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Quantum computing based on one-photon polarisation states of light pulses propagating in a doped resonance medium

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***Abstract*. The propagation of one-photon light pulses in optical media doped with rare-earth atoms is considered under conditions close to the atomic optical resonance. The four-beam polarisation interaction M-scheme with two probe pulses and two pump pulses is analysed. The regimes of induced cross-interaction between probe optical pulses are studied and the conditions are found for controlling efficiently their group velocities in the medium, which is determined by the tunings of the atomic optical resonance. An algorithm is proposed for quantum information processing by coding and controlling the time delays of one-photon light pulses in this system.**

Keywords: *quantum computing, doped media, one-photon light pulses.*

1. Introduction

At present the schemes using the time-entangled qubit states with bases coded by one-photon light pulses, which are well distinguished by the time of their detection, attract increasing practical interest [1]. These states are often obtained by purely optical methods of transformation of the light field in a Mach – Zehnder interferometer, nonlinear crystals, etc. This in fact restricts the field of their application for quantum computing because such systems are static with respect to radiation propagating in them and cannot be integrated into schemes for quantum data processing (see, for example, [2]). At the same time, modern methods for quantum data processing and storage require a simple solution for the development of logic schemes capable of operating with a large number of qubits [3].

In this paper, we considered a new method for coding and processing quantum information based on the control of the relative time delays of propagating one-photon light pulses in an optical matrix doped with resonance atoms. We analysed the cross-interaction effects for two probe one-photon light pulses in the matrix in the presence of two control pump pulses (the M interaction scheme), which allows us to perform the corresponding transformations of the quantum polarisation states of probe photons.

2. Analysis of the M-interaction scheme in a doped medium

Figure 1 presents the energy level diagram for the ^{59}Pr atom [4]. The levels $|1\rangle$, $|2\rangle$, and $|3\rangle$ are the hyperfine structure sublevels of the $^3H_4$ state. The auxiliary $|4\rangle$ and $|5\rangle$ levels are related to the excited $^1D_2$ state. Light interacts with atomic system in the following way: two long enough pump pulses $\boldsymbol{E}_{c1}(z,t)$ and $\boldsymbol{E}_{c2}(z,t)$ of a certain (circular) polarisation $\sigma_{c1,c2}^{+(-)}$ slightly detuned by δ_1 and δ_2 from atomic resonances open the allowed transitions $|2\rangle \rightarrow |4\rangle$ and $|3\rangle \rightarrow |5\rangle$, respectively. After some time delay, shorter probe pulses $\boldsymbol{E}_{p1}(z,t)$ and $\boldsymbol{E}_{p2}(z,t)$ with an arbitrary circular polarisation $(\sigma_{p1,p2}^{+(-)})$ follow, which propagate at a small angle to each other. The frequencies of these pulses are exactly tuned to the resonance with the $|4\rangle \rightarrow |1\rangle$ and $|5\rangle \rightarrow |1\rangle$ transitions, respectively. The dispersion properties of the atomic medium in this M-scheme can be considerably changed under certain conditions, resulting in the appearance of cross-interaction effects for probe light pulses (cf. [5, 6]). The control parameters of the problem are the pump-pulse intensities $I_{c1(c2)}$ and their detunings δ_1 and δ_2 from the resonance.

The atomic – optical interaction Hamiltonian for the M-scheme under study has the form

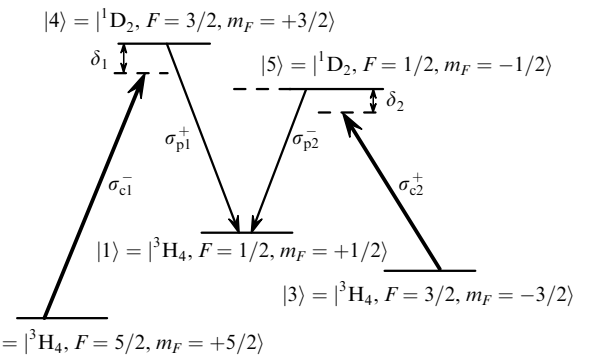

Figure 1. Energy level diagram of the ^{59}Pr atom.

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$$
\begin{aligned}
H &= H_{\text{free}} + H_{\text{int}},\\
H_{\text{free}} &= \hbar[\omega_{\text{M}}|2\rangle\langle 2| + (\omega_{\text{M}}+\omega_{\text{G}})|3\rangle\langle 3|\\
&\quad + (\omega_{\text{M}}+\omega_{\text{L}})|4\rangle\langle 4| + (\omega_{\text{M}}+\omega_{\text{L}}-\omega_{\text{G1}})|5\rangle\langle 5|],\\
H_{\text{int}} &= -\hbar\big[g_{\text{c1}}|4\rangle\langle 2|u_1 + g_{\text{c1}}^*|2\rangle\langle 4|u_1^* + g_{\text{c2}}|5\rangle\langle 3|u_2\\
&\quad + g_{\text{c2}}^*|3\rangle\langle 5|u_2^* + g_{\text{p1}}|4\rangle\langle 1|v_1 + g_{\text{p1}}^*|1\rangle\langle 4|v_1^*\\
&\quad + g_{\text{p2}}|5\rangle\langle 1|v_2 + g_{\text{p2}}^*|1\rangle\langle 5|v_2^*\big],
\end{aligned}
\tag{1}
$$

where $u_{1,2} = \exp[\mathrm{i}\boldsymbol{k}_{\text{c1(c2)}}z - \mathrm{i}\omega_{\text{c1(c2)}}t]$; $v_{1,2} = \exp[\mathrm{i}\boldsymbol{k}_{\text{p1(p2)}}z - \mathrm{i}\omega_{\text{p1(p2)}}t]$; ω_{M} is the $|1\rangle$ and $|2\rangle$ transition frequency; ω_{G} (ω_{G1}) is the $|2\rangle$ and $|3\rangle$ ($|4\rangle$ and $|5\rangle$ transition frequency; ω_{L} is the $|2\rangle$ and $|4\rangle$ transition frequency; $g_{\text{c1}} = |\mu_{24}|A_{\text{c1}}/\hbar$ and $g_{\text{c2}} = |\mu_{35}|A_{\text{c2}}/\hbar$ are the Rabi frequencies of the control pump pulses; $g_{\text{p1}} = |\mu_{14}|A_{\text{p1}}/\hbar$ and $g_{\text{p2}} = |\mu_{15}|A_{\text{p2}}/\hbar$ are the Rabi frequencies of probe pulses; μ_{mn} are the dipole moments of the corresponding transitions (we assume below that the dipole moments are the same for all optical transitions and equal to $\mu = 0.3\times10^{-32}$ C m [5]); and $A_{\text{c1(c2)}}$ and $A_{\text{p1(p2)}}$ are the slowly varying amplitudes of the corresponding optical pulses with frequencies $\omega_{\text{c1(c2)}}$ and $\omega_{\text{p1(p2)}}$ and the wave vectors $\boldsymbol{k}_{\text{c1(c2)}}$ and $\boldsymbol{k}_{\text{p1(p2)}}$, respectively.

We will use the adiabatic approximation for the atomic–optical interaction in the formalism of the density matrix ρ_{mn}, which is factorised to one-atomic states (see, for example, [7]). By using the Liouville equation for the averaged values of matrix elements ($\rho_{21} = \bar\rho_{21}v_1u_1^*$, $\rho_{41} = \bar\rho_{41}v_1$, etc.) with the initial conditions $\bar\rho_{11} = 1$, $\bar\rho_{mm} = 0$, and $m \in \{2..5\}$ and assuming that the action of pumping terminated beginning from the instant of the interaction of probe light pulses with the medium (i.e. $\bar\rho_{35} = \bar\rho_{24} = 0$), we obtain the system of coupled equations, which will determine the quantum dynamics of probe fields:

$$
\begin{aligned}
\dot{\bar\rho}_{41} &= \mathrm{i}g_{\text{p1}} + \mathrm{i}\bar\rho_{21}g_{\text{c1}} - \mathrm{i}\bar\rho_{45}g_{\text{p2}} - \bar\rho_{41}(\Gamma_1+\Gamma_4),\\
\dot{\bar\rho}_{21} &= -\mathrm{i}\bar\rho_{21}\delta_1 + \mathrm{i}\bar\rho_{41}g_{\text{c1}}^* - \bar\rho_{21}(\Gamma_1+\Gamma_2),\\
\dot{\bar\rho}_{45} &= -\mathrm{i}\bar\rho_{41}g_{\text{p2}}^* + \mathrm{i}\bar\rho_{15}g_{\text{p1}} - \bar\rho_{45}(\Gamma_4+\Gamma_5),\\
\dot{\bar\rho}_{51} &= \mathrm{i}g_{\text{p2}} + \mathrm{i}\bar\rho_{31}g_{\text{c2}} - \mathrm{i}\bar\rho_{54}g_{\text{p1}} - \bar\rho_{51}(\Gamma_1+\Gamma_5),\\
\dot{\bar\rho}_{31} &= -\mathrm{i}\bar\rho_{31}\delta_2 + \mathrm{i}\bar\rho_{51}g_{\text{c2}}^* - \bar\rho_{31}(\Gamma_1+\Gamma_3).
\end{aligned}
\tag{2}
$$

Here, $\delta_1 = \omega_{\text{c1}} - \omega_{\text{L}}$; $\delta_2 = \omega_{\text{c2}} + \omega_{\text{G}} + \omega_{\text{G1}} - \omega_{\text{L}}$; $\Gamma_m = \sum_{n=1,n\neq m}^{5}\gamma_{mn}$; γ_{mn} characterise relaxation in the system upon the $|m\rangle \to |n\rangle$ transition. We assume that $\gamma_{51} = \gamma_{41} = \gamma_{53} = \gamma_{42} = \gamma_{\text{opt}}$, $\gamma_{21} = \gamma_{31} = \gamma_{\text{mag}}$, and $\gamma_{32} = \gamma_{54} = \gamma_{\text{s}}$. The stationary solutions of Eqns (2) for the elements $\bar\rho_{41}$ and $\bar\rho_{51}$ of the density matrix corresponding to probe pulses (in the case of their cross-interaction) have the form

$$
\bar\rho_{41} = \frac{\dfrac{\mathrm{i}g_{\text{p1}}g_{\text{p2}}^2}{(\Gamma_4+\Gamma_5)A^*} + \mathrm{i}g_{\text{p1}}}{\dfrac{|g_{\text{c1}}|^2}{\mathrm{i}\delta_1+(\Gamma_1+\Gamma_2)} + \dfrac{|g_{\text{p2}}|^2}{(\Gamma_4+\Gamma_5)} + \dfrac{g_{\text{p1}}^2g_{\text{p2}}^2}{(\Gamma_4+\Gamma_5)^2A*} + (\Gamma_1+\Gamma_4)},
$$

$$
\bar\rho_{51} = \frac{-\dfrac{\mathrm{i}g_{\text{p2}}g_{\text{p1}}^2}{(\Gamma_4+\Gamma_5)B^*} + \mathrm{i}g_{\text{p2}}}{\dfrac{|g_{\text{c2}}|^2}{\mathrm{i}\delta_2+(\Gamma_1+\Gamma_3)} + \dfrac{|g_{\text{p1}}|^2}{(\Gamma_4+\Gamma_5)} + \dfrac{g_{\text{p1}}^2g_{\text{p2}}^2}{(\Gamma_4+\Gamma_5)^2B^*} + (\Gamma_1+\Gamma_5)},
\tag{3}
$$

where

$$
A = -\frac{|g_{\text{p1}}|^2}{(\Gamma_4+\Gamma_5)} - \frac{|g_{\text{c2}}|^2}{\mathrm{i}\delta_2+(\Gamma_1+\Gamma_3)} - (\Gamma_1+\Gamma_5),
$$

$$
B = \frac{|g_{\text{p2}}|^2}{(\Gamma_4+\Gamma_5)} + \frac{|g_{\text{c1}}|^2}{\mathrm{i}\delta_1+(\Gamma_1+\Gamma_2)} - (\Gamma_1+\Gamma_4).
$$

In the absence of one of the probe pulses, the problem is reduced to the Λ-interaction scheme. Solutions for $\bar\rho_{41}$ (for $g_{\text{p2}} = 0$) and $\bar\rho_{51}$ (for $g_{\text{p1}} = 0$) exactly correspond to the case of electromagnetic induced transparency [5]. Solutions (3) give the susceptibilities of the system

$$
\chi_{\text{p1}} = \frac{N|\mu|^2}{\hbar\varepsilon_0}\frac{\bar\rho_{41}}{g_{\text{p1}}} \quad\text{and}\quad \chi_{\text{p2}} = \frac{N|\mu|^2}{\hbar\varepsilon_0}\frac{\bar\rho_{51}}{g_{\text{p2}}},
$$

where N is the concentration of ^{59}Pr atoms and ε_0 is the dielectric constant. In this case, the refractive indices $n^{a(b)}$ and absorption coefficients $\alpha^{a(b)}$ of the doped crystal for two probe light pulses $\boldsymbol{E}_{\text{p1(p2)}}$ (a corresponds to the probe pulse p1, and b – to the probe pulse p2) can be written in the form $n^{a(b)} \simeq n_{0\text{M}} + \frac{1}{2}\operatorname{Re}\chi_{\text{p1(p2)}}$, $\alpha^{a(b)} = \alpha_{0\text{M}} + k_{\text{p1(p2)}}\times \operatorname{Im}\chi_{\text{p1(p2)}}$, where $n_{0\text{M}}$ and $\alpha_{0\text{M}}$ are the refractive index and absorption coefficient of the matrix, respectively. In the case $g_{\text{p1}} = 0$, we introduce the notation $n^b = n_\Lambda^b$, and $g_{\text{p2}} = 0$ – the notation $n^a = n_\Lambda^a$.

Figure 2 presents the three-dimensional dependence of the refractive index n^a on frequency detunings $\delta_{1(2)}$. The refractive index n^a has complicated dependences on both parameters δ_1 and δ_2 due to the cross-interaction between

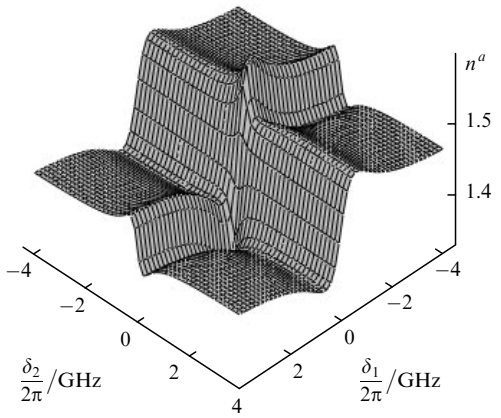

Figure 2. Frequency dependence of the refractive index n^a for a matrix doped with ^{59}Pr atoms (atomic concentration is 0.05 %) in the polarisation M-scheme of interaction with optical pulses. The parameters of doped atoms are $\gamma_{\text{opt}} = 43.5$ kHz, $\gamma_{\text{mag}} = 4$ kHz, $\gamma_{\text{s}} = 0.25$ kHz; parameters of light fields are $I_{\text{c1}} = 87.9$ kW cm^{-2} and $I_{\text{c2}} = 36.6$ kW cm^{-2} for pulse durations $\tau_{\text{c1}} = \tau_{\text{c2}} = 0.94$ μs, $I_{\text{p1}} = I_{\text{p2}} = 4.484$ W cm^{-2} for pulse durations $\tau_{\text{p1}} = \tau_{\text{p2}} = T = 258$ ps; parameters of the medium are: the length $L = 7.5$ μm and the refractive index of the matrix $n_{0\text{M}} = 1.45$.

probe light pulses in the system. This allows us to control the group velocities of propagating probe pulses to perform controllable quantum transformations with them (cf. [8]).

3. Coding and transformation of quantum information with one-photon light pulses in a doped medium

To perform quantum transformations taking into account the polarisation specific of the propagation of probe one-photon light pulses, we will use the notation

$$a^+_{1(2)}|\mathrm{vac}\rangle = |\sigma^{+(-)}_{\mathrm{p1}}\rangle, \quad b^+_{1(2)}|\mathrm{vac}\rangle = |\sigma^{+(-)}_{\mathrm{p2}}\rangle, \tag{4}$$

where $a^+_{1(2)}$ are the photon creation operators in the mode $\boldsymbol{E}_{\mathrm{p1}}$ in different polarisation states $|\sigma^{+(-)}_{\mathrm{p1}}\rangle$ with the right-hand (left-hand) circular polarisations, and similarly – for the field $\boldsymbol{E}_{\mathrm{p2}}$. In this case, the wave vector of the entangled state has the form [9]

$$|\psi\rangle = x_1|\sigma^-_{\mathrm{p1}}\rangle_a|\sigma^+_{\mathrm{p2}}\rangle_b + x_2|\sigma^-_{\mathrm{p1}}\rangle_a|\sigma^-_{\mathrm{p2}}\rangle_b + x_3|\sigma^+_{\mathrm{p1}}\rangle_a|\sigma^+_{\mathrm{p2}}\rangle_b$$

$$+ x_4|\sigma^+_{\mathrm{p1}}\rangle_a|\sigma^-_{\mathrm{p2}}\rangle_b, \tag{5}$$

where $x_i = \bar{x}_i \exp(-\mathrm{i}\varphi_i)$ are the complex amplitudes determining the probabilities of corresponding basis states and φ_i are their phases.

The $|4\rangle$ and $|1\rangle$ transition can occur only when photons of the probe field $\boldsymbol{E}_{\mathrm{p1}}$ have the right-hand circular polarisation $|\sigma^+_{\mathrm{p1}}\rangle$, while the $|5\rangle$ and $|1\rangle$ transition can occur when photons of the field $\boldsymbol{E}_{\mathrm{p2}}$ have the left-hand circular polarisation $|\sigma^-_{\mathrm{p2}}\rangle$ (see Fig. 1). Then, depending on the polarisation state of photons in probe-filed modes a and b, their delay times $\tau_{a(b)}$ during propagation in the medium will be different. These time delays are determined by the corresponding group velocities, which are in turn specified by the refractive index of the medium (see Fig. 2). The difference between these delays is manifested in experiments as the time difference $\Delta\tau_{ab}$ between two successive photocounts detected with a pair of photodetectors and a timer at the medium output. This relative delay can be written in the form

$$\Delta\tau_{ab} = \tau_a - \tau_b, \tag{6}$$

where $\tau_{a(b)} = Ln^{a(b)}_{\mathrm{g}}/c$; L is the length of the active overlap region of probe pulses in the medium; and

$$n^{a(b)}_{\mathrm{g}} = n^{a(b)} + \omega_{\mathrm{p1(p2)}}\frac{\mathrm{d}n^{a(b)}}{\mathrm{d}\omega_{\mathrm{p1(p2)}}}$$

are the group refractive indices.

Thus, the effect of induced cross-interaction is revealed only for the $|\sigma^+_{\mathrm{p1}}\rangle_a|\sigma^-_{\mathrm{p2}}\rangle_b$ state at the medium input, when both pulses are involved in the resonance atomic–optical interaction. For the $|\sigma^-_{\mathrm{p1}}\rangle_a|\sigma^+_{\mathrm{p2}}\rangle_b$ state at the medium input, the resonance interaction of light with impurity atoms is absent, and the propagation dynamics of probe pulses is determined only by the properties of the matrix, i.e. by the refractive index $n_{0\mathrm{M}}$. For the $|\sigma^-_{\mathrm{p1}}\rangle_a|\sigma^-_{\mathrm{p2}}\rangle_b$ state, we have $n^a = n_{0\mathrm{M}}$, $n^b = n^b_\Lambda$, and for the case $|\sigma^+_{\mathrm{p1}}\rangle_a|\sigma^+_{\mathrm{p2}}\rangle_b$, we have $n^a = n^a_\Lambda$, $n^b = n_{0\mathrm{M}}$.

The important result of our analysis, namely, the dependence of the group velocity of probe pulses on their polarisation state can be used for the relative time coding in which the basis states will be determined by the time delay $\Delta\tau_{ab}$ of photocounts expressed in durations of probe pulses (which we denote by T). We will use the following coding for the two-qubit register (taking into account the chosen initial polarisation state):

$$\begin{aligned}
|00\rangle &= \left(|\sigma^-_{\mathrm{p1}}\rangle_a|\sigma^+_{\mathrm{p2}}\rangle_b\right)_{\mathrm{in}} \rightarrow \Delta\tau^{00}_{ab} = 0,\\
|01\rangle &= \left(|\sigma^-_{\mathrm{p1}}\rangle_a|\sigma^-_{\mathrm{p2}}\rangle_b\right)_{\mathrm{in}} \rightarrow \Delta\tau^{01}_{ab} = T,\\
|10\rangle &= \left(|\sigma^+_{\mathrm{p1}}\rangle_a|\sigma^+_{\mathrm{p2}}\rangle_b\right)_{\mathrm{in}} \rightarrow \Delta\tau^{10}_{ab} = -2T,\\
|11\rangle &= \left(|\sigma^+_{\mathrm{p1}}\rangle_a|\sigma^-_{\mathrm{p2}}\rangle_b\right)_{\mathrm{in}} \rightarrow \Delta\tau^{11}_{ab} = -T.
\end{aligned} \tag{7}$$

Figure 3 shows the calculated dependences of the relative delays $\Delta\tau_{ab}$ of probe light pulses on the frequency detunings $\delta_{1(2)}$ of control pump pulses. Because the $|\sigma^-_{\mathrm{p1}}\rangle_a$ photon does not cause the resonance transition at the frequency ω_{p1}, the delay $\Delta\tau^{01}_{ab}$ depends only on the frequency detuning δ_2; similarly, the delay $\Delta\tau^{10}_{ab}$ depends only on δ_1. Because the relative delay $\Delta\tau^{11}_{ab}$ is produced by both pump pulses, it is determined by both parameters $\delta_{1(2)}$. Coding (7) can be performed by using frequency detunings $\delta_1/2\pi = 481.5$ MHz and $\delta_2/2\pi = 665$ MHz (see Fig. 3).

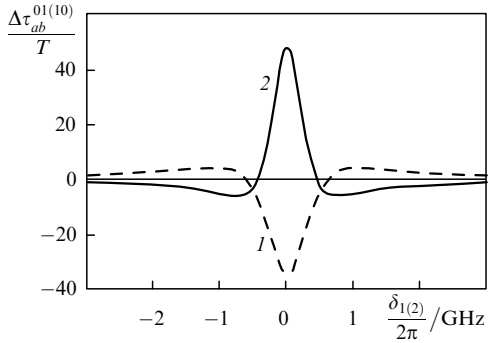

Figure 3. Frequency dependences of relative time delays $\Delta\tau_{ab}$ normalised to the probe pulse duration T for the states $|01\rangle = |\sigma^-_{\mathrm{p1}}\rangle_a|\sigma^-_{\mathrm{p2}}\rangle_b$ (*1*) and $|10\rangle = |\sigma^+_{\mathrm{p1}}\rangle_a|\sigma^+_{\mathrm{p2}}\rangle_b$ (*2*) of one-photon probe pulses propagating under conditions of atomic–optical interaction in a doped resonance medium. Parameters of the doped medium and probe pulses are as in Fig. 2; the pump parameters are $I_{\mathrm{c1}} = 41.75$ kW cm^{-2} and $I_{\mathrm{c2}} = 59.6$ kW cm^{-2}.

Figure 4 shows the two-cascade scheme for the preparation of the state of a two-qubit register followed by its processing. In particular, the time-bin-entangled state of a pair of qubits is prepared at stage 1. Then, at stage 2 this state is supplied to the input of the second doped crystal, which also realises the atomic–optical interaction M-scheme. In this scheme, pump pulses should be adjusted to provide the change in the mutual position of entangled qubits a and b in time, which is determined by the variation of the group velocities of probe pulses in the system. The realisation of this principal time scheme of coding corresponds to a certain logic transformation.

Consider, for example, the two-qubit logic operation XOR according to which the $|10\rangle$ state should be transformed to the $|11\rangle$ state, and vice versa, i.e. the probability

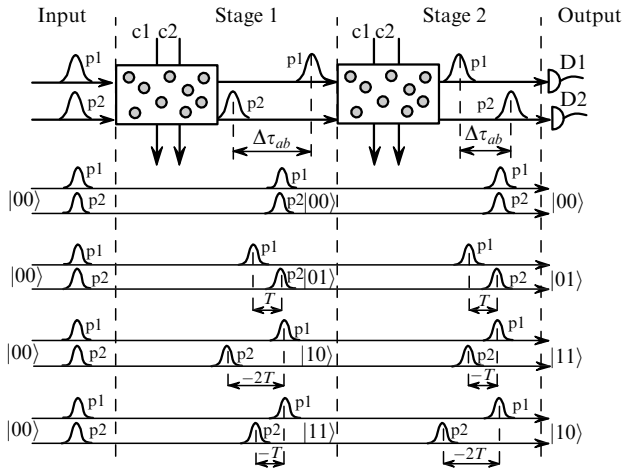

Figure 4. Principal scheme for relative time coding (stage 1) and realisation of the quantum logic XOR operation (stage 2) by using the cross-interaction of one-photon light pulses in the M-scheme of interaction in a two-stage doped resonance system.

amplitudes x_3 and x_4 in (5) should interchange places, x_1 and x_2 being invariable (see, for example, [2]). To do this, it is necessary to form at stage 2 (see Fig. 4) additional time delays:

$$\Delta\tau_{ab}^{01} = 0, \quad \Delta\tau_{ab}^{10} = T, \quad \Delta\tau_{ab}^{11} = -T. \tag{8}$$

Then, the final state of the system, detected by the shift of two photocounts of detectors, is specified as $\Delta\tau_{ab}^{00} = 0$, $\Delta\tau_{ab}^{01} = T$, $\Delta\tau_{ab}^{10} = -T$, and $\Delta\tau_{ab}^{11} = -2T$. Under the same conditions as for (7), requirement (8) can be fulfilled by choosing the control parameters $I_{c1} = 41.86$ kW cm^{-2}, $I_{c2} = 69.6$ kW cm^{-2}, $\delta_1/2\pi = 422.9$ MHz, and $\delta_2/2\pi = 732$ MHz. In the general case various adjustments in the system determined by these parameters specify possible elementary logic transformations of the two-qubit register, which makes the system in Fig. 1 universal for solving quantum computing problems.

4. Conclusions

The five-level four-beam scheme of the atomic–optical interaction presented in the paper can be extended to a multilevel scheme. In this case, an actual problem of parallel quantum data processing with an N-qubit register is solved by using transitions between numerous levels. For this purpose, of current interest are polyatomic molecules (imbedded, for example, into diamond matrices), which are characterised by numerous electronic transitions and provide the long decoherence time [10]. If photon-crystal optical fibres or other micro- and nanostructured optical media are used as solid matrices, it is promising to employ ultrashort light pulses, which can excite a variety of the electronic transitions (due to their broad spectrum), providing the realisation of logical schemes, but already with continuous observables [11].

In addition, upon multiphoton entanglement it is necessary to consider the spatial dynamics of atomic–optical interactions in terms of polariton states (see also [12]) formed in the medium. The minimal length of such a resonance medium required to obtain the interaction between any pair of N probe pulses is determined by the relation

$$TN < L/|v_g^i|_{\max}, \tag{9}$$

where $|v_g^i|_{\max}$ determines the maximum group velocity of propagating probe light pulses. Relation (9) imposes the restriction on the maximum volume of quantum information processed in such a scheme. Another restriction requires the fulfilment of the relation $T \ll 1/\gamma_{\text{opt}}$, which can be achieved by cooling the medium doped with resonance atoms. The coherent polariton states mentioned above, which appear due to atomic–optical interactions and can be associated with high-temperature states for the Bose–Einstein condensation [12], deserve a separate discussion.

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