LASERS BEAMS

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Optical diabols in singular laser beams*

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Abstract. **Polarisation singularities and optical diabols in laser beams are studied experimentally by the developed method of digital Stokes-polarimetry. Three possible morphological forms of singularities with circular polarisation (C points) are found and their statistics in the speckle pattern of photoinduced scattering is determined. Singular fields with controllable parameters are realised and used for studying topological transformations, creation and annihilation of singularities and optical diabols. Both possible types of distributions of the major and minor semiaxes of polarisation ellipses in the beam cross section around C points (optical diabols) are measured. It is shown that, in accordance with the theory, the annihilation of pairs of elliptical diabols is preceded by their transformation to hyperbolic diabols.**

Keywords: *optical polarisation singularities, C points, optical diabols, Stokes polarimetry, topological responses.*

1. Introduction

It is known that optical singularities are contained in all the transverse modes of laser radiation [1, 2]. Singularities in complex linearly polarised laser beams represent points on the wave front of the beam in which the radiation intensity is strictly zero and the wave phase becomes uncertain. The wave front has discontinuities at such points and acquires the helical form in their vicinity with the formation of an optical vortex on it [1, 2]. Singularities can also exist in beams with the inhomogeneous elliptic polarisation [1, 3 – 5]. Thus, the major and minor axes of the polarisation ellipse are equal at the points with circular polarisation (C points) [1] and the polarisation azimuth (the angle between the major axis of this ellipse and the coordinate axis) becomes uncertain (singular). A helical structure is formed around each of these points in the distribution of the polarisation azimuth over the beam cross section [1, 3], but a change in the polarisation azimuth during going around the point is not uniform as opposed to a change in the phase of scalar fields.

The study of the role of polarisation singularities in the formation of the general structure of the field [3, 6] has shown that a set of singularities in conjunction with saddles form the unified topological skeleton of the field, which determines qualitatively the polarisation of light at all points of the wave front. A change of at least one of the elements of the topological skeleton leads to the reconstruction of the polarisation structure of the entire surrounding field [3]. We proposed earlier the method to control anisotropic perturbations, which was used to study experimentally the response of the topological skeleton of a beam to polarisation perturbations, including the creation or annihilation of the C points [7]. It was found that the topological skeleton preserved its integrity at any perturbations.

New topological structures have been found recently in the theoretical analysis and experimental measurements of the transverse distribution of the lengths of axes of polarisation ellipses in singular beams [8, 9]. It has been shown that the transverse distributions of the major and minor semiaxes of polarisation ellipses in the vicinity of the C point forms surfaces converging to each other as the singularity is approached. The point contact of these surfaces occurs at the C point itself. As a result, the so-called optical diabol appears in the form of two cones with a common apex and common axis tilted to the laser beam axis. Similar structures are formed in the vicinity of 'umbilical points' in random optical fields [10] (an umbilical point is a point at which the eigenvalues of the matrix of the curvature of the transverse intensity distribution surface are degenerate).

In this paper, we perform for the first time a complex experimental study of polarisation singularities and optical diabols produced from them. The creation and annihilation of C-point pairs and accompanying diabols are studied. The mechanisms of creation and annihilation of optical diabols established in [9] are confirmed by the example of singular laser beams, whose parameters are changed either 'manually' or vary in real time during the development of photoinduced scattering of laser radiation in a lithium niobate crystal. These studies not only extend our concept of laser beams but also have important applications because polarisation singularities appear, for example, in laser beams with smooth wave fronts propagating in a turbulent atmosphere. In addition, polarisation singularities and optical diabols can be used as new sensitive sensors of optical imperfections of media and elements interacting with laser beams.

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2. Method of the complex measurement of properties of singular elliptically polarised laser beams

To study experimentally the annihilation and creation of C-point pairs, singular laser beams are required with structural parameters that are changed in time by force or vary in the natural way. They were synthesised by using the coaxial summation of two coherent orthogonally polarised waves (Fig. 1) with linear or circular polarisation in a Mach–Zehnder interferometer [11].

A linearly polarised beam from a He–Ne laser was directed to a Mach–Zehnder interferometer formed by beamsplitters (*1*) and (*6*) and mirrors (*2*) and (*9*). The signal and reference waves are formed in the upper and lower arms of the interferometer, respectively. Half-wave plate (*7*) rotates the polarisation vector of the reference wave by 90°, providing the orthogonality of polarisations of the reference and signal waves. Telescopic system (*8*) is used to form the spatial mode of the reference wave. Polarisers (*5*) and (*10*) provide the high degree of polarisation of interfering beams. The polarisation of the resulting beam is studied by using Stokes analyser (*12*) [11, 12]. Complete information on the beam polarisation at each point of its cross section is determined by measuring the intensity distribution in four linearly polarised and two circularly polarised components of the field [11] recorded with a CCD camera. Then, polarisation parameters are calculated at each point of the analysed region of the field by using a special software package for data processing [13].

Parameters of optical diabols at each point of the wave front of the field are determined by the lengths of the major (a) and minor (b) semiaxes of the polarisation ellipse and are not related directly to the phase characteristics of this field. The semiaxes of the ellipse are calculated from the measured non-normalised Stokes parameters S_i according to the expressions [8]

$$a = \sqrt{S_0 + \sqrt{S_1^2 + S_2^2}},$$
$$b = \sqrt{S_0 - \sqrt{S_1^2 + S_2^2}}, \quad (1)$$

As the signal wave with controllable polarisation parameters, a dipole of optical vortices (DOV) with charges +1 and −1 generated by computer-synthesised hologram (*3*) was used [14, 15]. The axes of quarter-wave plate (*11*) were initially oriented at an angle of 45° (135°) to the electric field strength vector $\boldsymbol{E}$ in the corresponding arms of the interferometer. As a result, the interfering waves acquired orthogonal circular polarisation. As plate (*11*) is turned through some angle around the optical axis with respect to its initial position, both beams forming the field under study lose circular polarisation and the polarisation structure of the output beam changes. It is known [11] that a sum of two waves with orthogonal circular polarisations and equal intensities is a linearly polarised wave (or elliptically polarised wave in the case of initial waves with different intensities). Because of a complicated phase structure of the optical dipole, the resulting beam acquires inhomogeneous elliptic polarisation in the cross section. Singular C points are initially formed in the localisation regions of optical vortices, where the intensity of one of the circularly polarised components is zero. By rotating plate (*11*), we could study in detail the processes of creation and annihilation of singularities and optical diabols and corresponding topological transformations.

Of principal interest is the study of the creation and evolution of singularities and optical diabols in the situation when parameters of the signal beam change in time due to its interaction with a photorefractive medium. Such singular fields were obtained by placing photorefractive $LiNbO_3$ crystal (*4*) in the upper arm of interferometer instead of the hologram. The field produced upon the photoinduced scattering of the laser beam represents a random speckle pattern with the large amplitude and phase modulation [16]. Because components of the electrooptical tensor of the crystal considerably differ from each other, the scattered radiation was linearly polarised. The sum of the orthogonally polarised speckle pattern and the plane reference wave forms a field with an intricate structure containing polarisation singularities appearing if the amplitudes of the reference wave and speckle pattern are equal and the phase difference between them is $\pi/2$. In our case, polarisation singularities are distributed randomly. Due to the interaction of the signal beam with the photorefractive medium,

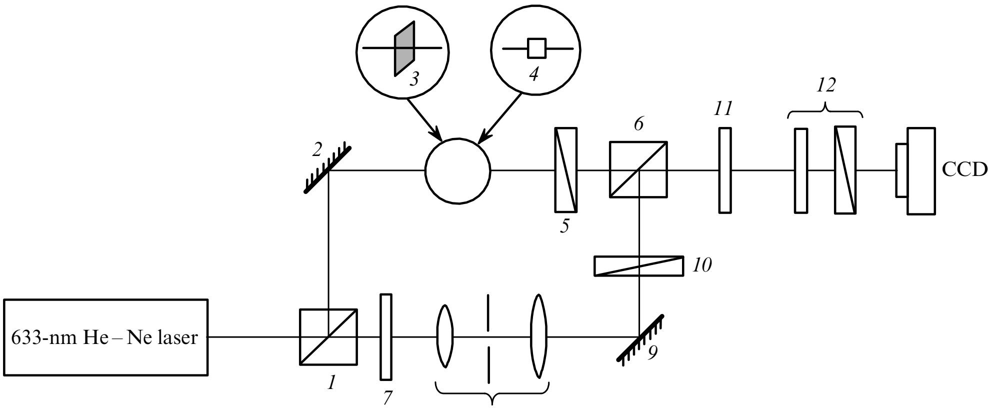

Figure 1. Scheme of the setup for formation of C points and optical diabols and studying their evolution with the help of a combined beam: (*1, 6*) beamsplitters; (*2, 9*) fold mirrors; (*3*) computer hologram; (*4*) lithium niobate photorefractive crystal; (*5, 10*) polarisers; (*7*) $\lambda/2$ plate; (*8*) telescope with the $12.5^{\times}$ magnification and a 20-μm point aperture in the focal plane; (*11*) $\lambda/4$ plate; (*12*) $\lambda/4$ plate and an analyser for measuring Stokes parameters.

the beam amplitude and phase structure change in time. Therefore, the polarisation structure of the resulting beam also changes and C-point pairs appear and annihilate.

Let us analyses the results obtained with the help of the methods described above.

3. Morphological forms of polarisation singularities and optical diabols in laser beams

A singularity of the wave front of a circularly polarised field corresponds to a pole of the Poincare sphere and is located at the intersection of the zero-value lines of the Stokes parameters S_1 and S_2 [13]. These parameters at all other points of the field do not vanish simultaneously. In this case, the polarisation azimuth at the C point is undetermined, i.e. a discontinuity appears in its distribution over the beam cross section. It has been shown in theoretical papers [1, 6] that only three morphological forms of the distribution of the polarisation azimuth over the beam cross section in the vicinity of C points can exist: Star, Lemon, and Monstar (Fig. 2). The polarisation azimuth α increases (decreases) by π with increasing the polar angle θ during a counterclockwise going around the C point. As a result, the topological singularity index, equal to the change in α in fractions of 2π after a complete going around the C point, is $1/2$ (Lemon or Monstar) or $-1/2$ (Star), respectively [1]. There exist several experimental methods for the recognition of the environment types of C points. The first of them is based on the analysis of signs of determinants proposed by Dennis [5], the second one is based on the determination of the number and position of points in which the polarisation azimuth satisfies the relation $\theta - \alpha = n\pi$ during going around the C point along a closed contour [12]. A set of such points as a function of the distance from the C point forms straight lines, along which the major axes of ellipses, with centres located on these straight lines, are oriented. These straight lines bend with distance from the singularity under the action of close polarisation inhomogeneities in the unified skeleton of the field.

Figure 2 shows examples of the three possible ordering types of polarisation ellipses around C points and two possible forms of optical diabols that we measured.

It is interesting to measure the statistical weights of the above-mentioned three morphological forms for random fields with a great number of C points. This allows one to judge independently the statistics of such fields. According to the theory [1], the frequencies S, L, M of the appearance of the C points of the corresponding type for the wave-front singularities with the Gaussian statistics are related by the expressions $S/(L+M) = 1$ and $M/(S+L) = 0.053$. Generally speaking, the statistics of the three morphological forms of C points in our case should differ from the presented values because polarisation singularities have new properties. Direct measurements of the statistics of C points of three types by using a sufficiently large data array are of great interest in this case. Such measurements were performed for 1056 C points in an elliptically polarised field obtained upon interference of a photoinduced speckle pattern with an orthogonally polarised plane reference wave. It was found that $S:L:M = 0.5:0.4:0.1$ with the average error of 2.3 %, which was determined by the fact that one of the C points in some C-point pairs was located outside the field region being tested. The realisation

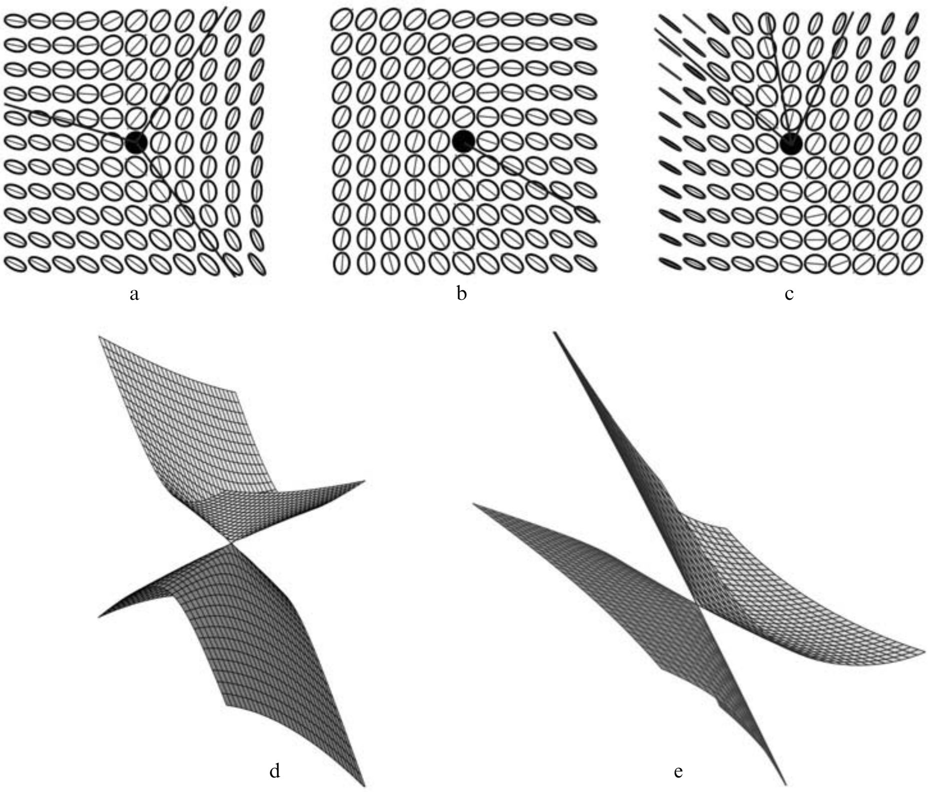

Figure 2. Possible types of distributions of the polarisation azimuth over the beam cross section in the vicinity of C points: Star (a), Lemon (b), and Monstar (c), and the possible forms of optical diabols: elliptic (d) and hyperbolic (e). Black circles are C points, black straight lines show directions along which $\theta - \alpha = n\pi$, where $n = -1, 0, 1, 2$.

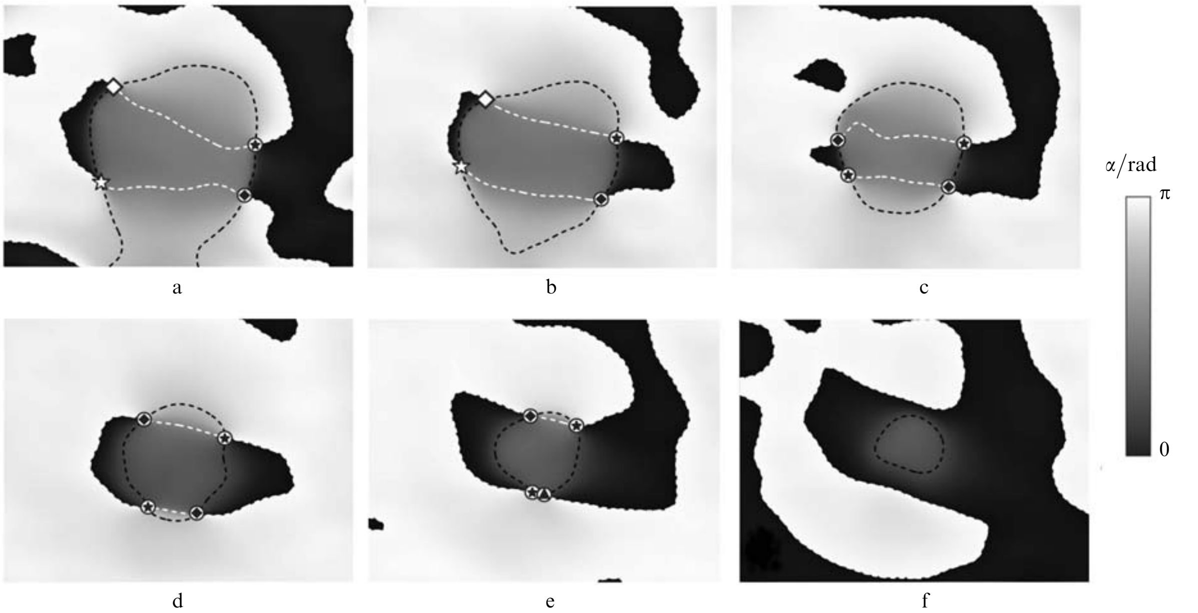

Figure 3. Fragments of the distribution of the polarisation azimuth in the cross section of a combined light beam with two C-point pairs for successive rotations (b – f) of a quarter-wave plate by 4° with respect to its initial position (a). Black and white dashed lines correspond to zero Stokes parameters S_1 and S_2, respectively. Asterisks, triangles, and rhombs are the C points of the types Star, Monstar, and Lemon, respectively, on which optical diabols are formed. The encircled C points correspond to singularities on which hyperbolic diabols are formed.

probability of the Monstar structures in singular fields is low because the structure of this type type is the intermediate form of the C-point environment appearing directly at the moment of creation (annihilation) of the C-point pair [1, 17, 18]. At the same time, the total measured frequency *M* (0.1) proved to be twice the value presented earlier (0.053) for the wave-front singularity.

The two measured forms of optical diabols (elliptic and hyperbolic) (Figs 2d, e) correspond mathematically to two possible forms of conical sections of diabols by the beam cross section. These sections for the elliptic diabol are ellipses, and cones are located on the opposite sides of the beam-cross section plane passing through the common apex of the cones (Fig. 2d). In the case of the hyperbolic diabol, cones are located simultaneously on both sides of the beam-cross section plane (Fig. 2e).

4. Evolution of pairs of singular C points and optical diabols with changing singular-field parameters

The evolution processes of creation and annihilation of C points and optical diabols are quite complicated and often proceed in many stages [9]. In this paper, we measured for the first time directly these processes with the help of singular fields with controllable parameters and in real time by using the changing speckle pattern of photoinduced scattering.

Consider the first case. Figure 3 demonstrates the annihilation process of a set of C points and optical diabols initiated by them with changing the field parameters. The upper pair of C points is located at the centres of DOVs, while the lower pair corresponds to satellites appeared at the DOV periphery. One can see that during the evolution of singularities two elliptic diabols transform to hyperbolic diabols (the left pair in Fig. 3c).

As the C points further approach each other, the singularity of the Lemon type transforms to the singularity of the Monstar type (Fig. 3e), which directly precedes the annihilation of a pair of singularities [1, 17, 18] and is caused by the continuity of the spatial distribution of the polarisation azimuth in the vicinity of the C point.

The theory shows [1, 18] that the direct annihilation of the elliptic and hyperbolic diabols is impossible. The transformation of elliptic diabols to hyperbolic ones occurs according to the rules of topological reactions presented in [9]. Figure 4 presents the contour maps of the distributions of the minor semiaxis of polarisation ellipses in the beam cross section. Figure 4a shows the vicinities of a pair of elliptic diabols. This pair transforms to a pair of hyperbolic diabols (Fig. 4b) with the participation of adjacent extrema and saddle points.

The experimental results presented here agree with the theoretical conclusion that singularities of the Lemon type

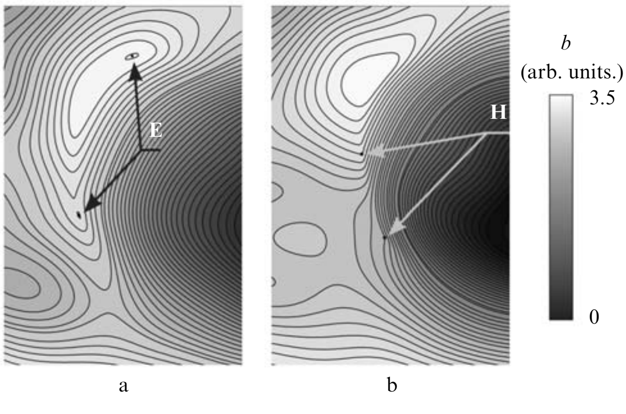

Figure 4. Distributions of the minor semiaxis *b* of the polarisation ellipse for beam polarisations changed by the successive rotation of a quarter-wave plate through 4°. The arrows show elliptic (E, a) and hyperbolic (H, b) diabols. The curves are the constant *b* value lines.

transform to the singularities of the Monstar type before the annihilation of the C-point pair. The assumption [1, 9] that the annihilation of the C-point pairs with the same type of diabols can occur is also confirmed.

5. Evolution of the C-point pairs and optical diabols with changing parameters of the singular field of radiation scattered in a photorefractive crystal

The presence of singularities in a beam leads to the additional rules of the transformation of the polarisation structure of the field caused by topological properties of singularities. They were studied with the help of time-dependent speckle patterns.

Consider the creation of a pair of C points and optical diabols initiated by them upon variation of the parameters of a random field. Figure 5 presents the typical stages of realisation of this process. At the left of Fig. 5 the contour maps of the distributions of the minor semiaxis of the polarisation ellipse in the beam cross section are presented and the position of C points and their type are indicated. At the right the distributions of the major and minor semiaxes of the polarisation ellipse are presented.

The pictures presented in Fig. 5 clearly demonstrate the intricate character of processes of creation and annihilation of optical diabols, which are accompanied by the reconstruction of the extema of the transverse distributions of semiaxes of the polarisation ellipse.

The moment of creation of C points is an instant event and its experimental study is a challenging problem. For this reason, we would have to analyse it by the field structure

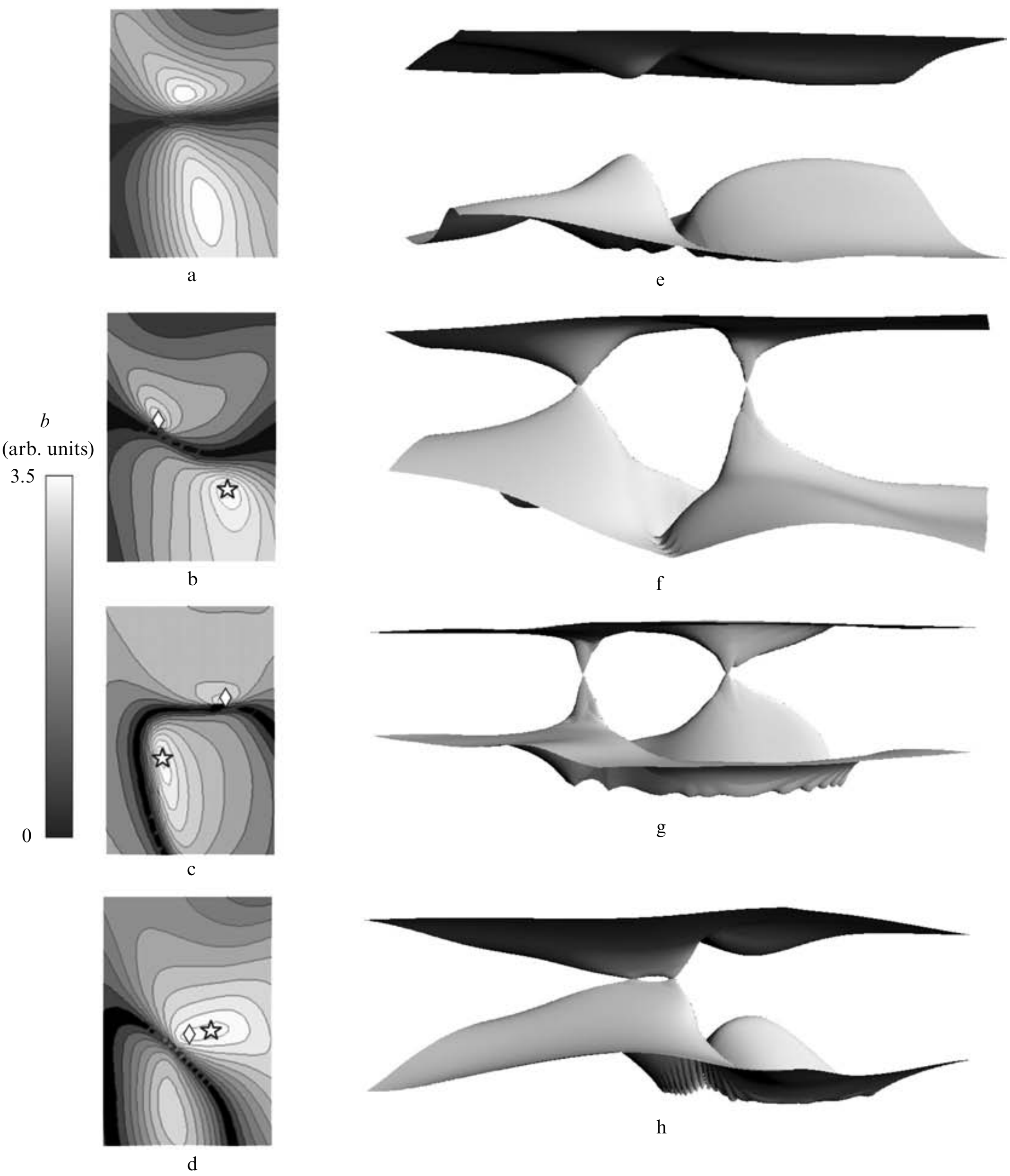

Figure 5. Fragments of the contour map of the distributions of the minor semiaxis of the polarisation ellipse in the cross section of a combined light beam (a – d) and distributions of the major and minor semiaxes of the polarisation ellipses for the same field regions in the volume representation (e – h). The asterisks and rhombs are the C points of the Star and Lemon types, respectively, on which optical diabols are formed.

before and after of the C point creation. We analysed the fragments of the field with sufficiently well isolated C points to avoid the identification of the movement of singularities as the creation of new singularities.

The creation of optical diabols is preceded by the appearance of a local maximum on the transverse distribution of the minor semiaxis and of the minimum on the surface of the transverse distribution of the major semiaxis (Figs 5a, e). Figures 5b, f present the same spatial region of the beam cross section after 1.5 min. One can see two created optical diabols separated by a distance of 150 μm. After next 1.5 min, the pair of optical diabols turned through an angle of 45° around the common axis (Figs 5c, g). Figures 5d, h demonstrate the formation of a structure required for the annihilation of diabols, in which one of the optical-diabol pairs is located in one extemium of the distribution. One can see that topological transformations of optical diabols are in good agreement with theoretical predictions [9].

6. Conclusions

By using the digital Stokes polarimetry, we have performed comprehensive measurements of the possible types of polarisation singularities and optical diabols in complex laser beams. The statistics of the morphological types of polarisation singularities in random speckle field has been studied. The spatial structure of optical diabols has been studied during their creation and deformation up to the moment preceding their annihilation. The high sensitivity of the structure and position of optical diabols to variations in objects transforming the optical field opens up the possibility of their using for the development of fundamentally new methods for precision diagnostics of optical imperfections of media and elements, the study of propagation of laser beams in the turbulent atmosphere, the coding and noise-suppressing data transfer by using laser beams.

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