of Q factors was set equal to zero in calculations, the resonator bandwidth was determined from the measured values of the fundamental relaxation frequency ω_r, the moduli of coupling coefficients were set equal to $m/2\pi = 333$ kHz, and their phase difference was $\vartheta = 0.648\pi$. For these values of parameters, the self-modulation frequency $\omega_m^0/2\pi$ calculated from expression (8) was 175 kHz, in good agreement with the experimental value.

The experimental data obtained in our paper cannot be described by the standard model, whereas the vector model with the polarisation factor $\beta = 0.7$ is in good agreement with experiments.

7. Conclusions

Our study has shown that the difference in polarisations of counterpropagating waves considerably affects the additional frequency of relaxation oscillations, the self-oscillation frequency, and the depth of intensity modulation of counterpropagating waves in the self-modulation lasing regime of the first kind. The features of self-modulation oscillations caused by the difference in polarisations of counterpropagating waves are well described by the vector model of a solid-state ring laser.

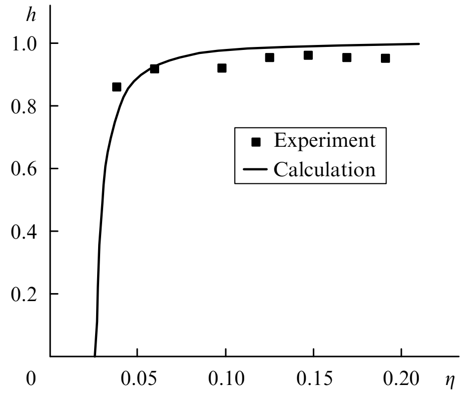

Figure 4. Dependence of the intensity self-modulation depth on the pump excess over the threshold for $\beta = 0.7$, $m/2\pi = 333$ kHz, $\vartheta = 0.648\pi$, and $\Delta = 0$.

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