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# Error statistics during the propagation of short optical pulses in a high-speed fibreoptic communication line

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***Abstract*.** **Simple analytic expressions are derived to approximate the bit error rate for data transmission through fibreoptic communication lines. The propagation of optical pulses is directly numerically simulated. Analytic estimates are in good agreement with numerical calculations.**

***Keywords***: *fibreoptic communication line, dynamics of optical pulses, error statistics.*

The quality of a communication line is estimated by the bit error rate (BER), which is the ratio of the number of error bits to the total number of transmitted bits. The determination of the BER is the fundamental problem in the construction of any optical communication system. The tails of the probability density distributions of unit and zero bits are often approximated in the literature by Gaussians. In this case, the calculation of the BER requires the knowledge of the $Q$ factor, which is defined by the expression $Q = (\mu_1 - \mu_0)/(\sigma_1 + \sigma_0)$, wheree $\mu_1$ and $\mu_2$ are the average values of unit and zero bits, respectively, and $\sigma_1$ and $\sigma_2$ are their root-mean-square deviations. The BER is calculated from the expression

$$\mathrm{BER} = \frac{1}{2}\mathrm{erfc}\frac{Q}{\sqrt{2}} \approx \frac{\exp(-Q^2/2)}{Q\sqrt{2\pi}}. \quad (1)$$

The Gaussian approximation is simple, but the accuracy of prediction of the probability error in this model is low.

In this paper, the results of direct numerical simulations of the error statistics in a fibreoptic communication line with a 40 Gbit s$^{-1}$ transmission are presented and simple analytic BER approximations are obtained. A communication line is considered with a periodic section having the configuration

SMF (85 km) + EDFA + DCF (14.85 km) + EDFA.

where SMF is a standard single-mode fibre; DCF is a dispersion-compensating fibre; and EDFA is an erbium-doped fibre amplifier. The parameters of optical fibres are presented in Table 1. Erbium amplifiers had the noise factor of 4.5 dB and the gain of 13.4 dB to compensate completely the attenuation of optical signals over the periodic section length. The average dispersion coefficient of the optical line was $-0.4$ ps nm$^{-1}$ km$^{-1}$. The communication line contained 31 sections and an additional SMF compensating the accumulated dispersion. The length of the additional SMF (72.6 km) was selected to provide the maximum $Q$ factor at the communication line end. The dispersion accumulated in 31 periodic sections was $-1238.1$ ps nm$^{-1}$ and the total dispersion of the additional SMF was 1234.2 ps nm$^{-1}$. Thus, the average dispersion of the entire communication line was almost zero. Gaussian pulses of duration 7.5 ps and peak power 5 mW were used as unit bits. The data transmission rate in one frequency channel was 40 Gbit s$^{-1}$.

**Table 1.** Parameters of optical fibres.

| Parameter | SMF | DCF |
|---|---|---|
| Attenuation at 1550 nm/dB km$^{-1}$ | 0.2 | 0.65 |
| Effective mode area/μm$^2$ | 80 | 19 |
| Dispersion coefficient/ps nm$^{-1}$ km$^{-1}$ | 17 | $-100$ |
| Dispersion slope/ps nm$^{-2}$ km$^{-1}$ | 0.07 | $-0.41$ |
| Nonlinear refractive index/m$^2$ W$^{-1}$ | $2.7 \times 10^{-20}$ | $2.7 \times 10^{-20}$ |

The dynamics of optical pulses is described by the generalised nonlinear Schrödinger equation for the complex envelope $A$ of the electromagnetic field [1]:

$$\mathrm{i}\frac{\partial A}{\partial z} - \frac{\beta_2(z)}{2}\frac{\partial^2 A}{\partial t^2} + \sigma(z)|A|^2 A = \mathrm{i}\Big[-\gamma(z) + \sum_{k=1}^{N} r_k \delta(z - z_k)\Big]A. \quad (2)$$

Here, $z$ is the distance along the communication line; $t$ is time; $|A|^2$ is power; $\beta_2$ is the group dispersion velocity parameter; $\sigma = 2\pi n_2/(\lambda_0 A_{\mathrm{eff}})$ is the Kerr nonlinearity coefficient ($\sigma$ and $\beta_2$ are represented as functions of $z$ to take into account their change on passing from one fibre type to the other); $n_2$ is the nonlinear refractive index; $\lambda_0$ is the carrier wavelength; $A_{\mathrm{eff}}$ is the effective area of the fibre mode; $\gamma(z)$ is the signal attenuation coefficient; and $z_k$ and $r_k$ are the position and gain of the $k$th amplifier, respectively,

In dispersion-controlled systems, optical fibres with chromatic dispersion of opposite signs are used to provide the control of the dispersion broadening of pulses. If the average dispersion of a communication line is zero, the

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signal shape in the linear case and the absence of attenuation and noise is completely reconstructed at the line end [1]. The model of the generalised nonlinear Schrödinger equation describing the propagation of optical pulses takes into account the following effects responsible for the signal distortion: the Kerr nonlinearity, dispersion broadening, amplified spontaneous emission (ASE), and fluctuations of the positions of individual bits (the so-called Gordon–Haus effect [2]). Aside from the Gordon–Haus effect, the jitter of pulses is also caused by electrostriction [3] and polarisation mode dispersion. The discussion of these effects is beyond the scope of the model considered here.

Statistical data processing was performed after the propagation of optical signals over a distance of 3000 km. A rectangular optical filter with the transmission bandwidth $B_{\rm op} = 100$ GHz and a third-order Butterworth electric filter with the transmission bandwidth $B_{\rm el} = 50$ GHz were mounted in front of a detector.

The signal degradation is mainly caused by ASE and nonlinearity. The tails of the distributions functions of zero and unit bits for a model taking into account only the influence of ASE on the signal were found in [4]. The output signal in this model is formed by adding ASE to the input signal. It was shown [4] that the statistics of zero bits is determined by the ratio of the optical filter bandwidth to the electric filter bandwidth on a detector. The analytic expression

$$w_1(y) = \frac{1}{2}\sqrt{\frac{M}{\pi I_0}}\left[\frac{I_1}{(y-I_0)^3}\right]^{1/4} \times \exp\left[-\frac{M}{I_0}(\sqrt{y-I_0}-\sqrt{I_1})^2\right] \quad (3)$$

obtained in [4] quite accurately describes the probability density of unit bits in the model neglecting nonlinearity. Function (3) is a good approximation for $y > 1.1I_0$. Here, $M$ is the number of degrees of freedom and $I_0$ and $I_1$ are the average values of the electric current corresponding to zero and unit bits, respectively.

The main result of this paper is analytic expressions accurately approximating the probability densities of the distribution function of zero and unit bits $w_0(y)$ and $w_1(y)$. We approximated the function $w_1(y)$ by the expression

$$w_1(y) = \frac{1}{2}\sqrt{\frac{\bar{M}}{\pi I_0}}\left(\frac{I_1}{(y+d-I_0)^3}\right)^{1/4} \times \exp\left[-\frac{\bar{M}}{I_0}(\sqrt{y+d-I_0}-\sqrt{I_1})^2\right]. \quad (4)$$

The parameters $\bar{M}$ and $d$ are selected from the sampling of zero and unit bits on the detector obtained in the numerical experiment to minimise the difference of results corresponding to the analytic probability density function and obtained in the numerical experiment. In this paper, the method of least squares was used. Figure 1 presents the logarithm of a theoretical curve obtained from the sampling consisting of 7736 current values on the detector (corresponding to unit bits) and the logarithm of the probability density function constructed by using the sampling containing 127464 values. Good agreement is observed between the analytic curve with parameters obtained by using a small sampling and the probability density function constructed by using a sampling of a considerably greater volume.

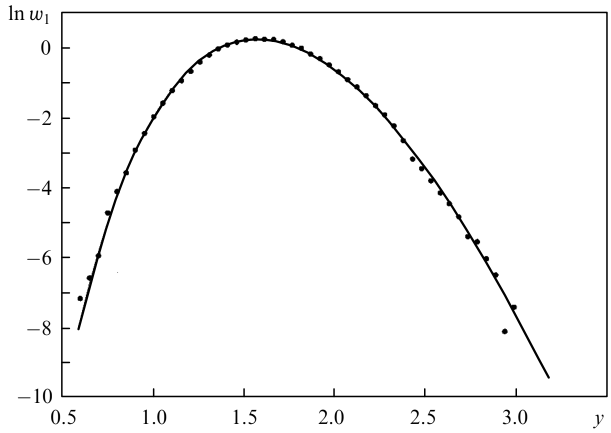


**Figure 1.** Dependence of the logarithm of the probability density $\ln w_1$ of unit bits on the current $y$ (in relative units) in the detector and its analytic approximation (solid curve); $\bar{M} = 4.83$, $d = 0.148$, $I_0 = 0.137$, $I_1 = 1.62$.

Expression (4) differs from (3) in that the parameter $M$ in the latter is determined only by the properties of optical and electric filters. The parameter $\bar{M}$ in (4) is selected from the sampling of unit bits obtained on the detector and, in addition, expression (4) contains the additional parameter $d$, which takes into account the distortion of the initial signal by nonlinearity.

Figure 2 presents the logarithm of the calculated probability density $w_0(y)$ for zero bits as a function of the detector current. One can see that the tail of the probability density of zero bits has the exponential asymptotics. Sampling for the statistical processing of zero bits was performed for 128024 bits. Note that the statistical estimate of the parameters of the linear approximation of the function $(-7.5y + 2.01)$ was obtained from the sampling consisting of 7624 zero bits.

The effects of nonlinear interaction and the spontaneous emission of amplifiers on the output signal were compared by considering the propagation of optical pulses in the case of ideal amplifiers without noise [5].

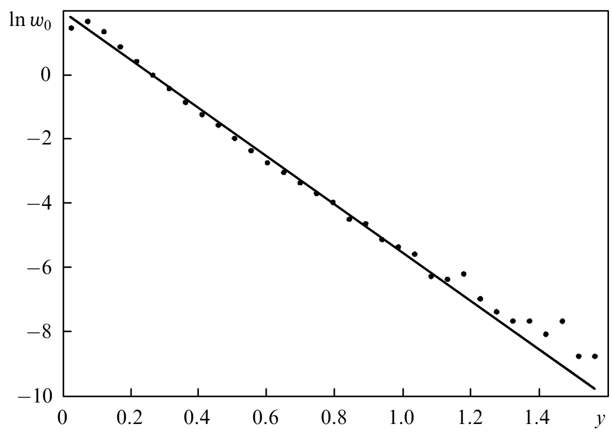


**Figure 2.** Dependence of the logarithm of the probability density $\ln w_0$ of zero bits on the current $y$ in the detector (dots) and its linear approximation $(-7.5y + 2.01)$ (straight line).

Figure 3 presents the probability densities of zero (a) and unit (b) bits for models without nonlinearity or with ideal amplifiers. A comparison of the curves in Fig. 3a shows that nonlinearity leads to the erroneous recognition of unity instead of zero to a greater extent than ASE, i.e. it proves to be the main factor deteriorating the zero bit signals. At the same time, a comparison of the curves in Fig. 3b shows that unit bit signals are mainly distorted by the noise of erbium amplifiers.

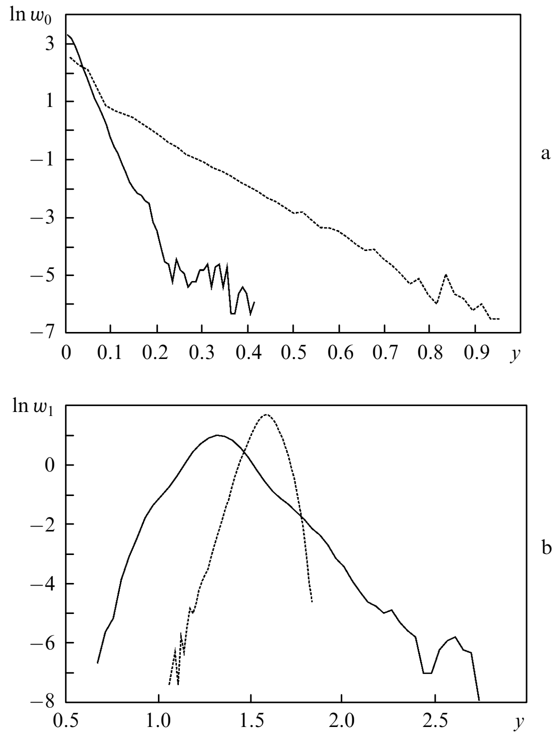


**Figure 3.** Dependences of the logarithm of the probability density for zero (a) and unit (b) bits on the current $y$ for the model without nonlinearity (solid curve) and the model with ideal amplifiers (dashed curve).

Figure 4 presents BERs obtained in the numerical experiment and their analytic approximation as a function of the level corresponding to the unit current bit. One can see that the exponential approximation for the probability density of zero bits and approximation (4) for unit bits are good approximations for estimating BER.

Note that the position of the maximum of the probability density of unit bits in the model taking into account the ASE and Kerr nonlinearity coincides with that for the model with ideal amplifiers. This is explained by the fact that the contribution from the interaction of the signal with noise is small compared to the contribution from the interaction between symbols. The parameter $d$ in expression (4) characterises the shift of the maximum of the probability density of unities on the detector caused by nonlinearity compared to the average current of unit bits on the source.

The analytic expressions obtained for the tails of the probability density for zero and unit bits considerably reduce the sampling volume required for the accurate determination of the BER.

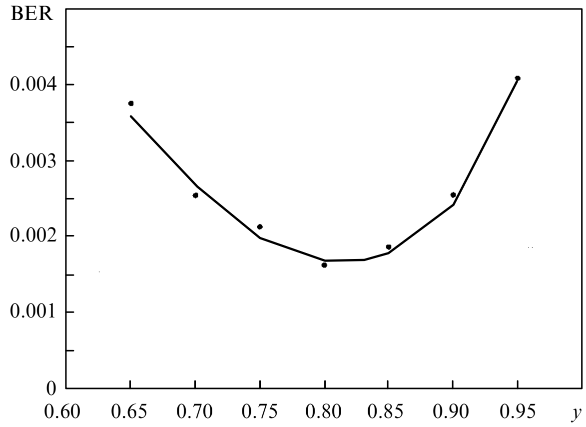


**Figure 4.** Bit error rate obtained in the numerical experiment (dots) and its analytic estimate (solid curve).

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