Quantum Electronics **38** (9) 877 – 883 (2008)

OPTICAL WAVEGUIDES

PACS numbers: 42.82.Et; 42.79.Sz
DOI: 10.1070/QE2008v038n09ABEH013604

Application of the theory of coupled waves for analysis of inclined reflectors in optical waveguides

E.A. Kolosovsky, A.V. Tsarev

***Abstract*. A new method for analysing the transmission and scattering of the guided TE mode in an inclined reflector located in an optical waveguide is proposed and studied. The reflection of an inhomogeneous optical beam from the inclined reflector is described semi-analytically for the first time by using the theory of coupled waves and taking into account the interrelation and transformation of energy between all the waves of the discrete and continuous spectra of the optical 2D-waveguide (even and odd guided, radiation, and evanescent waves). The results of calculations of the propagation of light through the inclined reflector in the form of a thin (10 – 500 nm) homogeneous strip obtained by our method and by the finite difference time domain (FDTD) method are in excellent quantitative agreement. The calculation rate of our method considerably (by one – two orders of magnitude) exceeds that of the FDTD method and our method has a better accuracy.**

Keywords: *integrated optics, optical waveguide, method of coupled modes, finite difference time domain method, nanophotonics.*

1. Introduction

Inclined nanoreflectors are used in optical waveguides as beamsplitters in devices for optical data processing [1, 2] or as basic elements in multirefection filtering technology [3 – 6]. The reflection of an inhomogeneous optical beam from single reflectors was earlier investigated by the finite difference time domain (FDTD) method [1 – 4]. The FDTD methods are universal, but they require large computational resources and do not provide the adequate accuracy in calculations of light energy losses upon weak reflection to the side from a single reflector, which is very important for optical signal processing with the help of new filters and multiplexers based on multibeam effects [3 – 6].

In this paper, we developed a fundamentally new approach based on the theory of coupled waves (modes) [7 – 9], which was first used here to analyse the total field during the propagation of an optical beam in an optical waveguide containing an inclined reflector. Earlier, the equations of coupled waves were considered, as a rule, only for two 'main' interacting modes [10]. Now, the propagation, specular reflection, and scattering of an inhomogeneous optical beam from an inclined reflector is described for the first time by using the theory of coupled modes taking into account the interrelation and transformation of energy between all the waves of the discrete and continuous spectra of an optical waveguide (even and odd guided, radiation, and evanescent waves). To simplify the problem, a three-dimensional (3D) optical waveguide is replaced by its two-dimensional (2D) analogue by using the effective refractive index method [11]. In this case, the TM polarisation of the 3D waveguide corresponds to the TE polarisation of the equivalent 2D waveguide. To demonstrate the possibilities of the method, we considered an important case of the reflection of the fundamental TE_0 mode from a narrow homogeneous nanostrip oriented at an angle to the axis of a homogeneous two-dimensional waveguide.

E.A. Kolosovsky, A.V. Tsarev Institute of Semiconductor Physics, Siberian Branch, Russian Academy of Sciences, prosp. Akad. Lavrent'eva 13, 630090 Novosibirsk, Russia; e-mail: kolos@thermo.isp.nsc.ru, tsarev@isp.nsc.ru

Received 7 April 2007; revision received 23 November 2007
Kvantovaya Elektronika **38** (9) 877 – 883 (2008)
Translated by M.N. Sapozhnikov

2. Use of the method of coupled waves to analyse the reflection of a guided mode from a strip inclined reflector crossing an optical waveguide

2.1 Formulation of the problem and basic equations

Consider a typical waveguide structure with a single inclined reflector (Fig. 1). We assume that the ends of the reflector project beyond the waveguide boundary to provide the complete capture of the 'tails' of the radiation field of the incident TE_0 mode. The electromagnetic field at any point of the structure is sought as a superposition of the forward and backward waves of the unperturbed optical waveguide. For each type of such forward and backward waves, we can obtain from Maxwell's equations [7] an exact integro-differential system relating the amplitudes of all the interacting waves in the interaction region oriented along the z axis:

$$\frac{\mathrm{d}A(\beta,z)}{\mathrm{d}z} = -\mathrm{i}\sum_{p,q}\int B(\beta',z)\kappa(\beta,\beta',z)\exp(\pm\mathrm{i}\xi z)\mathrm{d}\beta',$$
$$\frac{\mathrm{d}B(\beta,z)}{\mathrm{d}z} = \mathrm{i}\sum_{p,q}\int A(\beta',z)\kappa^*(\beta,\beta',z)\exp(\pm\mathrm{i}\xi z)\mathrm{d}\beta'. \quad (1)$$

Here, $A(\beta,z)$ are the changing amplitudes of the incident (forward) modes of the discrete and continuous spectra, which are the eigenmodes of the waveguide without a

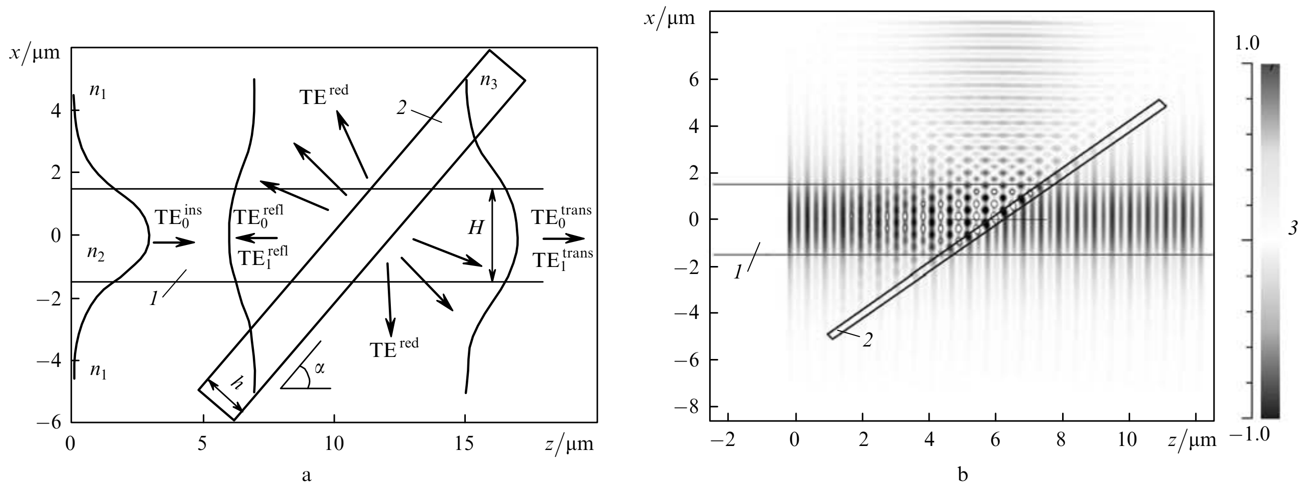

Figure 1. Illustrations to the calculation of fields in a waveguide with an inclined reflector by the method of coupled waves (a) and the FDTD method for $h = 300$ nm and $n_3 = 2.4$ (b): (*1*) optical waveguide; (*2*) inclined reflector; (*3*) intensity scale for the normalised electric field.

reflector, including the amplitude $A_0(z)$ of the fundamental TE_0 wave; $B(\beta, z)$ are the changing amplitudes of the natural backwards waves, including the amplitudes $B_n(z)$ of reflected discrete TE_n waves; β and β' are the longitudinal wave numbers of the eigenmodes; ξ are all possible pair combinations of the differences of the longitudinal wave vectors β and β' of the interacting waves, both for guided modes and modes of the continuous spectrum (radiation and evanescent, forward and backward);

$$\kappa(\beta, \beta', z) = \omega \int_{-\infty}^{+\infty} \Delta\varepsilon(x, z) E(\beta, x) E^*(\beta', x) \mathrm{d}x \tag{2}$$

is the coupling coefficient [7], which is proportional to the overlap integral for the transverse fields $E(x, \beta)$ and $E(x, \beta')$ of any two eigenmodes of the total set of modes of the given waveguide; $\Delta\varepsilon(x, z) = \varepsilon(x, z) - \varepsilon_0(x, z)$ is the difference of the permittivities caused by the presence of the reflector; $\varepsilon(x, z)$ and $\varepsilon_0(x, z)$ are the permittivities of the waveguide with the reflector and without it, respectively; ω is the frequency of light with the wavelength λ_0 in vacuum and the wave vector $k_0 = 2\pi/\lambda_0$; the subscripts p and q denote summation over the types of the waves, their directions (forward and backward) and parity, because the modes of the continuous spectrum are degenerate in the wave number.

The integration limits in (1) will be indicted below. Note that the theory of coupled waves is not at all a variant of the perturbation theory. It was developed independently and was presented by Marcuse for the most general case of anisotropic waveguides in 1975 [12]. It should be emphasised that because the formalism of coupled modes is rigorous [7], we will not use any approximations or expansions.

We will solve the infinite system of equations (1) by using the original numerical-analytical approach [8, 9]. Consider its basic principles. The intermediate integral functions $P(\beta, \beta', z)$ and $Q(\beta, \beta', z)$

$$P(\beta, \beta', z) = \int \kappa(\beta, \beta', z) \exp(\pm \mathrm{i}\xi z) 1 \mathrm{d}z,$$
$$Q(\beta, \beta', z) = \int \kappa(\beta, \beta', z) \exp(\pm \mathrm{i}\xi z) z \mathrm{d}z, \tag{3}$$

are introduced for each type of the waves – discrete and continuous, forward and backward, radiation and evanescent, even and odd.

The integration limits in (3) and (1) for forward or backward waves related to the index β are different (from 0 to z or from $z_{\max}$ to z). The integral functions $\kappa(\beta, \beta', z)$, $P(\beta, \beta', z)$, and $Q(\beta, \beta', z)$ are continuous. It is important that they can be represented analytically exactly for a homogeneous waveguide in the case of a reflector with plane boundaries. The explicit expressions for κ, P, and Q, which are combinations of exponentially trigonometric expressions, we omit here. Figure 2 presents the typical dependences of coupling coefficients on β and z, which were constructed based on the known fields [7] of a two-mode film optical waveguide. The parameters of the structure were as follows: the waveguide width was $H = 3.8$ μm, the reflector thickness was $h = 0.65$ μm, the reflector inclination angle was $\alpha = 60°$, and the refractive indices of the medium and waveguide were $n_1 = 2.2$, and $n_2 = 2.21$, respectively. Note that a special attention should be paid to the matching of intermediate functions (3) at the interfaces of media with different refractive indices.

The interaction region is divided along the z axis into a finite number (20 – 50) of parts z_k, and $A_0(z)$ is represented in the form of a piecewise linear function with unknown (complex) inclination coefficients a_k for each linear interval:

$$A_0(z) = a_k z + b_k. \tag{4}$$

According to the condition of the problem, the incident mode is the TE_0 mode. Therefore, the amplitudes of all the incident waves (except one) in the beginning of the interaction region ($z = 0$) are zero: $A(\beta, 0) = 0$, $A_0(0) = 1$. For backward waves, vice versa, the amplitudes are zero at the end of the interaction region, $B(\beta, z_{\max}) = 0$ because no reflected waves should be behind this region ($z > z_{\max}$).

First we separate the main part of the solution of system (1). The terms containing the function $A_0(z)$ are grouped to a separate subsystem, which, taking into account (3), (4), and the boundary conditions pointed out above, can be integrated over z:

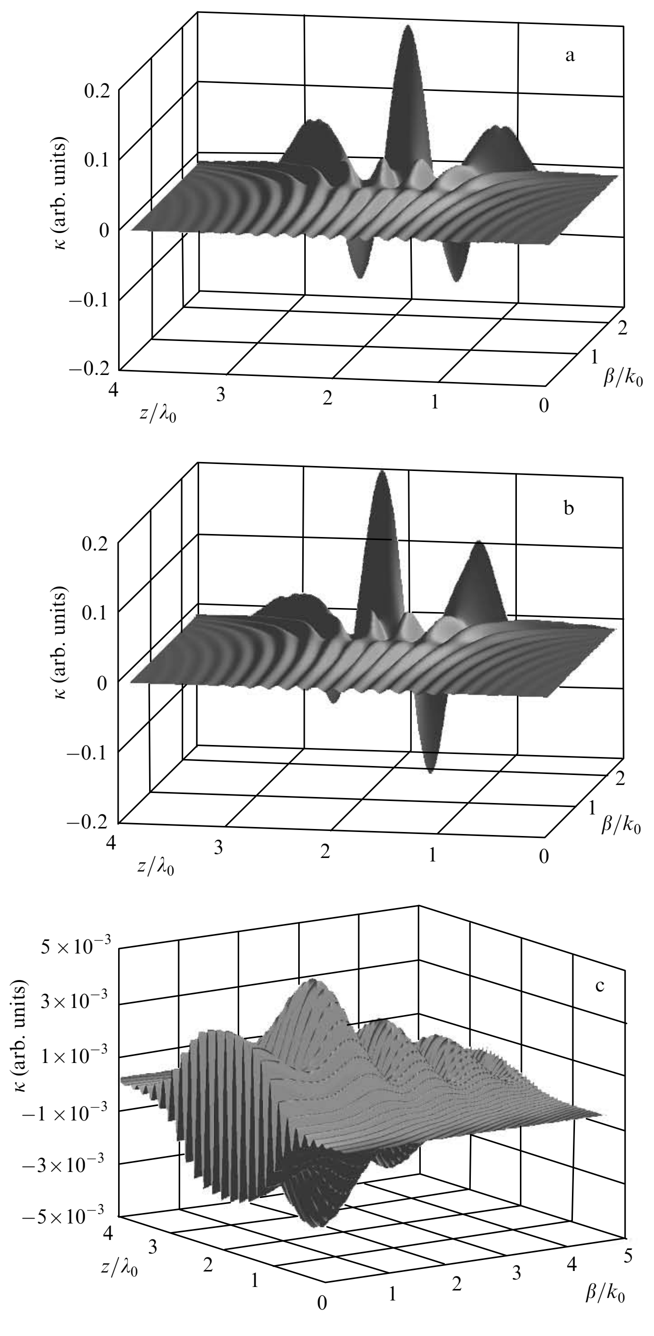

Figure 2. Typical form of the coupling coefficient $\kappa(\beta, z)$ of the TE_0 wave with continuous-spectrum modes in an optical waveguide with an inclined reflector for even (a) and odd (b) radiation modes and for even evanescent modes (c).

$$A(\beta,z) = -\mathrm{i}\sum_{p,q}\left\{\sum_{j=1}^{k}\{a_{j-1}[Q(\beta,\beta',z_j)-Q(\beta,\beta',z_{j-1})] + b_{j-1}[P(\beta,\beta',z_j)-P(\beta,\beta',z_{j-1})]\} + a_k[Q(\beta,\beta',z) - [Q(\beta,\beta',z_k)] + b_k[P(\beta,\beta',z)-P(\beta,\beta',z_k)]\right\}. \quad (5)$$

Here, $z_k < z < z_{k+1}$ and the free index β' (wave number) should be set equal to the longitudinal wave number of the discrete TE_0 mode. A similar expression can be written for the complex amplitudes $B(\beta, z)$ of backward waves, the only difference being that summation over the numbers k of linear parts of the interaction region is performed in the reverse order, from the end of the interaction region to its beginning. Taking into account summation over p and q, the right-hand side of (5) contains ten types of functions P and Q, of which eight belong to the continuous spectrum of the waves (radiation and evanescent, forward and backward, even and odd) and two – to the discrete spectrum of guided forward and backward modes. But in fact, expression (5) is a simple linear combination of functions P and Q with coefficients a_k, because b_k are expressed in terms of a_k linearly as $b_1 = 1 + a_1(z_1 - z_0)$, $b_2 = 1 + a_1(z_1 - z_0) + a_2(z_2 - z_1)$, etc.

To determine the slope a_k of the piecewise linear function $A_0(z)$, we write out the total equation

$$\frac{\mathrm{d}A_0(z)}{\mathrm{d}z} = -\mathrm{i}\sum_{p}\left\{\int A(\beta',z)\kappa(\beta,\beta',z)\exp(-\mathrm{i}\xi z)\mathrm{d}\beta' + \int B(\beta',z)\kappa(\beta,\beta',z)\exp(\mathrm{i}\xi z)\mathrm{d}\beta'\right\} \quad (6)$$

separately for the amplitude of the TE_0 wave from (1). The free index β in this equation should be set equal to the longitudinal wave number of the discrete TE_0 mode. Unlike (1), here the amplitudes $A(\beta, z)$ of the forward waves are taken out as a separate term and the index p denotes the remaining summation over the types of waves and parity. Because the left-hand side of expression (6) is $\mathrm{d}A_0(z)/\mathrm{d}z = a_k$, after the substitution of (5) into (6), this expression gives the algebraic system of linear equations of the N order equal to the number of regions z_k with the totally definite right-hand side c_k, whose explicit form is omitted here:

$$\sum_{i=1}^{N} M_{ki}a_i = c_k. \quad (7)$$

Despite the infinite number of modes of different types in the continuous spectrum, they are all integrally coupled with each other via the linear system of equations (7), which has a comparatively small order N. Without prejudice to the concept of the method, we can assume that the elements of the matrix M_{ki} are calculated at the middle point of each interval with the coordinate $z = (z_k + z_{k-1})/2$. The limits of integration over β' in (6) and (7) are different and depend on the wave type. In the case of radiation modes, it is not necessary to cut off the spectrum because their longitudinal wave number varies within finite limits, while the upper infinite integration limit for the evanescent waves with the imaginary β' is specified by the value in accordance with the required accuracy (the modulus of this value proved to be comparable with the real limit of β). Note that the density of points upon integration can be considerably decreased by arranging them not uniformly but dividing into groups containing 2–4 points and displacing them to quadrature nodes inside each of the groups. It is known [13] that summation with Gaussian quadrature weights is equivalent to the integration of the third–seventh order polynomials. Then, it is sufficient to fix 100–150 points in each region of the continuous spectrum and consider only this restricted number of spectral components.

Thus, the main part of the solution of system (1) for all the components of continuous and discrete spectra $A(\beta, z)$ and $B(\beta, z)$ is completely expressed analytically as linear combination (5) of functions P and Q with coefficients a_k

determined (numerically) by solving the algebraic system of linear equations (7). The component of the spectrum of the TE_0 mode is determined in turn self-consistently by solving (6) and represents simple piecewise linear function (4). We can say that all the components of the spectrum are expressed in terms of one main component $A_0(z)$, while the piecewise linear component A_0 is expressed in terms of other components, and any small changes in $A(\beta, z)$ and $B(\beta, z)$ cause changes in the elements of linear system (7) and slopes a_k. The main part of the solution of system (1) separated in such a way has a clear physical meaning.

It is known from the theory of coupled waves that the formalism of coupled waves is conservative and always preserves the total energy of the interacting waves, which can be redistributed only between the spectral components in the interaction region. Because according to the condition of the problem, the incident and only nonzero component is the TE_0 wave, the found solution concerns the case when the energy exchange involves only this wave as the main energy 'supplier'. The mutual energy exchange without the participation of the main component is neglected temporarily because the interaction of any pair of spectral components with amplitudes considerably smaller than A_0 is the lower-order quantity. However, it is important that (6) is the total non-truncated equation relating all spectral components.

Let us continue our solution. By substituting the main part of (5) into system (1) and integrating by parts over z, we obtain the expression for the addition to the solution, which can be calculated numerically by different methods:

$$r(\beta, z) = -\mathrm{i} \sum_{p,q} \int \left[P(\beta, \beta', z) A(\beta', z) - \int_0^z P(\beta, \beta', z') \frac{\mathrm{d}A(b', z')}{\mathrm{d}z'} \mathrm{d}z' \right] \mathrm{d}\beta', \tag{8}$$

where p and q still mean summation over the types, directions and parity of the waves, but terms with the amplitude A_0 in the sum are absent. The integration over z' in (8) for backward waves should be performed from $z_{\max}$ to z, and the limits of integration over β' depend on the type of waves. Thus, the complete solution of the system of equations (1) for coupled waves is decomposed into the analytic part in the form of superposition (5) of continuous functions P and Q and additional part (8), which is calculated numerically at the middle nodes z_k of the grid. By substituting the refined solution into (6), we obtain again the linear system of algebraic equations (7) for coefficients a_k. It is not necessary to calculate (8) with a high accuracy because the addition $r(\beta, z)$ can be further refined by solving again linear system (7) with the same matrix M_{ki} but with different right-hand sides c_k. In this case, the solutions $A(\beta, z)$ and $B(\beta, z)$ obtained at the previous step are substituted into addition (8).

Let us summarise the above discussion. The problem of the propagation and scattering of an inhomogeneous wave in a waveguide with an inclined reflector is considered in the spectral representation by using the theory of coupled waves. By introducing additional analytic functions, the system of infinite integro-differential equations for coupled waves can be transformed to a low-dimensional algebraic linear system and the main part of the general solution can be expressed in terms of the linear combination of the introduced functions. It became possible because in the case of a reflector with plane boundaries in a homogenous waveguide, the integral functions P and Q can be represented analytically exactly. In fact, the functions P and Q are the integrated 'parts' of system (1), on which all the further solution, including the addition r (8), is based. Linear system (7) is constructed from equation (6) for the amplitude A_0 of the incident TE_0 wave, which is represented in the form of simple piecewise linear function (4) with the slope a_k determined from (7) self-consistently with the amplitudes of all the above-mentioned types of the waves entering the right-hand side of (6). The general solution for the functions $A(\beta, z)$ and $B(\beta, z)$ is represented in the form of the main part of the solution (written as a linear combination of analytic functions P and Q with a finite number of the same coefficients as in $A_0(z)$ and some addition r (8), which is expressed in terms of the functions P and Q and can be found by different methods and interpolated.

The reflection coefficient R and transmission coefficient for the guided modes can be found exactly by solving equation (1) for $z = 0$ and $z = z_{\max}$. The side reflection coefficient R_0 for the radiation waves can be found from the Poynting vector or from the overlap integral for the found field and the incident-wave field.

Thus, we have managed to consider completely the self-consistent transformation of waves on an inclined reflector involving the modes of the infinite spectrum without using any approximations and have obtained in fact analytic solution. This provides not only the high calculation accuracy and rate, which cannot be achieved by other numerical (finite-difference) methods, but also makes possible selective and separate construction of the optical fields of incident and reflected waves (guided, radiation, and evanescent) only at the required site at a specified distance from the reflector. The spectral representation, in our opinion, has certain advantages over finite-difference methods. The finite-difference methods solve the problem of light propagation by direct simulations, step-by-step, inside a finite region. The direct simulation of wave processes, including light propagation, is performed with rapidly oscillating functions. To avoid the 'lost of the phase', numerical integration should be performed by using small enough steps both in time and spatial coordinates. This requirement leads to the increase in the calculation time, resulting in the increasing error at a large number of steps. For this reason, numerical methods are not so efficient for solving wave problems as they are for electrostatic, gravitational or ballistic problems. The coefficients in the spectral representation are smooth functions separated from rapidly oscillating wave cofactors, and the requirement of the restriction of the interaction region is removed.

2.2 Results of numerical simulation for a film waveguide

Consider as an example the reflection of the TE_0 mode from an inclined reflector ($h = 0.25$ μm, the angle $\alpha = 65°$, $n_3 = 1.33$) in a four-mode waveguide ($H = 3$ μm, $n_1 = 2$, $n_2 = 2.2$). Figure 3a shows the exact solution of system (1) for discrete modes. The upper curve, corresponding to the incident TE_0 mode, is the smoothest one, with $A_0(z = 0) = 1$ at the beginning of the interaction region. The curves and dots indicate the solutions corresponding to different numbers N (50 and 25) of sites z_k, respectively. One can see that the number $N = 25$ is sufficient. After the

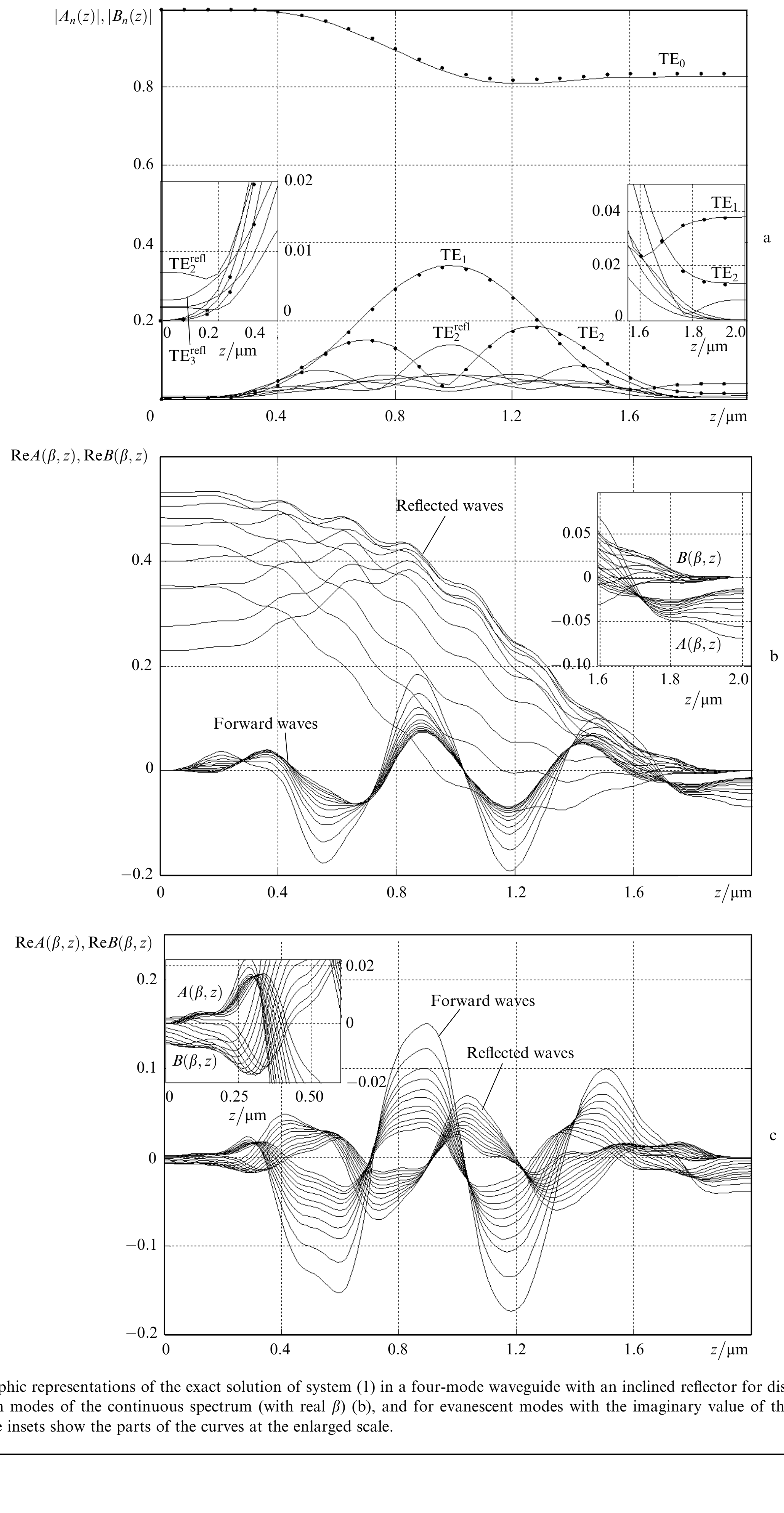

Figure 3. Graphic representations of the exact solution of system (1) in a four-mode waveguide with an inclined reflector for discrete modes (a), for some radiation modes of the continuous spectrum (with real β) (b), and for evanescent modes with the imaginary value of the longitudinal wave number β. The insets show the parts of the curves at the enlarged scale.

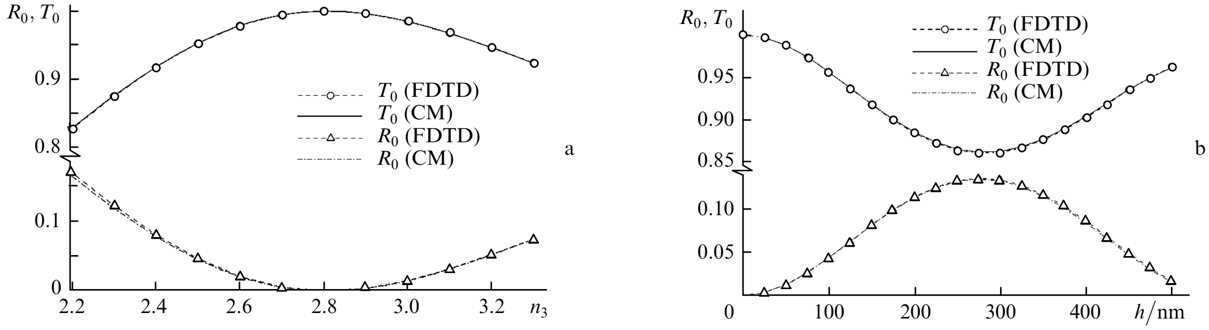

Figure 4. Transmission coefficient T_0 of the TE_0 mode and the coefficient of reflection upward R_0 from a strip reflector tilted at an angle of 45° to the waveguide axis as functions of the refractive index n_3 of the reflector ($h = 150$ nm) (a) and thickness h of the reflector ($n_3 = 2.4$) (b). CM: calculation by the method of coupled modes; FDTD: calculation by the FDTD method.

propagation of the TE_0 wave through the inclined reflector, the forward and reflected TE_1, TE_2, TE_3 modes and the scattered field of the continuous-spectrum modes appear, as shown schematically in Fig. 1a. The amplitudes of forward discrete modes at the end of the interaction region are modulo 0.8239, 0.0378, 0.0136, and 0.0075 and for backward waves at the beginning of the interaction region they are 0.0019, 0.0020, 0.0069, and 0.0030, respectively, in the order of increasing mode number. These values remain invariable to the left and right of the interaction region. The deficient energy ($\sim 32\,\%$) after propagation through the inclined reflector is compensated by the continuous-spectrum modes forming the scattered field. Figures 3b and 3c show the exact solution of system (1) for some amplitudes (real part) of the continuous spectrum: radiation and evanescent, forward (for which $A(\beta, 0) = 0$) and backward ($B(\beta, z_{\max}) = 0$). One can see that all they represent smooth functions.

2.3 Comparison of the obtained results with FDTD numerical experiments

The correctness of our approach is convincingly confirmed by a comparison of calculations performed by the method of coupled waves and the FDTD method. We have analysed many variants of the propagation of light in optical waveguides containing inclined reflectors of different thicknesses with different refractive indices. As an example, Fig. 4 presents the results of calculation of the transmission (T_0) and reflection (R_0) coefficients for the fundamental mode in a single-mode waveguide in the silicon-on-insulator structure at a wavelength of 1.5 μm. The waveguide width was $H = 3$ μm, the refractive indices of the waveguide and environment were $n_2 = 2.81$ and $n_1 = 2.8$. The FDTD calculations were performed by using the FullWave software package developed by RSoft Design Group Inc. [14] for photonics. One can see that both these methods correctly describe the interference nature of inclined reflectors; however, the method of coupled waves provides the one – two orders of magnitude higher calculation rate. For example, the time required to calculate one dot in Fig. 4 by the method of coupled waves is about 12 s compared to 2.1 or 36 min for the FDTD method with the finite-difference step of 0.02 or 0.01 μm, respectively.

It is fundamentally important that the method of coupled modes provides the high accuracy of determining scattering losses ($\alpha_{\rm loss} = 1 - T_0 - R_0$), which cannot be achieved in the best finite-difference methods, including commercial FDTD packages. Unlike analytic methods, the FDTD method has a finite error, which is inherent to different extents in all direct simulation methods. In simulations of the propagation of light in media, this error gives the absolute error of measuring the coefficients T_0 and R_0 of the order of one percent; however, this accuracy is insufficient for describing the operation of multibeam filtering elements having hundreds of reflectors and using multiple interference effects [3 – 6].

Thus, the new numerical-analytical realisation of the method of coupled modes is not only interesting in the scientific respect, but can be used also in many applies optical problems. In particular, due to its clearness, high accuracy and fast response, this method is convenient for the quantitative description of a new class of multibeam acoustooptic and thermooptic elements in devices for optical data processing [3 – 6].

3. Conclusions

We have analysed in the two-dimensional case the propagation of the guided TE_0 mode through a homogeneous strip reflector tilted an angle to the axis of a film optical waveguide. The propagation of an inhomogeneous optical beam through the inclined reflector and reflection of the beam from the reflector have been considered for the first time semi-analytically by using the theory of coupled modes taking into account the interrelation and transformation of energy between all the waves of the discrete and continuous spectra of the optical waveguide (even and odd guided, radiation, and evanescent modes). By introducing additional analytic functions, we have managed to transform the system of infinite integro-differential equations for coupled waves to the low-dimensional algebraic linear system and have expressed the main part of the general solution in terms of a linear combination of the introduced functions. The results of calculations of the propagation of light through an inclined reflector obtained by our method and the FDTD method with the help of the commercial FullWave software package [14] are in excellent quantitative agreement. The calculation rate provided by our method considerably (by one – two orders of magnitude) exceeds that of the FDTD method and our method has a better accuracy. We have proposed the unique algorithm for calculating complex optical elements with inclined nanoreflectors (including an important case of very low reflection coefficients for a single reflector), which has no so far analogues in the efficiency, accuracy, and speed of simulation.

Acknowledgements. The authors thank RSoft Design Group Inc. for granting the licence and the support for the Rsoft Photonic CAD Suite 8.0 software package (including FullWave) for numerical simulations of integro-optical elements. This work was supported by the Russian Foundation for Basic Research (Grant Nos 05002-08118-0fi and 08-02-121145-ofi).

References

1. Li Lixia, Nordin Gregory P., English Jennifer M., Jiang Jianhua. *Opt. Express*, **11**, 282 (2003).
2. Goldring D., Alperovich E., Levy U., Mendlovic D. *Opt. Express*, **13**, 2931 (2005).
3. Tsarev A.V., Passaro V.M.N., Magno F., in *Silicon Photonics*. Ed. by V.M.N. Passaro (Trivandrum, Kerala, India: Research Signpost, 2006) Ch. 3, p. 47.
4. Tsarev A.V. *Kvantovaya Elektron.*, **37**, 393 (2007) [*Quantum Electron.*, **37**, 393 (2007)].
5. Tsarev A.V., Kolosovsky E.A. *Avtometriya*, **42**, 93 (2006).
6. Tsarev A.V. *Nano-Mikrosist. Tekh.*, **4**, 51 (2007).
7. Tamir T. (Ed.) *Integrated Optics* (Berlin: Springer-Verlag, 1983; Moscow: Mir, 1985).
8. Kolosovsky E.A., Petrov D.V., Yakovkin I.B. *Opt. Commun.*, **60**, 280 (1986).
9. Kolosovsky E.A., Petrov D.V., Yakovkin I.B. *Avtometriya*, **3**, 69 (1987).
10. Yariv A., Yeh P. *Optical Waves in Crystals: Propagation and Control of Laser Radiation* (New York: Wiley, 1984; Moscow: Mir, 1987).
11. Chiang K.S. *Appl. Opt.*, **25**, 2169 (1986).
12. Marcuse D. *Bell Syst. Techn. J.*, **54**, 985 (1975).
13. Kalitkin N.N. *Chislennye metody* (Numerical Methods) (Moscow: Nauka, 1978).
14. www.rsoftdesign.com.