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Wind profile recovery from intensity fluctuations of a laser beam reflected in a turbulent atmosphere

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***Abstract*.** **An algorithm for the wind profile recovery from spatiotemporal spectra of a laser beam reflected in a turbulent atmosphere is presented. The cases of a spherical wave incident on a diffuse reflector of finite size and a spatially limited beam reflected from an infinite random surface are considered.**

Keywords: *optical turbulence, laser beam, reflection, wind profile.*

1. Introduction

Speckle methods for studying materials and dynamics of liquid or gas flows have been developed in many papers. The intensity fluctuations of laser radiation scattered by or transmitted through a diffuse surface (suspended particles) detected in these methods give information on the motion of a diffuse object itself (particles) or of its parts (deformations). The results of investigations performed in this field are presented, for example, in monographs [1 – 3].

The possibility of determining the wind velocity averaged (integrated) over the path in the atmosphere by measuring the spatiotemporal correlation function of turbulent intensity fluctuations of light scattered by a chaotic surface was analysed in fact in the only paper [4]. We are not aware of any publications considering the problem of measuring the wind velocity profile in the atmosphere from turbulent fluctuations of reflected optical radiation by illuminating a scattering surface.

In this paper, we analyse the possibilities of the wind profile recovery from turbulent intensity fluctuations of laser radiation propagated in the atmosphere after reflection. Expressions for the spatiotemporal correlation function of intensity are obtained and an algorithm for the wind profile recovery is proposed.

2. Formulation of the problem

Let us assume that a laser source located in the plane $x' = x_0$ illuminates a diffusely scattering surface located in the plane $x' = x$ and the reflected radiation is received by a computer-aided detector array in the source plane $x' = x_0$. The sequences of realisations of two-dimensional intensity distributions with a repetition rate $f = 1/\tau$ (τ is the time between exposures) in the digitised form are subjected to the spectral correlation processing. The scheme is presented in Fig. 1.

We assume that the field of the laser source in the plane $x' = x_0$ is specified by the Gaussian model

$$U_0(\boldsymbol{\rho}) = U_0 \exp\left[-\frac{\rho^2}{2a^2} - \mathrm{i}k\frac{\rho^2}{2F}\right]$$

$$= U_0 \exp\left[-\frac{\rho^2}{2a^2}\left(1 + \mathrm{i}\Omega\frac{L}{F}\right)\right], \qquad (1)$$

where $\Omega = ka^2/L$; a and F are the beam radius and the radius of curvature of the phase front at the centre of transmitting aperture, respectively; U_0 is the field amplitude on the beam axis; $k = 2\pi/\lambda$; and $L = x - x_0$ is the beam path length. The field incident on the reflecting surface in the plane $x' = x$ is written in the form [5]

$$U(x, \boldsymbol{r}) = \int \mathrm{d}\boldsymbol{\rho}' U_0(\boldsymbol{\rho}') G(x, \boldsymbol{r}; x_0, \boldsymbol{\rho}'), \qquad (2)$$

where

$$G(x, \boldsymbol{r}; x_0, \boldsymbol{\rho}') = \frac{k}{2\pi\mathrm{i}(x - x_0)} \exp\left[\frac{\mathrm{i}k}{2(x - x_0)}(\boldsymbol{r} - \boldsymbol{\rho}')^2\right]$$

$$\times \lim_{N\to\infty}\left[\frac{k}{2\pi\mathrm{i}(x - x_0)}\right]^{N-1} \int \mathrm{d}\boldsymbol{S}_1 ... \mathrm{d}\boldsymbol{S}_{N-1} \exp\left[\frac{\mathrm{i}k}{2(x - x_0)} \times\right.$$

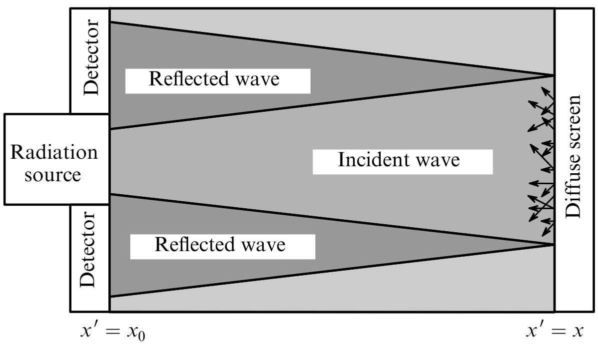

Figure 1. Geometry of the problem.

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$$\times \sum_{j=1}^{N-1} \boldsymbol{S}_j^2 + \mathrm{i}\frac{k}{2}\int_{x_0}^{x} \mathrm{d}x'\varepsilon_1\left(x', (1-\xi)\boldsymbol{\rho}' + \xi\boldsymbol{r} + \sum_{j=1}^{N-1} v_j(\xi)\boldsymbol{S}_j\right)\Bigg] \quad (3)$$

is the Green function [6, 7]; $\varepsilon_1(x', \boldsymbol{\rho}', \boldsymbol{r}, \boldsymbol{S}_j)$ is the part of the permittivity of the medium fluctuating due to turbulence; $\boldsymbol{\rho}'$, $\boldsymbol{r}$, and $\boldsymbol{S}_j$ are two-dimensional vectors in the transverse plane; integration is performed along the beam propagation direction; $\xi = (x' - x_0)/(x - x_0)$; and

$$v_j(\xi) = \sin(j\pi\xi)\left[\sqrt{2}N\sin\left(\frac{j\pi}{2N}\right)\right]^{-1}.$$

The wave formed on the reflecting surface due to scattering is described by the expression [5]

$$U_{\mathrm{r}}(x, \boldsymbol{r}') = \int \mathrm{d}\boldsymbol{r} V(\boldsymbol{r}, \boldsymbol{r}')U(x, \boldsymbol{r}),$$

where $V(\boldsymbol{r}, \boldsymbol{r}')$ is the reflection coefficient. Taking into account the reciprocity theorem for the Green function of the wave propagating in the forward and backward directions, the field received in the plane $x' = x_0$ has the form

$$U_{\mathrm{R}}(x_0, \boldsymbol{\rho}) = \int \mathrm{d}\boldsymbol{r}' U_{\mathrm{r}}(x, \boldsymbol{r}')G(x, \boldsymbol{r}; x_0, \boldsymbol{\rho})$$

$$= \int \mathrm{d}\boldsymbol{r}\mathrm{d}\boldsymbol{r}'\mathrm{d}\boldsymbol{\rho}' U_0(\boldsymbol{\rho}')V(\boldsymbol{r}, \boldsymbol{r}')G(x, \boldsymbol{r}; x_0, \boldsymbol{\rho}')G(x, \boldsymbol{r}'; x_0, \boldsymbol{\rho}) .$$

This gives the radiation intensity in the plane $x' = x_0$

$$I_{\mathrm{R}}(x_0, \boldsymbol{\rho}) = U_{\mathrm{R}}(x_0, \boldsymbol{\rho})U_{\mathrm{R}}^*(x_0, \boldsymbol{\rho}) = \int \mathrm{d}\boldsymbol{r}_{1,2}\mathrm{d}\boldsymbol{r}'_{1,2}\mathrm{d}\boldsymbol{\rho}'_{1,2}U_0(\boldsymbol{\rho}'_1)$$

$$\times U_0^*(\boldsymbol{\rho}'_2)V(\boldsymbol{r}_1, \boldsymbol{r}'_1)V^*(\boldsymbol{r}_2, \boldsymbol{r}'_2)G(x, \boldsymbol{r}_1; x_0, \boldsymbol{\rho}'_1)$$

$$\times G^*(x, \boldsymbol{r}_2; x_0, \boldsymbol{\rho}'_2)G(x, \boldsymbol{r}'_1; x_0, \boldsymbol{\rho})G^*(x, \boldsymbol{r}'_2; x_0, \boldsymbol{\rho}), \quad (4)$$

and we obtain the product of radiation intensities at points with radius vectors $\boldsymbol{\rho}_1$ and $\boldsymbol{\rho}_2$ at the instants $t = 0$ and $t = \tau$

$$I_{\mathrm{R}}(x_0, \boldsymbol{\rho}_1, 0)I_{\mathrm{R}}(x_0, \boldsymbol{\rho}_2, \tau) = \int \mathrm{d}\boldsymbol{r}_{1-4}\mathrm{d}\boldsymbol{r}'_{1-4}\mathrm{d}\boldsymbol{\rho}'_{1-4}U_0(\boldsymbol{\rho}'_1)U_0^*(\boldsymbol{\rho}'_2)$$

$$\times U_0(\boldsymbol{\rho}'_3)U_0^*(\boldsymbol{\rho}'_4)V(\boldsymbol{r}_1, \boldsymbol{r}'_1)V^*(\boldsymbol{r}_2, \boldsymbol{r}'_2)V(\boldsymbol{r}_3, \boldsymbol{r}'_3)$$

$$\times V^*(\boldsymbol{r}_4, \boldsymbol{r}'_4)\left[\frac{k}{2\pi(x - x_0)}\right]^8$$

$$\times \exp\left\{\frac{\mathrm{i}k}{2(x - x_0)}\left[(\boldsymbol{r}_1 - \boldsymbol{\rho}'_1)^2 - (\boldsymbol{r}_2 - \boldsymbol{\rho}'_2)^2 + (\boldsymbol{r}'_1 - \boldsymbol{\rho}_1)^2\right.\right.$$

$$- (\boldsymbol{r}'_2 - \boldsymbol{\rho}_1)^2 + (\boldsymbol{r}_3 - \boldsymbol{\rho}'_3)^2 - (\boldsymbol{r}_4 - \boldsymbol{\rho}'_4)^2 + (\boldsymbol{r}'_3 - \boldsymbol{\rho}_2)^2$$

$$\left.\left.- (\boldsymbol{r}'_4 - \boldsymbol{\rho}_2)^2\right]\right\} \lim_{N\to\infty}\left[\frac{k}{2\pi(x - x_0)}\right]^{8(N-1)}$$

$$\times \int \mathrm{d}\boldsymbol{a}_{1\ldots N-1}\mathrm{d}\boldsymbol{b}_{1\ldots N-1}\mathrm{d}\boldsymbol{c}_{1\ldots N-1}\mathrm{d}\boldsymbol{e}_{1\ldots N-1}\mathrm{d}\boldsymbol{f}_{1\ldots N-1}\mathrm{d}\boldsymbol{g}_{1\ldots N-1}\mathrm{d}\boldsymbol{h}_{1\ldots N-1}$$

$$\times \mathrm{d}\boldsymbol{t}_{1\ldots N-1}\exp\left[\frac{\mathrm{i}k}{2(x - x_0)}\sum_{j=1}^{N-1}(a_j^2 + b_j^2 - c_j^2 - e_j^2 + f_j^2 + g_j^2\right.$$

$$\left.- h_j^2 - t_j^2)\right]\exp\left\{\frac{\mathrm{i}k}{2}\int_{x_0}^{x}\mathrm{d}x'[\varepsilon_1(x', \boldsymbol{\rho}'_1, \boldsymbol{r}_1, \boldsymbol{a}_j; 0) -\right.$$

$$- \varepsilon_1(x', \boldsymbol{\rho}'_2, \boldsymbol{r}_2, \boldsymbol{c}_j; 0) + \varepsilon_1(x', \boldsymbol{\rho}_1, \boldsymbol{r}'_1, \boldsymbol{b}_j; 0) - \varepsilon_1(x', \boldsymbol{\rho}_1, \boldsymbol{r}'_2, \boldsymbol{e}_j; 0)$$

$$+ \varepsilon_1(x', \boldsymbol{\rho}'_3, \boldsymbol{r}_3, \boldsymbol{f}_j; \tau) - \varepsilon_1(x', \boldsymbol{\rho}'_4, \boldsymbol{r}_4, \boldsymbol{h}_j; \tau)$$

$$\left.+ \varepsilon_1(x', \boldsymbol{\rho}_2, \boldsymbol{r}'_3, \boldsymbol{g}_j; \tau) - \varepsilon_1(x', \boldsymbol{\rho}_2, \boldsymbol{r}'_4, \boldsymbol{t}_j; \tau)]\right\}. \quad (5)$$

The second spatiotemporal statistical intensity moment is obtained after averaging (5). We assume that the surface is diffuse, with the reflection coefficient varying randomly in time and space in the general case. We also assume that fluctuations of the reflection coefficient and refractive index in the atmosphere are independent. Then, averaging in (5) over the reflection coefficient and turbulent fluctuations ε_1 can be performed separately. By assuming that the correlation time of the reflection coefficient is shorter than the time τ between exposures, we obtain the expression

$$\langle V(\boldsymbol{r}_1, \boldsymbol{r}'_1)V^*(\boldsymbol{r}_2, \boldsymbol{r}'_2)V(\boldsymbol{r}_3, \boldsymbol{r}'_3)V^*(\boldsymbol{r}_4, \boldsymbol{r}'_4)\rangle$$

$$= \langle V(\boldsymbol{r}_1, \boldsymbol{r}'_1)V^*(\boldsymbol{r}_2, \boldsymbol{r}'_2)\rangle\langle V(\boldsymbol{r}_3, \boldsymbol{r}'_3)V^*(\boldsymbol{r}_4, \boldsymbol{r}'_4)\rangle \quad (6)$$

for the fourth statistical moment of the reflection coefficient [5], where

$$\langle V(\boldsymbol{r}_i, \boldsymbol{r}'_i)V^*(\boldsymbol{r}_l, \boldsymbol{r}'_l)\rangle$$

$$= (4\pi/k^2)|A(\boldsymbol{r}_i)|^2\delta(\boldsymbol{r}_i - \boldsymbol{r}_l)\delta(\boldsymbol{r}_i - \boldsymbol{r}'_i)\delta(\boldsymbol{r}_l - \boldsymbol{r}'_l); \quad (7)$$

$A(\boldsymbol{r}_i)$ are the amplitudes of the reflection coefficient and angle brackets denote ensemble averaging.

We will perform the averaging over turbulent fluctuations of the permittivity [the last exponential in (5)] by assuming that the probability density for the integral of the fluctuating part of the permittivity over the beam path is described by a Gaussian [8]

$$\langle\exp(\mathrm{i}\varphi)\rangle = \exp\left(-\frac{1}{2}\langle\varphi^2\rangle\right). \quad (8)$$

We represent the square of the integrand in the last exponential in (5) appearing upon averaging as a sum of squares of differences of functions ε_1 in pairs with the sign '+' in front of them if functions ε_1 have different signs and the sign '−' if these functions have the same signs. As a result, we obtain in the last exponential in (5) a sum of twenty-eight spatiotemporal structural functions of the permittivity of the type

$$\langle[\varepsilon_1(x', \boldsymbol{\rho}_i, \boldsymbol{r}_l, \boldsymbol{a}_j; t_1) - \varepsilon_1(x', \boldsymbol{\rho}'_k, \boldsymbol{r}'_m, \boldsymbol{b}_j; t_2)]^2\rangle, \quad (9)$$

in which averaging is performed by assuming that the fluctuations of ε_1 over the coordinate x' are delta-correlated [8],

$$\langle[\varepsilon_1(x', \boldsymbol{\rho}_i, \boldsymbol{r}_l, \boldsymbol{a}_j; t_1) - \varepsilon_1(x', \boldsymbol{\rho}'_k, \boldsymbol{r}'_m, \boldsymbol{b}_j; t_2)]^2\rangle$$

$$= 2\pi\int \mathrm{d}\boldsymbol{\kappa}\Phi_\varepsilon(\xi, \boldsymbol{\kappa})\Bigg[1 - \exp\Bigg\{\mathrm{i}\boldsymbol{\kappa}\Bigg[(\boldsymbol{\rho}_i - \boldsymbol{\rho}'_k)(1 - \xi) + (\boldsymbol{r}_l - \boldsymbol{r}'_m)\xi$$

$$+ \sum_{j=1}^{N-1} v_j(\xi)(\boldsymbol{a}_i - \boldsymbol{b}_j) + \boldsymbol{V}(x')(t_1 - t_2)\Bigg]\Bigg\}\Bigg] =$$

$$= H(\boldsymbol{\rho}_i - \boldsymbol{\rho}'_k, \boldsymbol{r}_l - \boldsymbol{r}'_m, \boldsymbol{a}_j - \boldsymbol{b}_j, t_1 - t_2), \tag{10}$$

where $\boldsymbol{\kappa}$ is the two-dimensional vector of the spatial frequency; $\Phi_\varepsilon(\xi, \boldsymbol{\kappa})$ is the three-dimensional vector of the permittivity fluctuations; and $\boldsymbol{V}(x')$ is the wind velocity. The averaging in (10) was performed by using the Taylor hypothesis of the frozen turbulence [9]

$$\varepsilon_1(x', \boldsymbol{\rho}; \tau) = \varepsilon_1(x', \boldsymbol{\rho} - \boldsymbol{V}(x')\tau; 0). \tag{11}$$

After substitution of expressions (6) and (7) into (5), averaging over an ensemble of turbulent fluctuations of the permittivity by using (9) and (10) and integrating by using the properties of delta functions, the second spatiotemporal statistical intensity moment can be represented in the form

$$\langle I_{\mathrm{R}}(x_0, \boldsymbol{\rho}_1; 0) I_{\mathrm{R}}(x_0, \boldsymbol{\rho}_2, \tau)\rangle$$

$$= \left(\frac{4\pi}{k^2}\right)^2 \int \mathrm{d}\boldsymbol{R}\mathrm{d}\boldsymbol{r}\mathrm{d}\boldsymbol{\rho}'_{1-4} \left|A\left(\boldsymbol{R} + \frac{\boldsymbol{r}}{2}\right)\right|^2 \left|A\left(\boldsymbol{R} - \frac{\boldsymbol{r}}{2}\right)\right|^2$$

$$\times U_0(\boldsymbol{\rho}'_1) U_0^*(\boldsymbol{\rho}'_2) U_0(\boldsymbol{\rho}'_3) U_0^*(\boldsymbol{\rho}'_4) \left[\frac{k}{2\pi(x - x_0)}\right]^8$$

$$\times \exp\left\{\frac{\mathrm{i}k}{2(x - x_0)}\left[\boldsymbol{\rho}_1'^2 - \boldsymbol{\rho}_2'^2 + \boldsymbol{\rho}_3'^2 - \boldsymbol{\rho}_4'^2 - 2\boldsymbol{R}(\boldsymbol{\rho}'_1 - \boldsymbol{\rho}'_2\right.\right.$$

$$\left.\left.+ \boldsymbol{\rho}'_3 - \boldsymbol{\rho}'_4) - \boldsymbol{r}(\boldsymbol{\rho}'_1 - \boldsymbol{\rho}'_2 - \boldsymbol{\rho}'_3 + \boldsymbol{\rho}'_4)\right\} \lim_{N\to\infty} \left[\frac{k}{2\pi(x - x_0)}\right]^{8(N-1)}$$

$$\times \int \mathrm{d}\boldsymbol{a}_{1\ldots N-1} \mathrm{d}\boldsymbol{b}_{1\ldots N-1} \mathrm{d}\boldsymbol{c}_{1\ldots N-1} \mathrm{d}\boldsymbol{e}_{1\ldots N-1} \mathrm{d}\boldsymbol{f}_{1\ldots N-1} \mathrm{d}\boldsymbol{g}_{1\ldots N-1} \mathrm{d}\boldsymbol{h}_{1\ldots N-1}$$

$$\times \mathrm{d}\boldsymbol{t}_{1\ldots N-1} \exp\left[\frac{\mathrm{i}k}{2(x - x_0)} \sum_{j=1}^{N-1} \left(a_j^2 + b_j^2 - c_j^2 - e_j^2 + f_j^2 + g_j^2\right.\right.$$

$$\left.\left. - h_j^2 - t_j^2\right)\right] \exp\left\{-\frac{\pi k^2}{2} \int_{x_0}^{x} \mathrm{d}x' [H(\boldsymbol{\rho}'_1 - \boldsymbol{\rho}'_2, \boldsymbol{r}, \boldsymbol{a}_j - \boldsymbol{c}_j, 0)\right.$$

$$- H(\boldsymbol{\rho}'_1 - \boldsymbol{\rho}_1, \boldsymbol{r}, \boldsymbol{a}_j - \boldsymbol{b}_j, 0) + H(\boldsymbol{\rho}'_1 - \boldsymbol{\rho}_1, \boldsymbol{r}, \boldsymbol{a}_j - \boldsymbol{e}_j, 0)$$

$$- H(\boldsymbol{\rho}'_1 - \boldsymbol{\rho}'_3, \boldsymbol{r}, \boldsymbol{a}_j - \boldsymbol{f}_j, -\tau) + H(\boldsymbol{\rho}'_1 - \boldsymbol{\rho}'_4, \boldsymbol{r}, \boldsymbol{a}_j - \boldsymbol{h}_j, -\tau)$$

$$- H(\boldsymbol{\rho}'_1 - \boldsymbol{\rho}_2, \boldsymbol{r}, \boldsymbol{a}_j - \boldsymbol{g}_j, -\tau) + H(\boldsymbol{\rho}'_1 - \boldsymbol{\rho}_2, \boldsymbol{r}, \boldsymbol{a}_j - \boldsymbol{t}_j, -\tau)$$

$$+ H(\boldsymbol{\rho}'_2 - \boldsymbol{\rho}_1, \boldsymbol{r}, \boldsymbol{c}_j - \boldsymbol{b}_j, 0) - H(\boldsymbol{\rho}'_2 - \boldsymbol{\rho}_1, \boldsymbol{r}, \boldsymbol{c}_j - \boldsymbol{e}_j, 0)$$

$$+ H(\boldsymbol{\rho}'_2 - \boldsymbol{\rho}'_3, \boldsymbol{r}, \boldsymbol{c}_j - \boldsymbol{f}_j, -\tau) - H(\boldsymbol{\rho}'_2 - \boldsymbol{\rho}'_4, \boldsymbol{r}, \boldsymbol{c}_j - \boldsymbol{h}_j, -\tau)$$

$$+ H(\boldsymbol{\rho}'_2 - \boldsymbol{\rho}_2, \boldsymbol{r}, \boldsymbol{c}_j - \boldsymbol{g}_j, -\tau) - H(\boldsymbol{\rho}'_2 - \boldsymbol{\rho}_2, \boldsymbol{r}, \boldsymbol{c}_j - \boldsymbol{t}_j, -\tau)$$

$$+ H(0, \boldsymbol{r}, \boldsymbol{b}_j - \boldsymbol{e}_j, 0) - H(\boldsymbol{\rho}_1 - \boldsymbol{\rho}'_3, \boldsymbol{r}, \boldsymbol{b}_j - \boldsymbol{f}_j, -\tau)$$

$$+ H(\boldsymbol{\rho}_1 - \boldsymbol{\rho}'_4, \boldsymbol{r}, \boldsymbol{b}_j - \boldsymbol{h}_j, -\tau) - H(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2, \boldsymbol{r}, \boldsymbol{b}_j - \boldsymbol{g}_j, -\tau)$$

$$+ H(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2, \boldsymbol{r}, \boldsymbol{b}_j - \boldsymbol{t}_j, -\tau) + H(\boldsymbol{\rho}_1 - \boldsymbol{\rho}'_3, \boldsymbol{r}, \boldsymbol{e}_j - \boldsymbol{f}_j, -\tau)$$

$$- H(\boldsymbol{\rho}_1 - \boldsymbol{\rho}'_4, \boldsymbol{r}, \boldsymbol{e}_j - \boldsymbol{h}_j, -\tau) + H(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2, \boldsymbol{r}, \boldsymbol{e}_j - \boldsymbol{g}_j, -\tau)$$

$$- H(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2, \boldsymbol{r}, \boldsymbol{e}_j - \boldsymbol{t}_j, -\tau) + H(\boldsymbol{\rho}'_3 - \boldsymbol{\rho}'_4, \boldsymbol{r}, \boldsymbol{f}_j - \boldsymbol{h}_j, 0) -$$

$$- H(\boldsymbol{\rho}'_3 - \boldsymbol{\rho}_2, \boldsymbol{r}, \boldsymbol{f}_j - \boldsymbol{g}_j, 0) + H(\boldsymbol{\rho}'_3 - \boldsymbol{\rho}_2, \boldsymbol{r}, \boldsymbol{f}_j - \boldsymbol{t}_j, 0)$$

$$+ H(\boldsymbol{\rho}'_4 - \boldsymbol{\rho}_2, \boldsymbol{r}, \boldsymbol{h}_j - \boldsymbol{g}_j, 0) - H(\boldsymbol{\rho}'_4 - \boldsymbol{\rho}_2, \boldsymbol{r}, \boldsymbol{h}_j - \boldsymbol{t}_j, 0)$$

$$\left. + H(0, \boldsymbol{r}, \boldsymbol{g}_j - \boldsymbol{t}_j, 0)]\right\}. \tag{12}$$

Expression (12) is rigorous within the framework of the modern theory of propagation of short waves in random media. It represents in the general form the spatiotemporal correlation function of intensity fluctuations of a Gaussian beam reflected from a diffuse surface under arbitrary turbulent conditions of propagation in the atmosphere. The only restriction is the assumption that the reflection coefficient of the surface (6) is constant in the calculation of correlations of the sequence of exposures (frames) of the intensity distributions.

Below, we consider the regime of a weak optical turbulence in the approximation of the Kolmogorov spectrum $\Phi_\varepsilon(\boldsymbol{\kappa})$, when the parameter $\beta_0^2 = 1.23 C_n^2 k^{7/6} x^{11/6}$ characterising the turbulent conditions of propagation (x is the beam path length, C_n^2 is the structural characteristic of fluctuations of the refractive index) does not exceed unity. The parameter β_0^2 stands in front of the integral in the last exponential in (12). Under the condition $\beta_0^2 < 1$, this exponential can be expanded into a Taylor series by retaining only the two first terms of the series.

Let us distinguish two diffraction propagation regimes: (i) a spherical wave illuminates a diffuse reflector of finite size and (ii) a spatially limited beam is reflected from an infinite diffuse surface. This allows us to simplify considerably expression (12) for the correlation function and construct the algorithm for recovering the wind velocity profile.

The initial field is specified in calculations by using model (1), and the amplitude of the reflection coefficient is written in the form [5]

$$A(\boldsymbol{R}) = A_0 \exp\left\{-\frac{R^2}{2a_{\mathrm{r}}^2}\right\}, \tag{13}$$

where $\boldsymbol{R}$ is the two-dimensional vector lying in the reflection plane; A_0 the amplitude at the centre of the reflector; and a_{r} is the effective radius of the reflector corresponding to the e^{-1} level of the squared amplitude.

3. Spherical wave – reflector of finite size

By substituting (1) and (13) into (12) and assuming that $\Omega \to 0$, which corresponds to the passage to the spherical weave regime, and expanding the last exponential in (12) into a series by retaining the first two terms, we find the spatiotemporal correlation function of intensity

$$K_I(x_0, \boldsymbol{\rho}_1, \boldsymbol{\rho}_2; \tau) = \langle I_{\mathrm{R}}(x_0, \boldsymbol{\rho}_1, 0) I_{\mathrm{R}}(x_0, \boldsymbol{\rho}_2, \tau)\rangle$$

$$- \langle I_{\mathrm{R}}(x_0, \boldsymbol{\rho}_1, 0)\rangle \langle I_{\mathrm{R}}(x_0, \boldsymbol{\rho}_2, \tau)\rangle \tag{14}$$

in the form

$$K_I(x_0, \boldsymbol{\rho}_1, \boldsymbol{\rho}_2; \tau) = A \int \mathrm{d}\boldsymbol{\kappa} \int_{x_0}^{x} \mathrm{d}x' \Phi_\varepsilon(x', \boldsymbol{\kappa}) \exp\left(-\frac{\xi^2 a_{\mathrm{r}}^2 \boldsymbol{\kappa}^2}{2}\right) \times$$

$$\times \sin^2\left[\frac{L}{2k}\boldsymbol{\kappa}^2\xi(1-\xi)\right]\{1+\exp[\mathrm{i}\boldsymbol{\kappa}\boldsymbol{\rho}_1(1-\xi)]\}$$

$$\times\{1+\exp[-\mathrm{i}\boldsymbol{\kappa}\boldsymbol{\rho}_2(1-\xi)]\}\exp[\mathrm{i}\boldsymbol{V}(\xi)\boldsymbol{\kappa}\tau], \quad (15)$$

where A is a numerical constant. Expression (15) for $\tau = 0$ coincides with results [5] obtained for the spatial correlation function of intensity for a spherical wave reflected from a diffuse reflector in a turbulent atmosphere for $\beta_0^2 < 1$.

Let us assume that $\boldsymbol{\rho}_1 = 0$ and consider the difference of values of the correlation function at points with radius vectors $\boldsymbol{\rho}_2$ and $-\boldsymbol{\rho}_2$

$$D_I(x_0,\boldsymbol{\rho},\tau) = K_I(x_0,0,\boldsymbol{\rho},\tau) - K_I(x_0,0,-\boldsymbol{\rho},\tau) =$$

$$= 4\mathrm{i}A\int \mathrm{d}\boldsymbol{\kappa}\int_{x_0}^{x}\mathrm{d}x'\Phi_\varepsilon(x',\boldsymbol{\kappa})\exp\left(-\frac{\xi^2a_\mathrm{r}^2\boldsymbol{\kappa}^2}{2}\right)$$

$$\times\sin^2\left[\frac{x-x_0}{2k}\boldsymbol{\kappa}^2\xi(1-\xi)\right]\sin[\boldsymbol{\kappa}\boldsymbol{\rho}(1-\xi)]\exp[\mathrm{i}\boldsymbol{V}(\xi)\boldsymbol{\kappa}\tau]. \quad (16)$$

It follows from (16) that, as the size of a scattering surface is increased ($a_\mathrm{r} \to \infty$), fluctuations of the reflected spherical wave decrease to zero due to averaging, in accordance with results [5].

Consider now the spatiotemporal spectrum in the region of spatial frequencies q_z, $q_y > 0$,

$$\tilde{F}_I(\boldsymbol{q},\omega) = \frac{1}{(2\pi)^3}\int\mathrm{d}\boldsymbol{\rho}\int\mathrm{d}\tau\exp(\mathrm{i}\boldsymbol{\rho}\boldsymbol{q}-\mathrm{i}\omega\tau)D_I(x_0,\boldsymbol{\rho},\tau)$$

$$= 2A\int_{x_0}^{x}\mathrm{d}x'\Phi_\varepsilon\left(x',\frac{\boldsymbol{q}}{1-\xi}\right)\exp\left(-\frac{\xi^2a_\mathrm{r}^2q^2}{2(1-\xi)^2}\right)$$

$$\times\sin^2\left(\frac{x-x_0}{2k}q^2\frac{\xi}{1-\xi}\right)\frac{1}{(1-\xi)^2}\delta\left(\omega-\frac{\boldsymbol{V}(\xi)\boldsymbol{q}}{1-\xi}\right). \quad (17)$$

For the power Kolmogorov spectrum of permittivity fluctuations $\Phi_\varepsilon(x',\boldsymbol{\kappa}) = 0.132C_n^2(x')\boldsymbol{\kappa}^{-11/3}$, the relation $\Phi_\varepsilon(x',\boldsymbol{q}/(1-\xi)) = \Phi_\varepsilon(x',\boldsymbol{q})(1-\xi)^{11/3}$ is valid. Let us set $\boldsymbol{q} = q_i\boldsymbol{e}_i$ ($\boldsymbol{e}_i$ is the unit vector of the Cartesian coordinate system), $\boldsymbol{V}(\xi)\boldsymbol{q} = V_i(\xi)q_i$, and $\alpha = \omega/q_i$ is the ratio of the time and spatial frequencies. Then, the normalised spectrum

$$f(q_i,\alpha) = \frac{\tilde{F}_I(q_i\boldsymbol{e}_i,\alpha q_i)}{0.132Aq_i^{-17/3}}$$

can be written in the form

$$f(q_i,\alpha) = 2\int_{x_0}^{x}\mathrm{d}x'C_n^2(x')\exp\left[-\frac{\xi^2a_\mathrm{r}^2q^2}{2(1-\xi)^2}\right]$$

$$\times\sin^2\left(\frac{x-x_0}{2k}q_i^2\frac{\xi}{1-\xi}\right)(1-\xi)^{5/3}q_i\,\delta\left[\alpha-\frac{V_i(\xi)}{1-\xi}\right]. \quad (18)$$

Let us calculate the integral

$$g(\gamma,\alpha) = \int_0^\infty f(q_i,\alpha)\exp(\mathrm{i}\gamma q_i^2)\mathrm{d}q_i = \frac{k}{2L}\int_{x_0}^{x}\mathrm{d}x'C_n^2(x')$$

$$\times\delta\left[\alpha-\frac{V_i(\xi)}{1-\xi}\right]\frac{(1-\xi)^{5/3}}{z(\xi,\gamma)[1+\theta^2(\xi,\gamma)]}, \quad (19)$$

where

$$z(\xi,\gamma) = \frac{k}{L}\left[\frac{a_\mathrm{r}^2}{2(1-\xi)^2}-\mathrm{i}\gamma\right]$$

and

$$\theta(\xi,\gamma) = \frac{1-\xi}{\xi}z(\xi,\gamma),$$

and the parameter γ takes positive values. If the reflecting surface is small enough, so that $4\sqrt{3}(k/L)a_\mathrm{r}^2 < 1$, the absolute value of the last fraction in (19) has a maximum at

$$\gamma^2 = \frac{L^2\xi^2}{3k^2(1-\xi)^2}\Big[2-3[\mathrm{Re}\,\theta(\xi,\gamma)]^2$$

$$+\left\{1-12[\mathrm{Re}\,\theta(\xi,\gamma)]^2\right\}^{1/2}\Big] \quad (20)$$

and possibly at the point $\gamma = 0$. Thus, if the delta function in the integrand in (19) is nonzero at the only point in the integration region, this point is unambiguously determined

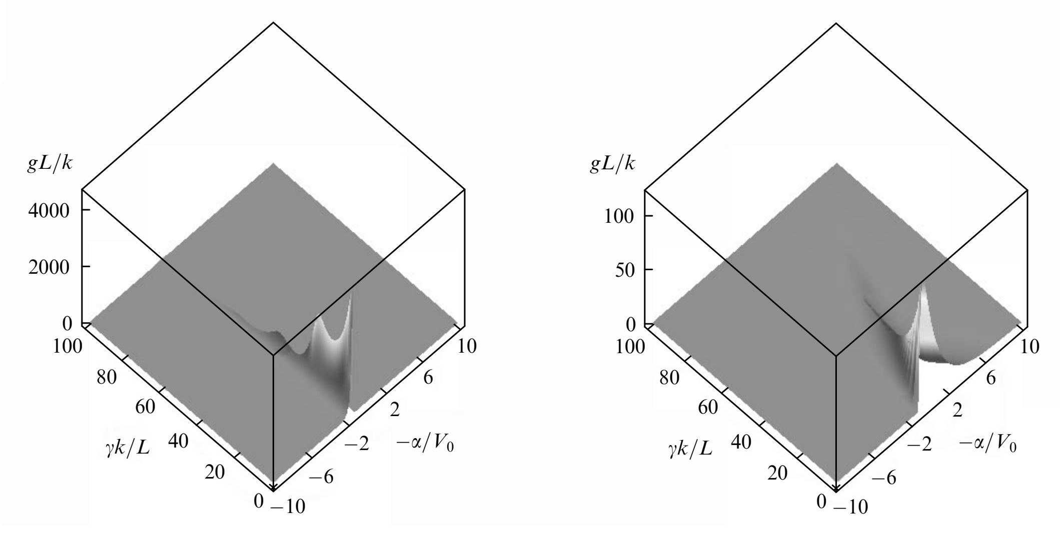

Figure 2. Behaviour of integral (19) for a constant wind velocity (a) and wind velocity changing linearly along the beam path (b).

by the position of maximum (20). Figure 2 shows the behaviour of integral (19) on the plane (γ, α) with a pronounced maximum in sections $\alpha = \mathrm{const}$ for the constant wind velocity $V_i(\xi) = V_0$ (Fig. 2a) and the wind velocity $V_i(\xi) = V_0(1 - 10\xi)$ changing linearly along the beam path (Fig. 2b). The structural characteristic was assumed constant and the effective radius of the diffuse surface was 1/10 radius of the first Fresnel zone $a_{\mathrm{r}} = 0.1(\lambda L)^{1/2}$

These features of integral (19) allow us to formulate the algorithm for recovering the wind velocity profile. We find spatiotemporal spectrum (17) from the recorded sequence of two-dimensional intensity distributions and calculate integral (19). By using relation (20), we find the coordinate α of the point on the beam path for each specified value of ξ for which the maximum of the modulus of integral (19) is separated. Then, by using the relation $\alpha = V_i(\xi)/(1 - \xi)$, which should be fulfilled for nonzero values of (19), we find the wind velocity $V_i(\xi)$ at the given point on the beam path. Having passed through the entire region of values of α at which integral (19) is nonzero, we construct the wind velocity profile. Note that the recovery of the velocity profile by this method is impossible at the ends of the path for

$$|\xi - 0.5| > \frac{1}{2}\left[1 - 4\sqrt{3}\left(\frac{k}{L}\right)a_{\mathrm{r}}^2\right]^{1/2}. \tag{21}$$

This is explained by the fact that both upon reflection from the diffuse surface and imaging, information on the wave phase is lost, and phase incursions from the ends of the path have no time to transform to noticeable intensity fluctuations.

An important feature of this algorithm is that the structural constant of the refractive index enters into (19) at a fixed value of α as a constant coefficient and its variation along the path does not affect the position of the maximum (20).

4. Spatially restricted beam – infinite surface

By substituting (1) and (13) into (12), assuming that $a_{\mathrm{r}} \to \infty$, and expanding the last exponential in (12) into a series by retaining the two first terms, we obtain the spatiotemporal correlation function (14) in the form

$$K_I(\boldsymbol{\rho}, \tau) = \frac{2\pi a^4}{L^4}\int \mathrm{d}\boldsymbol{\kappa}\int_{x_0}^{x} \mathrm{d}x' \Phi_\varepsilon(\xi, \boldsymbol{\kappa}) \exp[\mathrm{i}\boldsymbol{\kappa}\boldsymbol{V}(\xi)\tau]$$

$$\times \exp[\mathrm{i}(1-\xi)\boldsymbol{\kappa}\boldsymbol{\rho}] \sin^2\left[\frac{L\boldsymbol{\kappa}^2}{k}\xi(1-\xi)\right]\exp\left[-\boldsymbol{\kappa}^2\frac{\xi^2 a_{\mathrm{rb}}^2}{2}\right], \tag{22}$$

where

$$a_{\mathrm{rb}}^2 = a^2\left(1 - \frac{L}{F}\right) + \frac{L^2}{a^2k^2}. \tag{23}$$

As in the case (15), expression (22) for $\tau = 0$ coincides with the expression for the spatial correlation function obtained in [5] for the condition $\beta_0^2 < 1$.

Consider now the spatiotemporal spectrum

$$F_I(\boldsymbol{q}, \omega) = \frac{1}{(2\pi)^3}\int \mathrm{d}\boldsymbol{\rho}\int \mathrm{d}\tau \exp(-\mathrm{i}\boldsymbol{\rho}\boldsymbol{q} - \mathrm{i}\omega\tau) K_I(\boldsymbol{\rho}, \tau) =$$

$$= \frac{2\pi a^4}{L^4}\int_{x_0}^{x} \mathrm{d}x' \Phi_\varepsilon\left(\xi, \frac{\boldsymbol{q}}{1-\xi}\right)\exp\left(-\frac{\xi^2 a_{\mathrm{rb}}^2 q^2}{2(1-\xi)^2}\right)$$

$$\times \sin^2\left(\frac{Lq^2}{2k}\frac{\xi}{1-\xi}\right)\frac{1}{(1-\xi)^2}\delta(\omega - \boldsymbol{V}(\xi)\boldsymbol{q}). \tag{24}$$

This expression coincides accurate to a constant factor with the expression for the spatiotemporal spectrum of a spherical wave reflected from a reflector of finite size (17) with the effective radius $a_{\mathrm{r}} = a_{\mathrm{rb}}$. This means that algorithm (18)–(20) for recovering the wind velocity profile is also valid in the beam-infinite reflector regime.

5. Conclusions

We have proposed the algorithm for the wind profile recovery from fluctuations of laser radiation reflected in a turbulent atmosphere. The expressions have been obtained for the spatiotemporal correlation function and spectrum of the optical wave reflected by a diffuse scatterer in the turbulent atmosphere. It has been shown that the recovery of the wind velocity profile based on the correlation spectral analysis of two-dimensional intensity distributions of reflected radiation is possible in the weak optical turbulence regime, and the recovery algorithm has been proposed for the cases of a spherical wave incident on a diffuse reflector of finite size and a spatially restricted beam reflected from an infinite random surface. An important feature of the algorithm is that variations in the structural characteristic of the refractive index along the beam path do not affect the wind velocity profile being recovered.

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