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Complexes of in-phase two-dimensional laser solitons*

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***Abstract*. The structure and motion of complexes of in-phase weakly coupled fundamental solitons in a wide-aperture class A laser with saturable absorption are analysed. The symmetry of the field distribution and its relation to the motion of the complex are studied. Due to the absence of wavefront dislocations in such complexes, the transverse radiation intensity and phase distributions are the symmetry objects, which simplifies analysis compared to the case when wavefront dislocations are present. Four types of the motion of soliton complexes are demonstrated: a motionless complex in the presence of two mirror symmetry axes; linear motion of the complex when only one mirror symmetry axis exists; rotation around a motionless centre of inertia in the absence of the mirror symmetry axis and in the presence of symmetry with respect to rotation through the angle $2\pi/M$ (M is an integer); and curvilinear (circular) motion of the centre of inertia and simultaneous rotation of the complex around the instantaneous position of the centre of inertia in the absence of symmetry elements.**

Keywords: *autosolitons, dissipative solitons, complexes, symmetry, motion.*

1. Introduction

Dissipative solitons, or autosolitons, are stable localised structures of a field in a homogeneous or weakly modulated nonconservative (with considerable energy exchange) nonlinear medium or system [1]. Optical autosolitons entering this class, predicted in wide-aperture nonlinear-optical systems in the 1980s [2, 3], have a number of specific properties that are related, for example, to diffraction phenomena typical for optics. Therefore, the study of optical autosolitons is of considerable scientific interest; in addition, they are very promising for applications in optical data processing [4]. A particular case of optical autosolitons is solitons in a wide-aperture laser with saturable absorption, predicted in [5]. The unusual mechanics for complexes of such laser solitons, in particular, the curvilinear motion of the centre of inertia, which is impossible for conservative solitons, was demonstrated in [6 – 8]. The type of motion of two-dimensional solitons in class A lasers (where the relaxation time of nonlinear medium is much shorter than the photon lifetime in the cavity) is determined by the symmetry of the transverse distributions of the radiation intensity and energy fluxes (the Poynting vector) [8].

In this paper, we analysed the motion of the simplest and at the same time nontrivial variant of such complexes, namely, weakly coupled in-phase fundamental (vortex-free) laser solitons with fixed (linear) polarisation. The simplifying feature is the absence in this case, unlike previous papers [6 – 10], wavefront dislocations, so that the radiation phase is everywhere a certain and unique function of transverse coordinates. Then, it is sufficient to analyse only the symmetry of the transverse radiation intensity and phase distributions. The corresponding replacement of the vector field (energy fluxes) by the scalar field (phase) noticeably simplifies analysis. Another argument in favour of the choice of in-phase complexes is that the steady motion of the centre of inertia is impossible within the framework of the known asymptotic approaches of the theory of weak interaction of laser solitons [9] (this motion is decelerated and eventually ceases). At the same time, we will demonstrate below numerically all the regimes of the motion of complexes which can be observed in the case of the weak interaction of solitons [8], in particular, their linear and curvilinear motions.

2. The model and basic relations

We will consider the same scheme of a laser with saturable absorption in the same approximation and notation as in our previous papers [2 – 8]. The laser cavity can be formed by two parallel plane mirrors or can be a ring cavity. The cavity contains a nonlinear amplifying and absorbing medium. Radiation propagates predominantly along the cavity axis z, and the cavity loss for oblique beams quadratically increases with increasing a small angle of incidence, for example, due to the angular dependence of the reflectance of cavity mirrors.

We assume that the polarisation of radiation is fixed and the field envelope slowly varies in time and space, so that the

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field is described by the scalar complex envelope of the electric strength E. The nonlinearity of the medium is assumed inertialless. The cavity has a large aperture and a small length (large Fresnel number), so that its transverse sizes in the ideal model are infinitely large and the scheme has no transverse inhomogeneities (inhomogeneity is possible only in the initial conditions). We assume that linear (caused by diffraction and mirror transmission) and nonlinear changes in the field envelope are small, which allows us to average the envelope in the longitudinal direction (the average field model). Under these conditions, the field can be described by the generalised complex Ginzburg – Landau equation

$$\frac{\partial E}{\partial t} = (\mathrm{i} + d)\Delta_\perp E + f(|E|^2)E. \tag{1}$$

Here, t is the dimensionless time in units of the decay time of the field in an empty cavity; d is the effective diffusion coefficient describing a weak angular selectivity of the loss ($0 < d \ll 1$); $\Delta_\perp = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is the transverse Laplace operator corresponding to the diffraction of radiation; the transverse coordinates x and y are expressed in units of the effective width of the Fresnel zone $w_\mathrm{F} = \{L_\mathrm{cav}/[2k(1-|R|)]\}^{1/2}$; L_cav is the cavity length; k is the wave vector in the linear medium; and R is the product of the amplitude reflectances of cavity mirrors. We assume that the function $f(|E|^2)$ is a real function of the field intensity $I = |E|^2$ (frequency detunings are neglected), so that $\mathrm{Im} f = 0$. This function describes the saturation of amplification and absorption and contains constant (nonresonance) losses. Within the framework of a two-level model of media with amplification and absorption, the function f can be written in the form

$$f(|E|^2) = -1 + \frac{g_0}{1+|E|^2} - \frac{a_0}{1+b|E|^2}, \tag{2}$$

where g_0 and a_0 are the linear gain and absorption coefficients, respectively; b is the intensity ratio for saturated gain and absorption; the term -1 in the right-hand side represents the nonresonance loss due to time normalisation. The radiation intensity is normalised to the gain saturation intensity. The initial field distribution is specified as a linear superposition of several separate symmetric solitons separated by a comparatively large distance (compared to the width of a single soliton). By varying the initial condition, we can construct stable complexes with any number of individual solitons. In this case, the distance between neighbouring solitons is close to the distance between them in a stable in-phase pair.

The master equation was solved by the splitting method by using the fast Fourier transform algorithm [5]. The parameters of nonlinear function (2) used in calculations were $a_0 = 2$, $b = 10$, $g_0 = 2.11$ and the effective diffusion coefficient was $d = 0.06$. The main results were preserved when parameters were varied within a certain range.

3. Symmetric analysis and results of calculations

The vector $\boldsymbol{R}_\mathrm{c}(t)$ of transverse coordinates of the centre of inertia of a localised complex and the instantaneous velocity $\boldsymbol{V}_\mathrm{c}(t)$ of its transverse motion are determined by the relations

$$\boldsymbol{R}_\mathrm{c}(t) = \frac{\int \boldsymbol{r}_\perp |E|^2 \mathrm{d}\boldsymbol{r}_\perp}{\int |E|^2 \mathrm{d}\boldsymbol{r}_\perp}, \quad \boldsymbol{V}_\mathrm{c}(t) = \frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{R}_\mathrm{c}. \tag{3}$$

Not only coordinates of the centre of inertia but also its instantaneous velocity of motion are completely determined by the distributions (at the same instant) of the intensity $I = |E|^2$ and transverse Poynting vector [1] [taking master equation (1) into account]. In the case under study, the radiation intensity does not vanish and the wavefront has no dislocations. Therefore, to define the propagation direction of the energy flux, it is sufficient to specify the phase distribution (the energy flux direction is orthogonal to the phase level lines). In this case, it is sufficient to consider the simultaneous symmetry of the instantaneous transverse intensity and phase distributions. More exactly, it is reasonable to consider symmetry which is stable with respect to small asymmetric perturbations (not increasing during the further evolution). Then, according to [8], we can formulate the conclusions:

(i) *There exists the axial symmetry of the intensity and phase distributions.* For this type of symmetry, the velocity of motion $\boldsymbol{V}_\mathrm{c}$ of the centre of inertia is directed along the symmetry axis (the transverse component of the velocity is zero). Therefore, the motion of the structure can be only linear and its rotation is impossible (the angular velocity is $\Omega = 0$). If the structure has two or more symmetry axes, both the motion of the centre of inertia and rotation of the structure are absent.

(ii) *There exist the symmetry of the intensity and phase distributions with respect to the rotation through the angle $\alpha = 2\pi/M$ ($M = 2, 3, \dots$), i.e. the symmetry axis of the Mth order exists (C_M).* In this case, the velocity vector $\boldsymbol{V}_\mathrm{c} = 0$, so that the centre of inertia is immobile, but the structure can rotate ($\Omega \neq 0$). The case $M = 2$ corresponds to the central symmetry and the case $M = \infty$ is realised for axially symmetric structures (single autosoliton).

In the presence or absence of these symmetries, the following four variants of motion of soliton structures are possible:

(i) *Two (or more) symmetry axes.* In this case, both the translational ($\boldsymbol{V}_\mathrm{c} = 0$) and rotational ($\Omega = 0$) motions of the structure are absent. Figures 1 and 2 demonstrate the examples of such immobile structures. The phase distribution is presented in more detail for a pair of weakly coupled solitons in Fig. 1c; hereafter, we consider complexes with the minimal equilibrium distance between solitons, which are more stable than in the case of larger equilibrium distances known from the theory.

It is useful to compare this distribution with a portrait of radiation energy fluxes [8, 10], which are represented by a family of lines orthogonal to a family of equal-phase lines. The phase has seven singularities: four extrema (two maxima and two minima) and three saddle points. The extrema in the portrait of energy fluxes correspond to nodes N, while the saddle points correspond to saddles S. The centres of individual solitons correspond to the phase maxima or nodes in the portrait of fluxes. In the case of the 'the mechanical analogy', the sign of the effective potential is opposite to that of the phase, so that the centre of each (fundamental) soliton corresponds to the local minimum of the potential. The energy fluxes sweep down to this minimum in a cell restricted by a closed curve

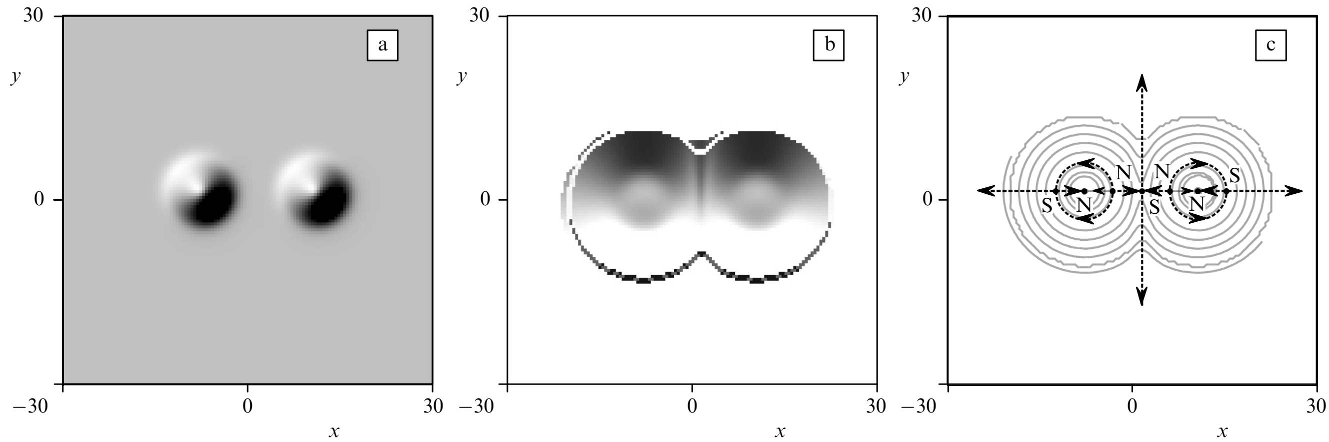

Figure 1. Intensity (a) and phase (b, c) distributions for a stable pair of weakly coupled in-phase fundamental laser solitons separated by a minimal equilibrium distance. The pair is at rest due to the presence of two mirror symmetry axes; N are nodes (phase maxima at the two most separated nodes and minima at the nearest nodes); S are saddle points, their separatrices being shown by the dashed arrows.

consisting of two approximate semicircles passing through peripheral saddles S and nodes N nearest to the centre of symmetry. These semicircles weakly distorted by the interaction of solitons correspond for the potential to the 'watershed' between radiation energy fluxes propagating to the centre of solitons (internal cells) and going away to infinity (external cells). In the middle between the centres of solitons, a central saddle is located. In this case, the separatrices of saddles divide the phase portrait into four cells symmetrical with respect to two orthogonal axes. The separatrices serve simultaneously as the symmetry axes of the intensity distribution, justifying the assignment of the structure to the given class. The analogous symmetry is also realised for immobile structures with a greater number of solitons (Fig. 2).

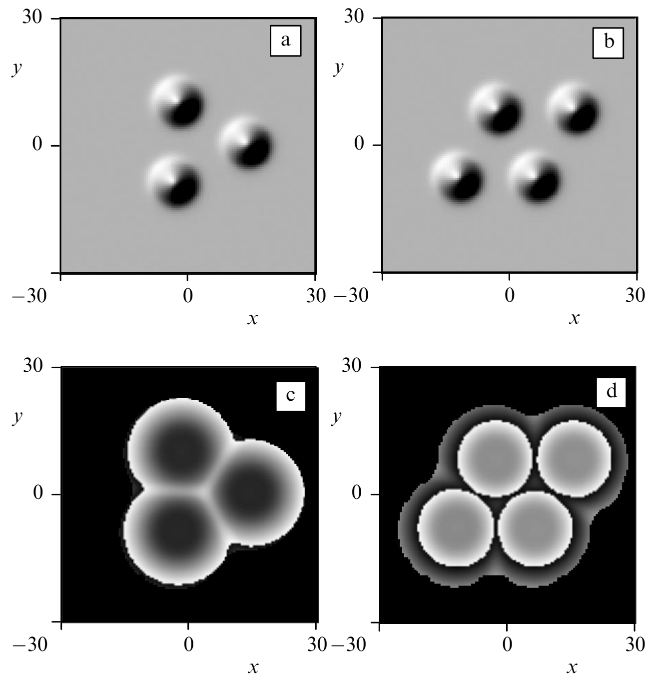

Figure 2. Intensity (a, c) and phase (b, d) distributions for stable complexes of weakly coupled in-phase fundamental laser solitons; complexes are immobile due to the presence of two mirror symmetry axes.

(ii) *The only symmetry axis*. According to the symmetry rules, such a structure can move only rectilinearly. The minimal number of solitons that can be used to obtain such structures with a weak in-phase coupling is equal to five. An example is shown in Fig. 3; the structure in this figure moves in the vertical direction at a velocity of $V_c \approx 0.0023$.

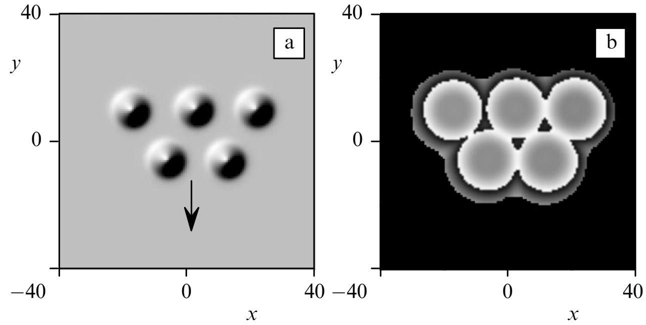

Figure 3. Intensity (a) and phase (b) distributions for a stable complex of five weakly coupled in-phase fundamental solitons; due to the presence of only one mirror symmetry axis, the structure moves rectilinearly at the constant velocity $V_c \approx 0.0023$.

(iii) *Symmetry to the rotation through the angle* $\alpha = 2\pi/M$ *(M = 2, 3,...)*. Such structures have the immobile centre of inertia coinciding with the centre of symmetry, but can rotate at some angular velocity. The minimal number of solitons from which such structures with a weak in-phase coupling can be constructed is equal to six. Figure 4 presents an example of a rotating structure with the central symmetry (the binary symmetry axis, $M = 2$) and Fig. 5 shows the rotation of a structure with the triad symmetry axis ($M = 3$).

(iv) *Structures without symmetry elements*. The minimal number of solitons required for constructing structures of this type is equal to seven. Figure 6 presents an example. Here, as in the general (nondegenerate) case, the absence of symmetry leads to the motion and rotation of the structure. The trajectory of the centre of inertia is a circle of radius

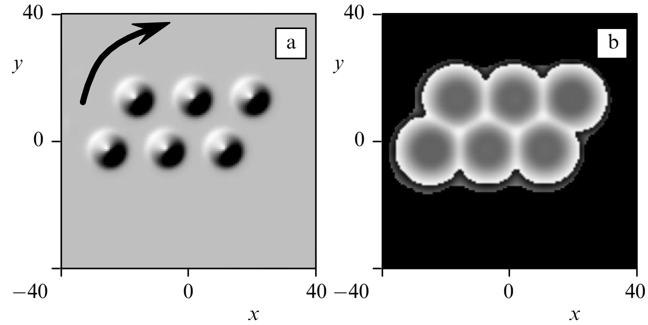

Figure 4. Intensity (a) and phase (b) distributions for a stable complex of six weakly coupled in-phase fundamental solitons; due to the presence of the central symmetry (the binary symmetry axis), the centre of inertia of the complex is immobile and the complex rotates in the direction shown by the arrow, with period $T \approx 135000$.

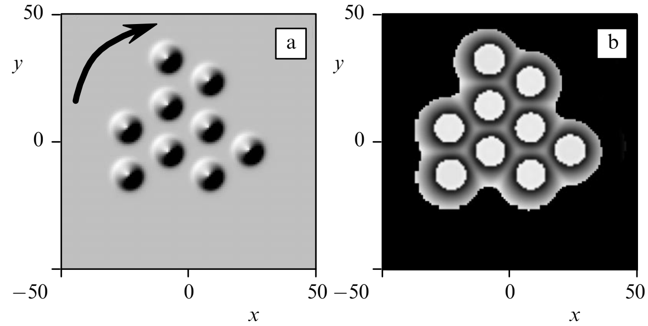

Figure 5. Intensity (a) and phase (b) distributions for a stable complex of weakly coupled in-phase fundamental solitons; due to the presence of the triad symmetry axis, the centre of inertia of the complex is immobile and the complex rotates in the direction shown by the arrow, with period $T \approx 210000$.

$R \approx 280$ (the trajectory in Fig. 6c differs from a circle due to the restricted calculation accuracy), the period of revolution along this circle is $T \approx 140000$. The structure performs a complete rotation for the same period (rotation of the type of the Moon movement around the Earth [6–8]).

4. Conclusions

Our calculations have shown that the four variants of motion are realised for complexes of weakly coupled in-phase solitons, including the translational motion at a constant velocity and circular motion of the centre of inertia at a constant linear velocity. Different types of motion correspond to the different symmetry of the transverse radiation intensity and phase distributions. The immobile soliton structures are stable at the same parameters, so that bifurcations accompanied by the disappearance of the stability of immobile solitons are absent. Because wavefront dislocations are also absent in this case, we can conclude that the motion is obviously caused by the asymmetry of the transverse field distribution. Therefore, the opinion that the wavefront dislocations play a decisive role in the dynamics of soliton complexes (see, for example, [9]) is not quite correct, and a more general criterion is related to the symmetry considerations. The latter are also substantial in the presence of dislocations. Indeed, symmetrically arranged dislocations do not cause the motion of a soliton complex. At the same time, the stable asymmetry of their arrangement leads to a considerable asymmetry of energy fluxes, and the velocities of motion of the complex noticeably exceed those observed in the absence of dislocations.

These results were obtained solving numerically master equation (1). In our opinion, at present the numerical results are most reliable. In particular, the known variant of the asymptotic theory of weak interaction of laser solitons [9] leads to the conclusion that the velocity of the centre of inertia of in-phase complexes in a stable regime is zero, which contradicts the simulation results. This conclusion suggests that distortions of the field profile of solitons due to their interaction with other solitons should be more exactly taken into account in the asymptotic theory. The possibility of introducing such corrections to the asymptotic theory is also confirmed by the fact that linear and angular velocities obtained for in-phase structures are considerably smaller than those obtained for complexes with solitons having different phases.

The motion of complexes of different types can find applications in optical data processing and logic operations

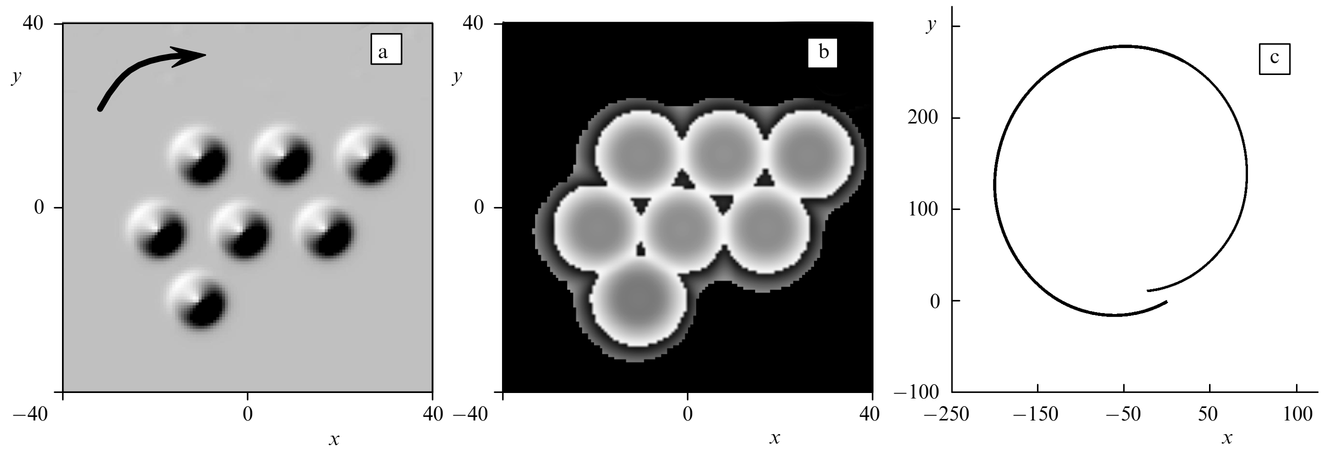

Figure 6. Intensity (a) and phase (b) distributions and the trajectory of the centre of inertia (c) for a stable asymmetric complex of weakly coupled in-phase fundamental solitons; the period of revolution of the centre of inertia along the circle $T \approx 140000$ coincides with the rotation period of the complex.

[4]. It seems that the most promising are semiconductor microcavities with quantum wells and dots [11]. The animation of the dynamics of a number of complexes of laser solitons can be found at web-site [12].

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