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Two schemes of logic gates for one-way quantum computing

A.M. Basharov, V.N. Gorbachev

***Abstract*.** **Two schemes of measurement-based gates are considered. The first scheme uses atomic cluster states generated by using the proposed cooperative process involving classical fields. The second scheme is based on the quantum correlation of biphotons and allows encoding classical information by the states of a quantum system.**

Keywords: *quantum computations, entangled states, quantum-measurement logic gates.*

1. Introduction

From the point of view of the successive performance of operations, the computation process is ordered in time because we distinguish, for example, the initial and final states. The time ordering plays an important role in the measurement-based models of quantum computers in which a quantum measurement is used as the basis mechanism. Indeed, due to the randomness of the quantum measurement outcome, the outcomes of measurements should determine the measurement platform of the next gate for determinate operation.

One-particle measurements and cluster states as well as entangled multiparticle states are the basic elements of one-way quantum computers (OQCs) proposed in [1, 2]. Based on OQCs with the help of cluster states prepared from polarised photons, the quantum search algorithm and Deutch – Josza algorithm were experimentally realised [3, 4]. Due to their properties, cluster states, being the universal resource for one-way computations [5], are of interest both for theoretical and experimental studies and they can be obtained in various physical systems. The properties of cluster states formed by polarised photons were experimentally studied in [6]. These states can be obtained in spin systems, in particular, in the Ising model, in optical gratings, Josephson junctions, etc. [7 – 10]. Cluster states in optical realisations of one-way computers discussed in [11] can be prepared from the EPR (Einstein – Podolsky – Rosen) pairs with the help of a 'fusion' operation based on the projection measurement [12, 13]. By using the squeezed light, the cluster states of continuous variables can be obtained [14] which allow universal calculations [15].

An OQC is based on a quantum-measurement gate formed by a cluster state. To perform the required operation, it is necessary first to write the initial data in the gate input and then to entangle all qubits and perform the measurement. This means that the entanglement should be performed by a controllable physical interaction. A variant of the controllable generation of graph states, in particular, cluster states was considered in [16] for a spin XY gate formed by the optical grating of neutral atoms.

Logic gates for one-way quantum computers can be considered from the point of view of teleportation. Indeed, the standard teleportation protocol allows one to transfer the unknown state from point A to point B, which can be treated from the point of view of computing as the unit operation performed by the gate. Teleportation gates were proposed in [17] for constructing universal fault-tolerant gates. The possibility of their using in computations was discussed in [18 – 20]. Thus, there exist two close models of a quantum computer based on measurements: the one-way quantum computer and the teleportation computer. The relation between them was considered in [21].

In this paper, we discuss two questions which are important for realisation measurement-based computers. The first question concerns the controllable interaction for preparing cluster states and performing time-ordered computations. We consider two cooperative processes, namely, the cooperative absorption and coherence exchange. They include classical fields, which are a convenient controllable parameter.

The optical absorption of a photon by a pair of atoms was experimentally observed in a crystal doped with Pr^{3+} ions [22]. This absorption occurs due to the interference between the dipole – dipole interaction and interaction between light and atoms [23]. We found the effective interaction Hamiltonians for these two cooperative processes, which include classical fields, and schemes of logical gates based on atomic cluster states.

The second question discussed in the paper concerns the problem of encoding classical information with the help of the quantum-system states. This is important because, for example, any quantum algorithm has the classical input.

As one of the solutions of this problem, we propose a measurement-based gate realised by using the EPR pair. The gate has the classical input and quantum output and transforms classical data to a quantum-system state. Com-

A.M. Basharov Russian Scientific Center 'Kurchatov Institute', pl. akad. Kurchatova 1, 123182 Moscow, Russia;
V.N. Gorbachev North-West Institute of Printing Arts, St. Petersburg State University of Technology and Design, per. Dzhambula 13, 191180 St. Petersburg, Russia; e-mail: valery.gorbachev@gmail.com

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pared to the OQC gates, our gate can use more complicated simultaneous measurements and can already perform some computational tasks. We consider the realisation of such a gate by using biphotons, which are the basis of quantum information processes and can be prepared experimentally [26–29]. Thus, the C-NOT operation was experimentally demonstrated on biphotons [30].

2. General scheme of a gate based on the projection measurement

Logic gates in the models of a one-way quantum computer and a teleportation computer have the same structure, which is determined by fundamental physical principles. These are a change in the quantum state with the help of the projection measurement and the deterministic operation performed by recover operators.

Let the observable A of some system in the state $|\Psi_{\rm in}\rangle$ be measured. Let $|a_k\rangle$ ($k = 1, 2\ldots$) be the eigenvectors of A. If the outcome j is obtained, the input state is projected to $|a_j\rangle$. Let us assume that there exits the unitary operator $U_{\rm G} : U_{\rm G}|\Psi_{\rm in}\rangle = |a_j\rangle$; then a gate appears which performed the operation $U_{\rm G}$. But this gate is not deterministic because the undesirable outcome $m \neq j$ exists, which appears with the probability $|\langle\Psi_{\rm in}|a_m\rangle|^2$. Let us introduce the unitary operator $R(m)$ correcting all the undesirable outcomes $R(m)|a_m\rangle = |a_j\rangle$; then, the gate will always perform the operation $U_{\rm G}$. This requires a set of retrieval operators R (byproduct operators in the literature on one-way quantum computers).

Operations performed by the gate can be realised by using either one-particle measurements (as in the model of a one-way computer) or with the help of simultaneous measurements in the entangled basis (as in the model based on teleportation). However, these two methods do not differ significantly because they both use the entanglement. Indeed, the measurement of the state Ψ in the entangled basis means that we entangle particles in the state Ψ before the measurement by using the appropriate unitary operation and then perform one-particle measurements. As an example, we consider below both types of measurements.

3. Cluster states

Cluster states (CSs) can be generated by the interaction Hamiltonian $V = \frac{1}{4}\sum_{a,b} g_{a,b}(t)(1-Z_a)(1-Z_b)$, where $Z_a = \sigma_{za}$ is the Pauli operator of a particle. Let the initial state n be superimposed, $|+\rangle_n = (1/\sqrt{2})^n \otimes_a (|0\rangle + |1\rangle)_a$, and then

$$\mathrm{CS} = T\exp\left\{-\frac{\mathrm{i}}{\hbar}\int_0^t \mathrm{d}t' V(t')\right\}|+\rangle_n = \otimes_{a,b}(C-Z)_{ab}|+\rangle_n, \quad (1)$$

where T is the time-ordered operator; here, we assumed that

$$\exp\left\{-\frac{\mathrm{i}}{\hbar}\int_0^t \mathrm{d}t' g_{ab}(t')\right\} - 1 = -2$$

and introduced the conditional phase shift operator $C-Z$, which is determined by the relation $|xy\rangle \to (-1)^{xy}|xy\rangle$ (x, $y = 0, 1$). The operator $C-Z$ transforms the superposition $|+\rangle_2$ to the entangled state $\phi_{ab} = (C-Z)|+\rangle_2$ which up to the local unitary transformation is equal to the Bell state. For $g_{a,b} = g\delta_{a,a+1}$ and $n = 2, 3, 4$, cluster states have the form

$$\Phi_2 = \frac{1}{\sqrt{2}}(|0,-\rangle + |1,+\rangle),$$

$$\Phi_3 = \sqrt{2}(|+,0,-\rangle - |-,1,+\rangle), \quad (2)$$

$$\Phi_4 = \frac{1}{2}(|0,-,0,-\rangle + |1,+,0,-\rangle - |0,+,1,+\rangle - |1,-,1,+\rangle),$$

where $|\pm\rangle = (1/\sqrt{2})(|0\rangle \pm |1\rangle)$. The specific feature of cluster states is revealed only for $n = 4$ because two- and three-particle CSs are unitary-equivalent to the Bell and GHZ (Grinberger–Horne–Zeilinger) states.

For our purposes, the following observation is important. By using the relation $HaZ_aH_a = X_a$ (H_a is the Hadamard transformation, $X_a = \sigma_{xa}$ is the Pauli operator), we find that the CS can be represented as

$$\mathrm{CS} = TG\exp\left(-\frac{\mathrm{i}}{\hbar}\int_0^t \mathrm{d}t' V_1(t')\right)|0\rangle, \quad (3)$$

where

$$G = \exp\left[-\frac{\mathrm{i}}{\hbar}\int_0^t \mathrm{d}t' \frac{1}{4}\sum_{ab} g_{ab}(t')(1 - Z_a - Z_b)\right] \otimes_a H_a,$$

and the Hamiltonian V_1 is defined by the relation

$$V_1 = \frac{1}{4}\sum_{ab} g_{ab}(t)(X_a \otimes X_b). \quad (4)$$

This equation means that, up to the local unitary transformation EG, the required CSs can be generated by the new Hamiltonian V_1 from the initial states $|0\rangle = \otimes_a|0\rangle a$. The found Hamiltonian V_1 describes the paired interaction between particles and can appear aside from the Ising model in various three- and four-particle processes including classical fields.

4. Cooperative processes

Consider the interaction between atoms a, b and the two modes of an electromagnetic field with frequencies satisfying the relations

$$\omega_a + \omega_b = \Omega_1, \quad (5)$$

$$\omega_a - \omega_b = \Omega_2, \quad (6)$$

where ω_r is the transition frequency in the atom $r = a$, b and Ω_p is the frequency of the mode $p = 1, 2$. These equations considered from the point of view of the law of conservation of energy have a simple meaning. Thus, Eqn (5) means that the energy of the absorbed photon is spent to excite the first and second atoms from the lower to upper level. This is the case of cooperative absorption or emission, when two atoms absorb or emit one photon in cooperation. Interaction (6) is known in problems of atomic collisions in gases as radiative scattering or coherence exchange [24, 25].

Note that these cooperative processes have a close analogue, namely, well-known three-photon parametric processes in a transparent medium with the quadratic nonlinearity, where photon frequencies are related by the expression $\omega_1 + \omega_2 = \omega_3$. Thus, the pump photon decomposes in frequency-division processes into two photons – a signal photon and an idler photon, $\omega_3 \to \omega_1 + \omega_2$, and instead of them a pair of excited atoms appears in cooperative process (5). The parametric frequency conversion process $\omega_1 - \omega_3 = \omega_2$ is analogous to cooperative coherence exchange process (6). Along with parametric three-photon processes, cooperative processes can be of interest for problems of the quantum theory of information.

Modes with frequencies Ω_1 and Ω_2 in cooperative processes (5) and (6) can be considered classically as the waves with the given amplitude. In this case, the effective interaction Hamiltonian found in Appendix has the form $v = \mathrm{i}\hbar k(f^\dagger - f)$. This Hamiltonian describes cooperative absorption by atoms a and b if $f = s_{10}(a)s_{10}(b)$, and the coherence exchange process if $f = s_{10}(a)s_{01}(b)$, where $s_{xy}(r) = |x\rangle_r\langle y|$ is the one-atom operator related to the atom $r = a, b$; $x,\ y = 0,\ 1$. The coupling constant k depends on the amplitude of a classical field, which we assume the given parameter.

By using the algebra $f, f^\dagger$, which looks like the algebra of one-atom operators, $f^2 = f^{\dagger 2} = 0$, $f^\dagger f f^\dagger = f^\dagger$ (except the relation $f^\dagger f + f f^\dagger \neq 1$), we can find the evolution operator

$$U = \exp(-\mathrm{i}\hbar^{-1} v t) = 1 + (f^\dagger f + f f^\dagger)$$

$$\times(\cos tk - 1) + (f^\dagger - f)\sin tk. \qquad (7)$$

Analysis of the evolution of two atoms determined by expression (7) shows that

(i) during cooperative absorption and coherence exchange, the entangled states $(\sqrt{2})(|00\rangle + |11\rangle)$ and $(\sqrt{2})(|01\rangle + |10\rangle)$ can be generated from the initial states $|00\rangle$ and $|01\rangle$ of atoms and

(ii) the $a \otimes b \to b \otimes a$ swapping operation can be realised based on the coherence exchange.

Let us assume that conditions (5) and (6) are satisfied simultaneously and modes at frequencies Ω_1 and Ω_2 are classical. Then, the effective Hamiltonian of the form $\vartheta_{ab} = h_{ab}(X_a \otimes X_b)$ appears which describes the interaction between atoms a and b, where the coupling constant h_{ab} is determined by the amplitudes of classical fields. The classical modes are the convenient control parameter. By using ϑ_{ab} as the operation fusing two atoms, we can obtain, according to (1), various cluster states.

Consider, for example, the generation of the state Φ_4. Let us take four atoms (a, b, c, d) with transition frequencies ω_s $(s = a, b, c, d)$ and two classical modes at frequencies Ω_1 and Ω_2. Let the atoms be located at various spatial points. Then, if three atoms have the same transition frequencies $\omega_b = \omega_c = \omega_d \neq \omega_a$ and resonance conditions (5) and (6) are satisfied, we can prepare, for example, the cluster 2D state (Fig. 1a). For this purpose, fusing operators of the form $\vartheta_{ab} + \vartheta_{ac} + \vartheta_{ad}$ are used. If the pairs of atoms have the same frequencies, $\omega_a = \omega_c$ and $\omega_b = \omega_d$, by using the fusing operators $\vartheta_{ab} + \vartheta_{bc} + \vartheta_{cd}$, we can prepare a linear cluster state (Fig. 1b). The 'box' state (Fig. 1c) can be obtained with the help of operators of the form $\vartheta_{ab} + \vartheta_{bc} + \vartheta_{cd} + \vartheta_{da}$.

The four-particle states presented above are equivalent to (1) up to the local unitary transformation G and can be used to construct various logic gates, which will be unitary-equivalent to the original gates proposed for one-way computers. Consider, for example, the C-NOT gate which can be realised based on the 2D state shown in Fig. 1a.

Recall how the original C-NOT gate operates [1]. Qubit 4 in this gate is controlling, qubit 2 is the target input, and qubit 3 is its output. First qubits 2 and 3 are prepared in the superposition state $|+\rangle$. The input state is written in qubits 1 and 4 and then the cluster state $S[|a\rangle_1 \otimes |+\rangle_2 \otimes |+\rangle_3 \otimes |b\rangle_4]$ is prepared, where, according to (1) and (3),

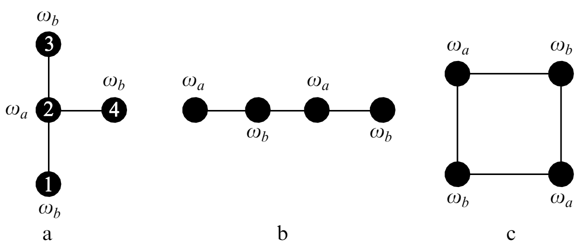

Figure 1. Four-atom cluster states: 2D state, which can be prepared if one atom has the transition frequency ω_a and three other atoms have the transition frequency ω_b; this state is used to realise the C-NOT operation on qubits 3 and 4, where 4 is the controlling qubit, and the input state of the controlling qubit is teleported from qubit 1 to qubit 3 (a), the linear cluster state (b), and the 'box' state (c).

$$S = \exp\left[\left(\frac{\mathrm{i}\pi}{4}\right)(3 - Z_1 - 3Z_2 + Z_3 + Z_4)\right] \otimes_{a=1}^{4} H_a$$

$$\times \exp\left[\left(\frac{\mathrm{i}\pi}{4}\right)(X_1X_2 + X_2X_3 + X_2X_4)\right] \otimes_{a=1}^{4} H_a. \qquad (8)$$

Then, the measurement of the observable X of qubits 1 and 2 is performed, which leads to the C-NOT operation between qubits 3 and 4. This process can be described in the following way:

$$S[|a\rangle_1 \otimes |+\rangle_2 \otimes |+\rangle_3 \otimes |b\rangle_4$$

$$= \sum_{x_1, x_2} |x_1\rangle_1 \otimes |x_2\rangle_2 R_{34}(C - X)_{43}|ab\rangle_{34}, \qquad (9)$$

where $|x_k\rangle$ is the eigenvector of the operator X_k $(k = 1,\ 2)$; R_{34} is the recover operator depending on the Pauli operators of the observables X, Z of qubits 3, 4 [1].

By using Eqn (9), we consider what will occur in our case when the cluster state required for the realisation of the C-NOT gate is generated by the Hamiltonian V_1 instead of V. Accurate to the insignificant phase factor $\exp(\mathrm{i}3\pi/4)$, we have

$$\exp\left[\left(\frac{\mathrm{i}\pi}{4}\right)(X_1X_2 + X_2X_3 + X_2X_4)\right]\left[|a'\rangle_1 \otimes |0\rangle_2 \otimes |0\rangle_3\right.$$

$$\left.\otimes|b'\rangle_4\right] = \sum_{u_1, u_2} |u_1\rangle_1 \otimes |u_2\rangle_2 R'_{34}(C - X)_{34}|a'b'\rangle_{34}, \qquad (10)$$

where $|u_k\rangle = H_k \exp(\mathrm{i}\theta_k Z_k)|x_k\rangle$ $(k = 1,\ 2)$; $\theta_1 = \pi/4$ and $\theta_2 = 3\pi/4$ are the eigenvectors of new observables $U_1 = U_2 = Y$; $R' = H_3H_4\exp[(\mathrm{i}\pi/4)(Z_3 + Z_4)]R_{34}H_3H_4$ is the new recover operator. Equation (10) means that the operation C-NOT appears between qubits 3 and 4. To perform this operation, the following procedure is needed. First qubits 2 and 3 are prepared in the $|0\rangle$ state, which is the eigenvector of the observable Z. The input data are written in qubits 1 and 4 and then all qubits are entangled by using the operation $\exp[(\mathrm{i}\pi/4)(X_1X_2 + X_2X_3 + X_2X_4)]$, which is generated by the Hamiltonian V_1. Then, the observables Y of qubits 1 and 2 are measured, and up to the unitary transformation R'_{34} the operation C-NOT appears between qubits 3 and 4, where qubit 4 is a target, while the state of the controlling qubit is teleported from qubit 1 to qubit 3. Note that, due to the classical modes in the interaction Hamiltonian generating the cluster state, we have the controllable preparation of the entangled state and the time-ordered gate operation.

To perform the required operation in the gate, it is necessary to write the initial data in its input. If these data are encoded by the quantum-system states, they can be written by using, for example, the swapping operation. This operation can be physically realised with the help of the considered cooperative coherence exchange process and is described by the Hamiltonian $\upsilon = \mathrm{i}\hbar k(f^{\dagger} - f)$, where $f = s_{10}(a)s_{01}(b)$, which leads to the $a \otimes b \to b \otimes a$ swapping operation. Thus, any output state of the quantum register can be written in the input of another quantum register. However, the input data can be classical, as for example, for any quantum algorithm. One of the possible solutions of this problem can be obtained by using gates based on the EPR pairs.

5. Biphoton-based gates

An EPR pair can be prepared in different ways, in particular, by using cooperative processes (5) or (6). However, we consider here the optical realisation of this pair based on biphotons. For this purpose, a biphoton in the maximally entangled state of the type $|b\rangle = (1/\sqrt{2})(|00\rangle + |11\rangle)$ is required, where the vectors $|0\rangle$ and $|1\rangle$ are the eigenvectors of the Pauli operator $\sigma_z = Z = |0\rangle\langle 0| - |1\rangle\langle 1|$. The logic states $|0\rangle$ and $|1\rangle$ can be associated with physical states, for example, with horizontally and vertically polarised photons or with the Fock states of light $|n\rangle$, where $n = 0, 1$. If a biphoton is in the $|b\rangle$ state, a strict correlation takes place between the result of measurement for one of the photons, for example, the idler photon and the state of the remaining signal photon. This fact proves to be important for our scheme. By considering the measurement of the observable Z, we find that due to the outcome $n = 0, 1$, the signal photon is projected to the $|n\rangle$ state. In other words, the measurement of the observable $Z \otimes 1$ projects the signal photon to one of the eigenvalues of the operator $1 \otimes Z$. Let us introduce instead of Z another observable $A = SZS^{\dagger}$ with the eigenvectors having the form $|A_n\rangle = S|n\rangle$, where S is the unitary operator. Then, the following observation is valid. The measurement of the observable $A \otimes 1$ performed over the biphoton projects the remaining photon to the $|B_n\rangle$ state, which is one of the eigenvectors of the observable $1 \otimes B$, where $B = KZK^{\dagger}$ and $K = (S^{\dagger})^T$.

This can be easily proved. By introducing a new basis $|A_n\rangle$, we find

$$|b\rangle = \frac{1}{\sqrt{2}} \sum_{nn'} |A_{n'}\rangle\langle A_{n'}|n\rangle|n\rangle$$

$$= \frac{1}{\sqrt{2}} \sum_{n'} |A_{n'}\rangle|B_{n'}\rangle = (S \otimes K)|b\rangle, \tag{11}$$

where $|B_{n'}\rangle = \sum_n \langle A_{n'}|n\rangle|n\rangle = K|n'\rangle$. Let us rewrite Eqn (11) in the form $(S \otimes 1)|b\rangle = (1 \otimes K^{\dagger})|b\rangle$, which means that any rotation S of the idler photon leads to the rotation K of the signal photon, and vice versa. The operator S is determined from the correlation of K.

Consider a gate consisting of a biphoton, the scheme for measuring the observable Z, and the recover operator R. Let us introduce the recover operator $R(n,s)|n\rangle = |s\rangle$, where $n, s = 0, 1$, which we represent in the form $R(n,s) = X^{n+s}$, where $X = |0\rangle\langle 1| + |1\rangle\langle 0|$. Then, the biphoton state will take the form

$$|b\rangle = \frac{1}{\sqrt{2}} \sum_{n=0,1} |n\rangle \otimes R(n,s)|s\rangle. \tag{12}$$

The gate operates as follows. The idler photon is measured in the basis of the observable Z. If the outcome $n = 0, 1$ is obtained, it follows from (12) that the signal photon is projected to the $R(n,s)|s\rangle$ state. Then, according to this result, the signal photon is subjected to the unitary operation $R^{\dagger}(n,s)$ and its state will be $|s\rangle$. The introduced binary variable s can be treated as the external classical signal. Therefore, our gate transforms classical information to the quantum state:

$$U_{\mathrm{G}} : s \to |s\rangle.$$

Such a transformation can be of interest because, for example, any quantum algorithm has the classical input and, therefore, begins with the transformation of classical information to the quantum-system states.

Equation (11) allows us to introduce a gate with a measurement platform more complicated than Z. Thus, by measuring the observable $A = SZS^{\dagger}$, we find that

$$|b\rangle = (S \otimes K) = \frac{1}{\sqrt{2}} \sum_n |A_n\rangle R'(n,s)K|s\rangle, \tag{13}$$

where $R'(n,s) = KR(n,s)K^{\dagger}$. This equation means that the gate, including the measurement of the observable A and the reconstructing operator $R'(n,s)$, performs the operation $s \to K|s\rangle$. Thus, we can perform various operations and obtain various states at the output. Let us show, for example, that our gates can operate with the superposition and entangled states, which are the main resource of any quantum computation. To generate a superposition, we assume that $S = K = H$, where H is the Hadamard transformation. Then, the required scheme will include a biphoton, the measurement $X = HZH$, and the recover operator $R' = HX^{n+s}H = Z^{n+s}$. Such a gate performs the transformation $s \to (1/\sqrt{2})(|0\rangle + (-1)^s|1\rangle)$.

To generate the entangled state, we can take two biphotons and the Bell measurement of two idler photons in the basis of the observable $A = SZ \otimes ZS^{\dagger}$, where $S = C_{12}(H \otimes 1)$, and C_{12} is the C-NOT operation between idler photons. The recover operator for this case has the form $R = (Z^{m+s} \otimes 1)(1 \otimes X^{n+p})$, where bits m, n, and s, p code the measurement result and a classical external signal. As a result, the gate performs the operation $s, p \to (1/\sqrt{2})\times(|0,p\rangle + (-1)^s|1, 1 \oplus p\rangle)$. The above-considered examples represent schemes for encoding classical information with the help of the superposition and entangled photon states.

Another example concerns the calculation of the function

$$f \to U_f : |x,y\rangle \to |x, y \otimes f(x)\rangle, \tag{14}$$

where $x, y = 0$, 1. This problem can be performed with the help of two biphotons, the measurement of the observable $A = U_f(Z \otimes Z)U_f^{\dagger}$, and the recover operator $R = X^a \otimes X^b$, where $a = n + x$, $b = m + f(n) + y + f(x)$, while two pairs of biphotons, n, m and x, y, encode the outcomes of measurements of the observable A and the classical input signal. In this case, the gate performs the operation $x, y \rightarrow |x, y \otimes f(y)\rangle$.

In the experiment, the biphoton state $|b_{HV}\rangle = (1/\sqrt{2})\times(|HH\rangle + |VV\rangle$ can be prepared. Although such a pure state appears only after postselection, the generation rate of such biphotons can be high. Thus, the probability of generation of one biphoton by a 200-mW femtosecond laser emitting 100-fs pulses at a pulse repetition rate of 100 MHz is 10^{-4} and the generation rate of biphotons is of the order of 10^4 s^{-1}. Such a rate is of interest for real quantum communications.

The polarisation of the signal and idler photons can be quite easily changed by using linear optical elements such as quarter- and half-wave plates, polarisation beamsplitters, Pockels cell, etc. Such transformations are described by the unitary operator

$$D = \begin{bmatrix} t^* & -r \\ r^* & t \end{bmatrix}, \tag{15}$$

where $|r|^2 + |t|^2 = 1$. In the general case the transformation of the operator D preserves invariant only a biphoton in the antisymmetric Bell state: $(D \otimes D)(1/\sqrt{2})(|HV\rangle - |VH\rangle = (1/\sqrt{2})(|HV\rangle - |VH\rangle)$. However, in the particular case of polarisation rotation, which is described by the operator D with coefficients $r = \sin\alpha$ and $t = \cos\alpha$, we obtain Eqn (11): $(D \otimes D)|b_{HV}\rangle = |b_{HV}\rangle$.

This means that the biphoton $|b_{HV}\rangle$ can be used in our gates. In this case, the gate will contain a polarisation beamsplitter, two detectors, and a Pockels cell. The gate operates as follows. The polarisation beamsplitter separates idler photons with the H and V polarisations, which are incident on detectors D_H and D_V. One bit of classical information n is obtained due to the measurement because only two outcomes exist, when one and only one of the detectors (D_H or D_V) detects a photon. There also exists an external classical signal, whose values $s = H$, V together with n arrive at the Pockels cell input. This cell plays the role of the recover operator R, which rotates the polarisation of the signal photon and prepares it the $|s\rangle$; state. By using the noncollinear generation regime, we can obtain the quantum register where a biphoton set is obtained by selecting spatially conjugated pairs.

From the physical point of view, the scheme constructed according to (12) represents a one-photon source that generated a photon by using a biphoton in the pure $|b\rangle$ state. In this case, the main role is played by the correlation between photons rather than by the entanglement of the state. This means that such a source can use the mixed biphoton state $B = (1/2)(|00\rangle\langle+00| + |11\rangle\langle 11|)$ and the measurement of the observable Z. The entangled state can be prepared simpler in experiments, but it has, however, the classical correlation. This leads to some peculiarities if the entangled biphoton state is used in logic gates. Thus, for example, gates cannot transform a classical signal to the quantum superposition. However, this problem can be solved differently by preparing first the one-photon polarisation state and then rotating polarisation with the help of the Pockels cell: $|H\rangle, s \rightarrow (1/\sqrt{2})(|H\rangle + (-1)^s|V\rangle)$.

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Appendix

The effective Hamiltonians for cooperative processes can be obtained in the second order of the perturbation theory in the interaction. Consider the interaction operator in the form $H = V_{\rm d} + W$, where $V_{\rm d} = -\sum_a d_a E(r_a)$ is the Hamiltonian of interaction between atoms and the field in the dipole approximation;

$$W = \sum_{a \neq c} \frac{(d_a, d_c)R^2 - 3(d_a, R)(d_c, R)}{R^5}$$

is the dipole–dipole interaction Hamiltonian. In the second order over H, the evolution operator

$$T\exp\left[-\frac{\rm i}{\hbar}\int_0^t {\rm d}t' H(t')\right]$$

has two terms of the form

$$P = \left(-\frac{\rm i}{\hbar}\right)^2 \int_{t_0}^{t} {\rm d}t_2 \int_{t_0}^{t_2} {\rm d}t_1 (V_{\rm d}(t_2)W(t_1) + W(t_2)V_{\rm d}(t_1)), \tag{A1}$$

containing multiparticle processes defined in (5) and (6). Let us introduce the interaction picture $d_a(t) = a_{mn}^a \exp({\rm i}\omega_{mn}^a t)$, where $a_{mn}^a = d_{mna}|m\rangle_a\langle n|$; d_{mna} is the matrix element of the $m \leftrightarrow n$ transition in the atom a; $\omega_{mn}^a = (E_{ma} - E_{na})/\hbar$ is the transition frequency; $E(r_a, t) = \sum_q u_q(r_a)e_q^a \exp(-{\rm i}\Omega_q t)$, $q = \pm k$; $e_q^a = u_k(r_a)$; $u_{-k}(r) = u_k^*(r)$ is the set of orthogonal functions; $\Omega_{-k} = -\Omega_k$; $c_k, c_{-k} = c_k^{\dagger}$ are the creation and annihilation operators of a photon with the wave vector k; $[c_k; c_m^{\dagger}] = \delta_{km}$; and summation over k is summation over polarisation. Then, ther dipole–dipole interaction Hamiltonian takes the form

$$W(t) = \sum_{a \neq c}\sum_{mnpr} a_{mn}^a a_{pr}^c \exp({\rm i}\omega_{mn}^a t + {\rm i}\omega_{pr}^b t)\mu_{ac},$$

where μ_{ac} is the coupling constant.

Let the frequencies satisfy the condition

$$\omega_{ps}^a + \omega_{rz}^c = \Omega_q. \tag{A2}$$

Then, by using the value of the integral

$$\int_0^t {\rm d}t_2 \int_0^{t_2} {\rm d}t_1 \exp({\rm i}yt_2 + {\rm i}xt_1) \approx \frac{t}{{\rm i}x}$$

for $x + y \approx 0$, we obtain $P = (-{\rm i}/\hbar)t\vartheta_1$, where the effective Hamiltonian ϑ_1 has the form

$$\vartheta_1 = -\frac{\rm i}{\hbar}\sum_{b,\, a \neq c,\, q}\sum_{mnpsrz} \left(a_{mn}^b \otimes a_{ps}^a \otimes a_{rz}^c \otimes e_q^b\right)\mu_{ac}$$

$$\times\left(\frac{1}{\omega_{ps}^a + \omega_{rz}^c} + \frac{1}{\omega_{mn}^b + \Omega_q}\right)\delta\left(\omega_{mn}^b + \omega_{ps}^a + \omega_{rz}^c + \Omega_q\right). \tag{A3}$$

This Hamiltonian describes the interaction of three atoms (a, b, c) and one mode (e_q). It can be reduced by setting $a = b$. Then, we obtain the effective Hamiltonian ϑ_2 describing already the interaction of two atoms and the electromagnetic field mode:

$$\vartheta_2 = \sum_{a \neq c,\, q} \sum_{m,s,r,z} f^{acq}_{msrz} \left(|m\rangle_a \langle s| \otimes |r\rangle_c \langle z| \otimes e^a_q \right) \times \delta\left(\omega^a_{ms} + \omega^c_{rz} - \Omega_q \right), \tag{A4}$$

where the coupling constant is

$$f^{acq}_{msrz} = -\frac{\mathrm{i}}{\hbar} \sum_n d_{mna}\, d_{nsa}\, d_{rzc}\, \mu_{ac} \times \left(\frac{1}{\omega^a_{ns} + \omega^c_{rz}} + \frac{1}{\omega^a_{mn} + \Omega_q} \right). \tag{A5}$$

Under resonance conditions (5) and (6), we can treat atoms as two-level systems with upper level 1 and lower level 0. Let $\omega^r_{10} = \omega_r$ be the transition frequency in the atom ($r = a$, b). Then, the effective Hamiltonian describing the cooperative absorption and coherence exchange for atoms a and b has the form

$$\begin{aligned} V_{ab1} &= \mathrm{i}\hbar^{-1} k_1 \left(s_{10}(a) s_{10}(b) c_1 - s_{01}(a) s_{01}(b) c^\dagger_1 \right), \\ V_{ab2} &= \mathrm{i}\hbar^{-1} k_2 \left(s_{10}(a) s_{01}(b) c_2 - s_{01}(a) s_{10}(b) c^\dagger_2 \right), \end{aligned} \tag{A6}$$

where $s_{xy}(r) = |x\rangle_r \langle y|$; $x, y = 0, 1$; $r = a, b$; and $c_m, c^\dagger_1$ are the photon annihilation and creation operators at frequency Ω_m ($m = 1, 2$).

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