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Dynamics of a Fabry – Perot cavity in the field of a plane gravitational wave

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***Abstract*. The interaction of a weak gravitational wave with a Fabry – Perot cavity is analysed beyond the long-wavelength approximation in the input-mirror locally Lorentzian frame of reference taking the light pressure into account. The generalised expressions are obtained for the coefficient of pondermotive optical rigidity, the motion law of the moving mirror of the cavity and the response function of the cavity. It is shown that the latter is a sum of two phase shifts of a circulating light wave: the phase incursion after reflection from the moving mirror and the phase incursion due to the direct interaction of gravitational and light waves in the cavity. The possibility of the resonance detection of high-frequency gravitational waves by using the optical rigidity effect is considered.**

Keywords: *gravitational waves, gravitational-wave detectors, Fabry – Perot cavity.*

1. Introduction

At present the search for gravitational-wave (GW) radiation from astrophysical sources is performed with the help of ground-based laser Michelson interferometers with arms of length from a few hundreds of meters (300 m in the Japanese TAMA-300 antenna and 600 m in the Anglo – German GEO-600 antenna) to a few kilometres (3 km in the Franco – Italian VIRGO antenna and 4 km in American LIGO antennas) [1, 2]. Variations of the separation between the interferometer mirrors caused by gravitational waves and fluctuations of the different nature limiting the sensitivity of GW detectors are recorded by the change in the interference pattern on a photodetector. The GW radiation from double systems of neutron stars and black holes, antisymmetric explosions of supernovas, rotating pulsars, etc., falls in the frequency range of ground-based interferometers ($f_{\rm GW} \approx 50 - 1000$ Hz). We will call gravitational waves in this frequency range the low-frequency waves. They satisfy the relation $L \ll \lambda_{\rm GW}$, where L is the interferometer arm length, which is called the long-wavelength approximation.

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To amplify the response to low-frequency GW signals, additional (input) mirrors are placed into the arms of the Michelson interferometers, transforming them to Fabry – Perot cavities. The optical resonance gain can be estimated by the order of magnitude as τ^*/τ, where τ^* is the relaxation time of the cavity and $\tau = L/c$ is the photon transit time along the interferometer arm (for LIGO, $\tau^* \approx 1.6 \times 10^{-3}$ s, $\tau \approx 1.3 \times 10^{-5}$ s). It was shown that in the operating frequency range of ground-based interferometers (i.e. in the long-wavelength approximation), the light pressure on moving mirrors (probe masses) in the detuned cavity efficiently transforms free probe masses to linear oscillators due to the pondermotive optical rigidity effect [3 – 13], providing the additional mechanical resonance gain at some frequencies.

In addition, it has been demonstrated in the literature that the response to the GW signal in a Fabry – Perot cavity is also amplified near the frequencies multiple of the free spectral range of the cavity (for LIGO, $f_{\rm FSR} = 37.5$k Hz) [14, 15]. The long-wavelength approximation is violated for gravitational waves at these frequencies because $\lambda_{\rm GW} = 2L$ for $f_{\rm GW} = f_{\rm FSR} = c/2L$. And although the sources of high-frequency GW radiation ($f_{\rm GW} > 10$kHz) are virtually unknown in modern astrophysics, some string cosmological models [16 – 18] predict the existence of the relict gravitational background in the frequency range $f_{\rm GW} \approx 10^{-6} - 10^{10}$ Hz. Because the search for and recording of the GW background are planned already with the Advanced LIGO antenna based on the use of the optical rigidity, the consideration of the possibility of the resonance detection of high-frequency waves is of current interest.

In this paper, we calculated and analysed the response of the Fabry – Perot cavity to a gravitational wave of an arbitrary frequency (i.e. beyond the long-wavelength approximation) taking into account the light pressure on mirrors, resulting in the effect of pondermotive optical rigidity. We also analysed the possibility of mechanical resonance detecting the GW signal near the frequency of the free spectral range of the cavity.

2. Electromagnetic wave in the gravitational wave field

The gravitational field in the general relativity theory is identified with the metric properties of space – time. The field of a weak plane +-polarised gravitational wave in the locally Lorentzian frame of reference of a physical body corresponds to the metric [19]

$$\mathrm{d}s^2 = g_{\alpha\beta}(x)\mathrm{d}x^\alpha \mathrm{d}x^\beta = -c^2\mathrm{d}t^2 + \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2 + \frac{1}{2c^2}\left(x^2 - y^2\right)\ddot{h}\,(c\,\mathrm{d}t - \mathrm{d}z)^2. \quad (1)$$

Here, $h = h(t - z/c)$; $|h| \ll 1$ is the GW function; and Greek indices run through 0, 1, 2, 3 or ct, x, y, z.

The wave equation for the z component of the four-dimensional potential of the electromagnetic field $A^\mu = (0, 0, 0, A)$ with the applied Coulomb gauge in metric (1) takes the form [20]

$$\frac{\partial^2 A}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 A}{\partial t^2} = \frac{1}{2}\frac{x^2}{c^2}\,\ddot{h}(t)\,\frac{1}{c^2}\frac{\partial^2 A}{\partial t^2}. \quad (2)$$

This equation describes quite accurately the propagation of a linearly polarised laser beam of radius ~ 10 cm at a wavelength of ~ 1 μm along the x axis in the field of a gravitational wave at the frequency $f_{\mathrm{GW}} = 10^2 - 10^5$ Hz (corresponding to $\lambda_{\mathrm{GW}} = 10^3 - 10^6$ m).

The solution of Eqn (2) obtained in [20] by the method of successive approximations accurate to the terms of the order of Ω/ω_0 has the form

$$A(x, t) = A_+(x, t) + A_-(x, t),$$
$$A_\pm(x, t) = A_{\pm 0}[1 + g_\pm(x, t)] \times \exp[-\mathrm{i}(\omega_0 t \mp k_0 x)] + \mathrm{c.c.}, \quad (3)$$

where

$$g_\pm(x, t) = \int_{-\infty}^{+\infty}\left\{\frac{1}{4}\,\omega_0\Omega\,\frac{x^2}{c^2}\,h(\Omega) \mp \mathrm{i}\frac{1}{2}\,k_0 x h(\Omega) + \frac{1}{2}\frac{\omega_0}{\Omega}\,h(\Omega)\left[\exp\left(\pm\mathrm{i}\Omega\frac{x}{c}\right) - 1\right]\right\}\exp(-\mathrm{i}\Omega t)\,\frac{\mathrm{d}\Omega}{2\pi}; \quad (4)$$

and $k_0 = \omega_0/c$. The function A_+ describes a light wave propagating in the positive direction of the x axis, and the function A_- describes a counterpropagating wave. The amplitudes $A_{\pm 0}$ and frequency ω_0 are obtained from the initial boundary problem, which is formulated and solved in the next section.

3. Electromagnetic wave in a Fabry – Perot cavity in the gravitational-wave field

Consider now a light wave circulating in a Fabry – Perot cavity of length L in the gravitational-wave field (Fig. 1). We will call one of the cavity mirrors the input mirror and the other – the moving mirror. Let us couple our frame of reference with the input mirror (in other words, we will use this mirror as a locally Lorentzian frame of reference) and assume that it partially transmits optical radiation with the corresponding amplitude coefficient $T \ll 1$, while the moving mirror reflects 100 % of radiation. For simplicity, we neglect optical losses in both mirrors.

Let a linearly polarised plane light wave $A_{\mathrm{in}}(x, t)$ be incident on the input mirror. We write it as a sum of a 'large' wave with the amplitude $A_{\mathrm{in}\,0}$ and frequency ω_0 and a 'small' addition $a_{\mathrm{in}}(x, t)$ corresponding to the optical noise (for simplicity, we consider only classical electromagnetic fields and their fluctuations):

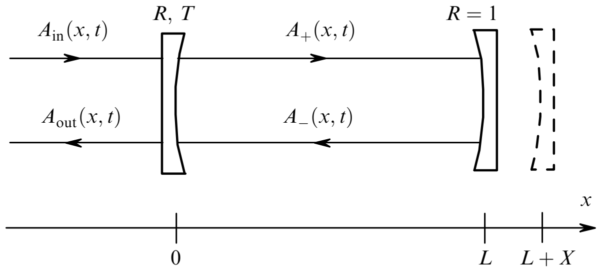

Figure 1. Fabry – Perot cavity with an absolutely reflecting moving mirror. The left mirror is fixed and has the amplitude transmission coefficient T. The isolated cavity without pumping has the fixed length L, the coordinate of the moving mirror is $x(t) = L + X(t)$, $X \ll L$.

$$A_{\mathrm{in}}(x, t) = A_{\mathrm{in}\,0}\exp[-\mathrm{i}(\omega_0 t - k_0 x)] + a_{\mathrm{in}}(x, t)\exp[-\mathrm{i}(\omega_0 t - k_0 x)] + \mathrm{c.c.},$$

$$a_{\mathrm{in}}(x, t) = \int_{-\infty}^{+\infty} a_{\mathrm{in}}(\Omega + \omega_0)\exp\left[-\mathrm{i}\Omega\left(t - \frac{x}{c}\right)\right]\frac{\mathrm{d}\Omega}{2\pi}.$$

We represent the optical field in the cavity as a sum of two waves propagating in the positive $[A_+(x, t)]$ and negative $[A_-(x, t)]$ directions along the x axis and divide each of the waves into three parts: a 'large' component with the amplitude $A_{\pm 0}$, a 'small' addition corresponding to the direct interaction of the gravitational and light waves [see expression (3)], and a 'small' unknown function $a_\pm(x, t)$ describing the optical noise and phase shift of a wave circulating in the cavity (we will neglect the terms of the type $h \times a_\pm \sim h^2$):

$$A_\pm(x, t) = A_{\pm 0}\exp[-\mathrm{i}(\omega_0 t \mp k_0 x)] + A_{\pm 0} g_\pm(x, t)\exp[-\mathrm{i}(\omega_0 t \mp k_0 x)] + a_\pm(x, t)\exp[-\mathrm{i}(\omega_0 t \mp k_0 x)] + \mathrm{c.c.},$$

$$a_\pm(x, t) = \int_{-\infty}^{+\infty} a_\pm(\Omega + \omega_0)\exp\left[-\mathrm{i}\Omega\left(t \mp \frac{x}{c}\right)\right]\frac{\mathrm{d}\Omega}{2\pi}.$$

The wave $A_{\mathrm{out}}(x, t)$ reflected from the cavity includes both the wave reflected from the input mirror and the wave emerged from the cavity. The latter contains information on the GW signal (and noises masking it). We represent the reflected wave also as a sum of the 'large' and 'small' components:

$$A_{\mathrm{out}}(x, t) = A_{\mathrm{out}\,0}\exp[-\mathrm{i}(\omega_0 t + k_0 x)] + a_{\mathrm{out}}(x, t)\exp[-\mathrm{i}(\omega_0 t + k_0 x)] + \mathrm{c.c.},$$

$$a_{\mathrm{out}}(x, t) = \int_{-\infty}^{+\infty} a_{\mathrm{out}}(\Omega + \omega_0)\exp\left[-\mathrm{i}\Omega\left(t + \frac{x}{c}\right)\right]\frac{\mathrm{d}\Omega}{2\pi}.$$

To find the relation between the optical fields written above, we impose the boundary conditions: the continuity condition for the vector potential on the surfaces of the input mirror and the condition that the tangential component of the electric field vector on the surface of the moving mirror in its intrinsic frame of reference is zero [21]:

$$A_{\mathrm{out}}(0,\, t) = RA_{\mathrm{in}}(0,\, t) + TA_-(0,\, t),$$

$$A_+(0,\,t) = TA_{\rm in}(0,\,t) - RA_-(0,\,t),$$

$$A_+[L + X(t-\tau),\,t-\tau] + A_-[L + X(t-\tau),\,t-\tau] = 0.$$

Here, $\tau = L/c$. The solution of this system of equations obtained by the method of successive approximations [20] has the form

$$A_{+0} = \frac{T}{1 - R\exp(2{\rm i}\omega_0\tau)}A_{\rm in\,0},$$

$$A_{-0} = -\frac{T\exp(2{\rm i}\omega_0\tau)}{1 - R\exp(2{\rm i}\omega_0\tau)}A_{\rm in\,0},$$

$$A_{\rm out\,0} = \frac{R - \exp(2{\rm i}\omega_0\tau)}{1 - R\exp(2{\rm i}\omega_0\tau)}A_{\rm in\,0}$$

in the zero-order approximation and

$$a_+(\Omega+\omega_0) = a_{\rm in}(\Omega+\omega_0)\frac{T}{1 - R\exp[2{\rm i}(\Omega+\omega_0)\tau]}$$

$$+ \frac{R\exp(2{\rm i}\omega_0\tau)A_{+0}}{1 - R\exp[2{\rm i}(\Omega+\omega_0)\tau]}{\rm i}[\delta\Psi_{\rm mir}(\Omega) + \delta\Psi_{\rm GW+EMW}(\Omega)],$$

$$a_-(\Omega+\omega_0) = -a_{\rm in}(\Omega+\omega_0)\frac{T\exp[2{\rm i}(\Omega+\omega_0)\tau]}{1 - R\exp[2{\rm i}(\Omega+\omega_0)\tau]} \tag{5}$$

$$+ \frac{A_{-0}}{1 - R\exp[2{\rm i}(\Omega+\omega_0)\tau]}{\rm i}[\delta\Psi_{\rm mir}(\Omega) + \delta\Psi_{\rm GW+EMW}(\Omega)],$$

$$a_{\rm out}(\Omega+\omega_0) = a_{\rm in}(\Omega+\omega_0)\frac{R - \exp[2{\rm i}(\Omega+\omega_0)\tau]}{1 - R\exp[2{\rm i}(\Omega+\omega_0)\tau]}$$

$$+ \frac{TA_{-0}}{1 - R\exp[2{\rm i}(\Omega+\omega_0)\tau]}{\rm i}[\delta\Psi_{\rm mir}(\Omega) + \delta\Psi_{\rm GW+EMW}(\Omega)]$$

in the first approximation.

Here, we introduced the convenient notation for phase shifts after single reflection according to their physical meaning:

$$\delta\Psi_{\rm mir}(\Omega) = 2k_0X(\Omega)\exp({\rm i}\Omega\tau)$$

corresponds to the phase shift of the light wave reflected from the mirror moving according to the law $X = X(t)$ [or $X(\Omega)$ in the spectral representation], which will be obtained below;

$$\delta\Psi_{\rm GW+EMW}(\Omega) = {\rm i}[g_-(L,\Omega+\omega_0) - g_+(L,\Omega+\omega_0)]$$

$$\times\exp({\rm i}\Omega\tau) \approx -k_0Lh(\Omega)\left(1 - \frac{\sin\Omega\tau}{\Omega\tau}\right)\exp({\rm i}\Omega\tau)$$

describes the phase shift of the light wave (EMW) due to its direct interaction with the gravitational wave calculated in the approximation $\Omega \sim \omega_{\rm GW} \ll \omega_0$. Note that this effect can be qualitatively interpreted as the appearance of the effective refractive index depending on the coordinate and time.

4. Equation of mirror motion

The equation of mirror motion in the gravitational-wave field (1) taking the light pressure into account has the form [20]

$$m\left[\frac{{\rm d}^2X}{{\rm d}t^2} - \frac{1}{2}L\ddot{h}(t)\right] = \frac{S}{8\pi}\left[\frac{1}{c^2}\left(\frac{\partial A}{\partial t}\right)^2 + \left(\frac{\partial A}{\partial x}\right)^2\right]^{(1)}_{x=L+X(t)}, \tag{6}$$

where m is the mirror mass; S is the laser-beam cross section area; and $A = A(x,t)$ is the electromagnetic field potential in the cavity.

It is convenient to solve Eqn (6) in the spectral representation by the method of successive approximations. We are interested only in the first approximation because the pressure force in the zero-order approximation is constant and can be compensated. The right-hand side of this equation (denoted by F) can be divided into two terms according to their physical meaning (by neglecting the fluctuation component of the force proportional to $a_{\rm in}$) [20]:

$$F(\Omega) = F_{\rm pm}(\Omega) + F_{\rm GW+EMW}(\Omega),$$

where

$$F_{\rm pm}(\Omega) = -\mathcal{K}(\Omega)X(\Omega) = -K(\Omega)X(\Omega) + 2{\rm i}\Omega\Gamma(\Omega)X(\Omega) \tag{7}$$

is the pondermotive component of the light pressure force;

$$F_{\rm GW+EMW}(\Omega) = \frac{1}{2}\mathcal{K}(\Omega)\left(1 - \frac{\sin\Omega\tau}{\Omega\tau}\right)Lh(\Omega) \tag{8}$$

is the correction to the force for the direct interaction of the gravitational and light waves;

$$K(\Omega) = \frac{4k_0SW_{\rm FP}R\exp(2{\rm i}\Omega\tau)\sin(2\omega_0\tau)}{1 - 2R\exp(2{\rm i}\Omega\tau)\cos(2\omega_0\tau) + R^2\exp(4{\rm i}\Omega\tau)} \tag{9}$$

is the optical rigidity coefficient [7, 8, 22];

$$\Gamma(\Omega) = \frac{SW_{\rm FP}}{c}\,\frac{1 - R^2\exp(4{\rm i}\Omega\tau)}{1 - 2R\exp(2{\rm i}\Omega\tau)\cos(2\omega_0\tau) + R^2\exp(4{\rm i}\Omega\tau)} \tag{10}$$

is the radiative friction coefficient [22]; and $W_{\rm FP} = k_0^2(A_{+0} \times A_{+0}^* + A_{-0}A_{-0}^*)/2\pi$ is the energy density of a light wave circulating in the Fabry–Perot cavity.

Note that the pondermotive force includes the terms describing the rigidity K and radiative friction Γ. The latter is the relativistic correction and appears taking into account the terms proportional to $\dot{X}/c$. Both these effects appear with a time delay (of the order of the relaxation time of the cavity). The division of the pondermotive force into the restoring force and friction force is conditional because both these terms have real and imaginary parts. The quantity $\Re(K)$ is usually called the optical rigidity. Because $2{\rm i}\Omega\Gamma \sim (\Omega/\omega_0)K$, the friction Γ in most cases is masked by the imaginary part of the rigidity and can be omitted below. The only exclusion is the case of the optical resonance $\omega_0\tau = \pi n$, $n = 0, 1, 2, ...,$ for which $K = 0$. In the detuned cavity, depending on the choice of the operating point in the resonance curve, the following cases are possible [3, 4]: either $\Re[K(\Omega)] > 0$ and $\Im[K(\Omega)] < 0$, or $\Re[K(\Omega)] < 0$ and $\Im[K(\Omega)] > 0$. These inequalities reflect the fact that the appearance of the pondermotive rigidity can be accompanied by the development of instability.

5. Law of mirror motion and the cavity response

The solution of Eqn (6) in the spectral representation, taking expressions (7)–(10) into account, has the form

$$X(\Omega)=\frac{1}{2}\frac{Lh(\Omega)}{m\Omega^2-\mathcal{K}(\Omega)}\left[m\Omega^2-\mathcal{K}(\Omega)\left(1-\frac{\sin\Omega\tau}{\Omega\tau}\right)\right], \qquad (11)$$

and the corresponding response of the detector (5) (we neglect the term proportional to $a_{\rm in}$ and describing fluctuations of the optical field) is

$$a_{\rm out}(\Omega+\omega_0)=-A_{\rm in0}\times\frac{T^2\exp(2{\rm i}\omega_0\tau)}{[1-R\exp(2{\rm i}\omega_0\tau)]\{1-R\exp[2{\rm i}(\Omega+\omega_0)\tau]\}}\times\frac{m\Omega^2}{m\Omega^2-\mathcal{K}(\Omega)}{\rm i}k_0Lh(\Omega)\frac{\sin\Omega\tau}{\Omega\tau}\exp({\rm i}\Omega\tau). \qquad (12)$$

In most cases, the replacement $\mathcal{K}(\Omega)\to K(\Omega)$ can be made in expressions (11) and (12).

It follows directly from the expressions obtained above that the interaction of gravitational waves at frequencies $\Omega=\omega_{\rm GW}\approx n\omega_{\rm FSR}$ ($\omega_{\rm FSR}=\pi c/L$ is the free spectral range of the cavity) with the cavity can be accompanied by the parametric excitation of additional optical modes.

6. Response of the detuned cavity near the FSR frequency

Let us analyse the response function of the cavity near the FSR frequency by introducing the notation $\Delta=\Omega-\omega_{\rm FSR}$, $|\Delta|\ll\omega_{\rm FSR}$ is the detuning from the FSR frequency, δ is the detuning from the selected operation mode of the cavity (i.e. $\omega_0=\pi n_0/\tau+\delta$, n_0 is fixed), $\gamma=(1-R)/(2\tau)$ is the half-width of the resonance curve, $\mathcal{E}_{\rm FP}$ is the total electromagnetic-field energy in the cavity. The expressions for the optical rigidity coefficient (9) and cavity response function (12) take the form

$$K(\Delta)=\frac{2\omega_0\mathcal{E}_{\rm FP}}{L^2}\frac{\delta}{\delta^2+(\gamma-{\rm i}\Delta)^2},$$

$$a_{\rm out}(\omega_{\rm FSR}+\Delta+\omega_0)\approx-A_{\rm in0}\frac{\gamma/\tau}{\gamma-{\rm i}\delta}\frac{1}{\gamma-{\rm i}(\delta+\Delta)}\times\frac{m\omega_{\rm FSR}^2}{m\omega_{\rm FSR}^2-K(\Delta)}\frac{\Delta}{\omega_{\rm FSR}}{\rm i}k_0Lh(\omega_{\rm FSR}+\Delta).$$

The condition of the resonance detecting regime is the equality $m\omega_{\rm FSR}^2-\Re[K(\Delta)]=0$. The ratio $\Re[K(\Delta)]/(m\omega_{\rm FSR}^2)$ can be estimated by the order of magnitude as $\sim10^{-4}$ [20] for the following values of parameters planned in Advanced LIGO detectors: $L=4$ km, $\omega_0/2\pi=3\times10^{14}$ Hz, $\delta/2\pi=100$ Hz, $\gamma/2\pi=1$ Hz (narrowband operation regime), $\mathcal{E}_{\rm FP}=20$ J, $m=40$ kg and $\omega_{\rm FSR}/2\pi=37.5$ kHz. Therefore, to obtain the mechanical resonance gain upon detection of high-frequency gravitational waves with $f_{\rm GW}\approx30$ kHz, it is necessary to increase the circulating optical power and reduce the mirror mass and the width of the resonance curve. Note that even when the equality $m\omega_{\rm FSR}^2-\Re[K(\Delta)]=0$ is fulfilled, the resonance gain will be limited by the value $m\omega_{\rm FSR}^2/\Im[K(\Delta)]$.

7. Conclusions

We have analysed the interaction of a weak plane +-polarised gravitational wave with a Fabry–Perot cavity in the input-mirror locally Lorentzian frame of reference taking the light pressure force into account beyond the framework of the long-wavelength approximation. We have obtained the generalised expressions for the optical rigidity coefficient, the law of mirror motion, and the cavity response to gravitational waves at arbitrary frequencies, including high-frequency waves predicted by some cosmological models. Based on these expressions, we have considered the possibility of the resonance detection of high-frequency gravitational waves and have shown that to obtain the mechanical resonance gain, it is necessary to increase the optical power circulating in the cavity and reduce the probe masses and the width of the resonance curve.

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