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Remote laser speckle interferometry: A speckle pattern formation model

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***Abstract*. A mathematical model of laser radiation scattering from a remote rough surface illuminated by a focused laser beam is proposed. The model can be used to estimate the mean number and size of speckles and their mean and maximum intensity. Fluctuations of the speckle intensity were observed upon focusing radiation in the object plane. The magnitude of fluctuations achieves 50 %–70 % and depends on the object roughness and the laser beam parameters.**

Keywords: *sensing, speckle interferometry, laser radiation, remote sensing.*

1. Introduction

The methods of remote laser diagnostics based on speckle interferometry such as Doppler anemometry, remote object recognition, and contactless defectoscopy find increasing applications in the last years. These methods have already found wide industrial applications [1 – 3].

The methods are based on the use of a standard interferometer with one of the mirrors replaced by a diffusely reflecting surface under study [3]. The reflected field is formed by many independent waves with different phases propagating in different directions. The interference of these waves in a detector plane produces the so-called speckle pattern.

The sensitivity of speckle-interferometers is usually enhanced by the heterodyne method. To obtain information on a surface by using the heterodyne method, it is necessary to record variations in the interference pattern in the region of one speckle. To provide the maximum sensitivity and maximum signal-to-noise ratio, it is necessary to match the speckle size with the area of a photodetector taking into account the speckle intensity. The characteristics of a speckle depend on the parameters of a probe laser beam, the characteristics of the reflecting surface, and the properties of the optical system used for illumination and detection. To optimise the parameters of a laser beam, a focusing system, and a photodetector, it is necessary to have information on the basic parameters of the speckle pattern.

By studying speckle patterns obtained from different diffusion surfaces under different illumination conditions, we found some new properties which have not been reported in the literature. This required the construction of a new mathematical model to describe the formation of speckle patterns observed upon diffuse scattering of radiation. The known theoretical models neglect the divergence of a laser beam [5 – 8], the difference of its degree of coherence from unity [5 – 13], and the possibility of its inaccurate focusing to an object [9 – 13]. In this paper, we propose a theoretical model that takes into account these characteristics of radiation in the diffraction approximation. The dependences of the speckle pattern parameters on the characteristics of a laser beam, focusing system, and an object predicted by the model are consistent with experimental data.

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2. Model of light scattering by a rough surface

Consider the formation and measurement of a speckle pattern in the scheme presented in Fig. 1. Within the framework of the statistical approach, the main characteristic of the radiation field propagating from an optical transmitter to a detector is its correlation function. This

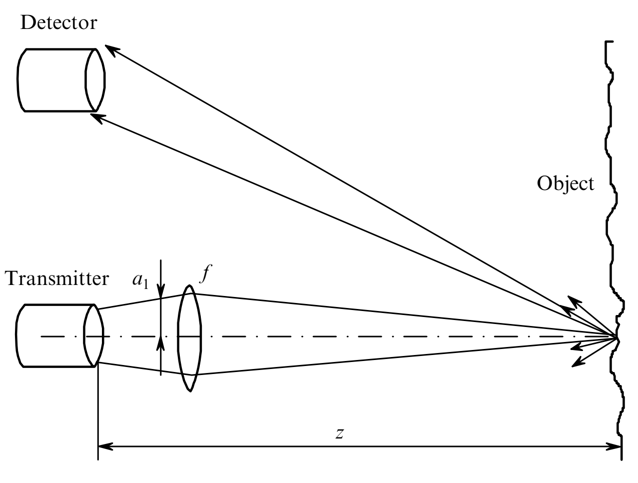

Figure 1. Optical scheme of formation and recording of speckle patterns (a_1 is the beam radius at the transmitter output; f is the focal distance of the optical system of the transmitter; z is the distance between the transmitter/detector and object.

function determines the main parameters of speckles: their size, number, and intensity [14].

Consider the propagation of radiation in two stages: first we find the characteristics of the field u_z on the surface under study in the diffraction approximation and then calculate these characteristics for a reflected wave at the input mirror of the detector. At the first stage, the correlation function of the field is described by the expression

$$\langle u_z(\boldsymbol{\rho})u_z^*(\boldsymbol{r})\rangle \approx \int \langle u_t(\boldsymbol{\rho}_1)u_t^*(\boldsymbol{r}_1)\rangle \times \exp\left\{ i\frac{k}{2z}\left[(\boldsymbol{\rho}_1-\boldsymbol{\rho})^2-(\boldsymbol{r}_1-r)^2\right]\right\} d\boldsymbol{\rho}_1 d\boldsymbol{r}_1, \quad (1)$$

where $\langle u_t(\boldsymbol{\rho}_1)u_t^*(\boldsymbol{r}_1)\rangle$ is the correlation function of the fields $u_t(\boldsymbol{\rho})$ at the transmitter output; the angle brackets denote statistical averaging over realisations; z is the distance between the transmitter/detector and object (along the optical axis of the system); $k = 2\pi/\lambda$ is the wave number; λ is the radiation wavelength; $\boldsymbol{\rho}_1$, $\boldsymbol{r}_1$ and $\boldsymbol{\rho}$, $\boldsymbol{r}$ are the coordinates in the transmitter and object planes, respectively.

Let us refine the form of correlation functions in expression (1). The field at the transmitter output can be written in the form

$$u_t(\boldsymbol{\rho}) = \exp[i\varphi_t(\boldsymbol{\rho})]\exp\left(-i\frac{k\boldsymbol{\rho}^2}{2f}\right)\exp\left(-\frac{2\boldsymbol{\rho}^2}{a_1^2}\right), \quad (2)$$

where a_1 is the laser beam radius at the transmitter output; $\varphi_t(\boldsymbol{\rho})$ is the field phase; and f is the focal distance of the transmitter optical system. In this case, the correlation function of the field at the transmitter output is described by the expression

$$\langle u_t(\boldsymbol{\rho}_1)u_t^*(\boldsymbol{r}_1)\rangle = \langle \exp\{i[\varphi_t(\boldsymbol{\rho}_1)-\varphi_t(\boldsymbol{r}_1)]\}\rangle \times \exp\left[-i\frac{k(\boldsymbol{\rho}_1^2-\boldsymbol{r}_1^2)}{2f}\right]\exp\left[-\frac{2(\boldsymbol{\rho}_1^2+\boldsymbol{r}_1^2)}{a_1^2}\right]. \quad (3)$$

We will assume that the random field of phase distortions is distributed according to the normal law and is stationary. The correlation function can be described by the Gaussian [4]

$$\langle \exp\{i[\varphi_t(\boldsymbol{\rho}_1)-\varphi_t(\boldsymbol{r}_1)]\}\rangle \approx \exp\left[-\frac{(\boldsymbol{\rho}_1-\boldsymbol{r}_1)^2}{c_1^2}\right], \quad (4)$$

where c_1 is the radius of correlations of the phase distortions of the field. By substituting (2)–(4) into (1) and integrating, we obtain

$$|\langle u_z(\boldsymbol{\rho})u_z^*(\boldsymbol{r})\rangle| \approx \exp\left[-\frac{(\boldsymbol{\rho}-\boldsymbol{r})^2}{c_z^2}\right]\exp\left[-\frac{2(\boldsymbol{\rho}^2+\boldsymbol{r}^2)}{a_z^2}\right], \quad (5)$$

where

$$c_z = \frac{c_1}{N_0}\sqrt{\gamma(z)} \quad (6)$$

is the correlation radius of the laser beam in the object plane; $N_0 = ka_1^2/(4z)$ is the Fresnel number;

$$\gamma(z) = 1 + \frac{a_1}{c_1^2} + N_0^2\left(1-\frac{z}{f}\right)^2$$

is the coefficient taking into account the diffraction and geometrical broadening of the laser beam; and

$$a_z = \frac{a_1}{N_0}\sqrt{\gamma(z)} \quad (7)$$

is the laser beam radius on the object surface.

Consider at the second stage the reflection of the optical wave from the object and its propagation to the detector. The correlation function of the field u_r in the detector plane can be written in the form

$$\langle u_r(\boldsymbol{\rho})u_r^*(\boldsymbol{r})\rangle \approx \int \langle u_z(\boldsymbol{\rho}_2)u_z^*(\boldsymbol{r}_2)\rangle \Gamma(\boldsymbol{\rho}_2,\boldsymbol{r}_2) \times \exp\left\{ i\frac{k}{2z}\left[(\boldsymbol{\rho}-\boldsymbol{\rho}_2)^2-(\boldsymbol{r}-\boldsymbol{r}_2)^2\right]\right\} d\boldsymbol{\rho}_2 d\boldsymbol{r}_2, \quad (8)$$

where $\Gamma(\boldsymbol{\rho}_2,\boldsymbol{r}_2)$ is the function describing field distortions appearing after reflection of light from the object surface.

We will describe the propagation of radiation from the object to detector by using the following assumptions:

(i) the height of irregularities greatly exceeds the laser radiation wavelength λ;

(ii) the correlation radius (characteristic scale) of irregularities greatly exceeds the wavelength λ;

(iii) the mutual shading of surface elements is absent;

(iv) the scattered field is studied in the Fresnel diffraction zone;

(v) multiple scattering is absent;

(vi) phase distortions introduced by irregularities are statistically independent of the beam distortions.

The topography of surface irregularities can be quite intricate. We will assume that the surface is isotropic and its relief is described by a random stationary function. The function describing field distortions after reflection of light from a rough surface can be written in the form

$$\Gamma(\boldsymbol{\rho}_2,\boldsymbol{r}_2) = \langle \exp[i\varphi(\boldsymbol{\rho}_2)]\exp[-i\varphi(\boldsymbol{r}_2)]\rangle,$$

where $\varphi(\boldsymbol{x}) = 2kh(\boldsymbol{x})$; $h(\boldsymbol{x})$ is the surface irregularity height. We will assume below that the function $\varphi(\boldsymbol{x})$ describes normally distributed stationary phase distortions with the dispersion σ^2. For typical irregularities, we have $\sigma^2 \gg 1$, and therefore the 'large' dispersion approximation [15] can be used, in which correlation functions are described by Gaussians:

$$\langle \exp\{i[\varphi(\boldsymbol{\rho}_2)-\varphi(\boldsymbol{r}_2)]\}\rangle \approx \exp[-(\boldsymbol{\rho}_2-\boldsymbol{r}_2)^2/c_w^2], \quad (9)$$

where c_w is the effective correlation radius of irregularities. Then, the correlation radius of phase distortions of a beam reflected from surface irregularities in the detector plane is calculated from the expression

$$c_{zz} = \left(\frac{1}{1/c_z^2+1/c_w^2}\right)^2. \quad (10)$$

Taking (9) into account, expression (8) can be integrated. In this case, the beam radius on the detector is

$$a_{2z} = \frac{a_z}{N_z}\sqrt{\gamma_z}, \qquad (11)$$

where

$$\gamma_z = 1 + \left(\frac{a_z}{c_{2z}}\right)^2 + N_z^2\left(1 - \frac{z}{f}\right)^2$$

is the coefficient taking into account the geometrical broadening and broadening due to incomplete coherence of the laser beam; $N_z = ka_z^2/(4z)$ is the Fresnel number for the laser beam on the surface; and $c_{2z} = (c_{zz}/N_z)\sqrt{\gamma_z}$ is the mean speckle size.

As mentioned above, the basic measurable characteristics of scattered light are the mean speckle size and its mean intensity I_{sp} normalised to the initial mean intensity $\bar{I}$ of the beam. Taking (10) and (11) into account, we obtain $I_{sp}/\bar{I} = (a_1/a_{2z})^2$. In this case, the maximum speckle intensity is approximately twice as large as the mean intensity. The number of speckles on the detector is determined by the expression

$$N_{sp} = 1 + \frac{a_{2z}^2}{c_{2z}^2} = 1 + \frac{a_z^2}{c_{zz}^2}.$$

The root-mean-square intensity fluctuation is

$$\sigma_I \approx \frac{I_{sp}}{\sqrt{N_{sp}}}.$$

This model can be used for calculating the main characteristics of the speckle pattern observed upon reflection of a laser beam from a rough surface (the number of speckles and their mean size, and the mean and maximum intensities of a speckle) taking into account characteristics of a radiation source and an object.

Numerical studies revealed an important feature in the dependence of the maximum intensity I_{max} of a single speckle on the focal distance f: if the beam size on a target was comparable with the correlation radius of irregularities, the monotonic behaviour of this dependence was violated (Fig. 2). Upon precise focusing on the target ($z = f$), the intensity I_{max} decreased down to 70 % of the maximally achievable (optimal) intensity. In this case, the optimal focal distance can differ from the distance z to the target by 20 % – 30 %. The intensity modulation depth considerably depends on the degree of beam aberration at the output of the focusing system (Fig. 2). The higher is the beam quality, the stronger is a decrease in its intensity. Because of a high sensitivity of the speckle intensity to the focal distance, the detecting system can pass to the unstable regime. The operation stability of the system as a whole can be increased due to an increase in the beam aberration at the source output or with the help of an adaptive optical system maintaining a signal at the absolute maximum.

The minimum of the dependence of the speckle intensity I_{max} on the focal distance observed in the case of precise focusing of the laser beam to the target appears due to a change in the diameter of the beam reflected from the target in the detector plane caused by the competition of the geometrical and diffraction divergences of the reflected beam. The local minimum of the dependence $I_{max}(f)$ appears when the diffraction divergence begins to exceed the geometrical one.

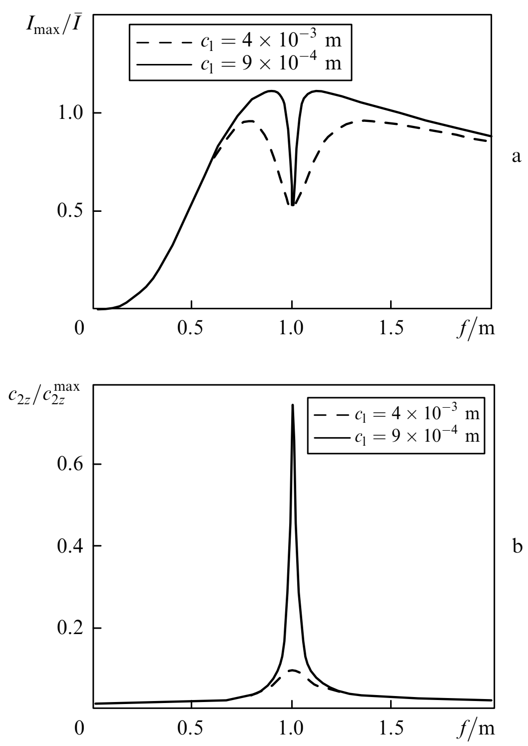

Figure 2. Dependences of the maximum intensity I_{max} of a speckle normalised to the mean radiation intensity $\bar{I}$ at the transmitter output and of the mean speckle size c_{2z} on the focal distance f for illuminating beams with different aberrations. The distance from a telescope to a target is $d = 1$ m.

3. Experimental results

The calculations were verified by using the experimental setup shown schematically in Fig. 3. We used a diode-pumped 300-mW cw Nd : YAG laser which operated in a nearly single-mode regime. The laser beam diameter on the output mirror was 1.5 mm. The beam diameter was enlarged in telescope (*2*) up to 9 mm, and the beam was directed with the help of mirrors (*3*) and (*4*) on object (*5*). Scattered radiation was detected with camera (*6*) (DALSA 1M-28, the sensitive area 10.9 × 10.9 mm, 1024 × 1024 pixels) and processed in PC (*7*). All the optical elements were mounted on a vibration isolated optical table.

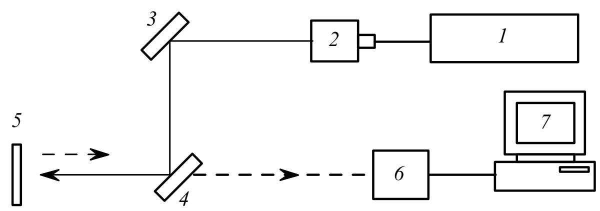

Figure 3. Scheme of the experimental setup: (*1*) laser; (*2*) telescope; (*3*, *4*) mirrors; (*5*) object; (*6*) measuring chamber; (*7*) PC.

Four specially manufactured aluminium plates (samples 1 – 4) with surfaces of different qualities were used as objects (*5*). The surface relief of each of the plates was measured with a mechanical profilograph and the autocorrelation functions of the roughness of their surface were calculated (Fig. 4).

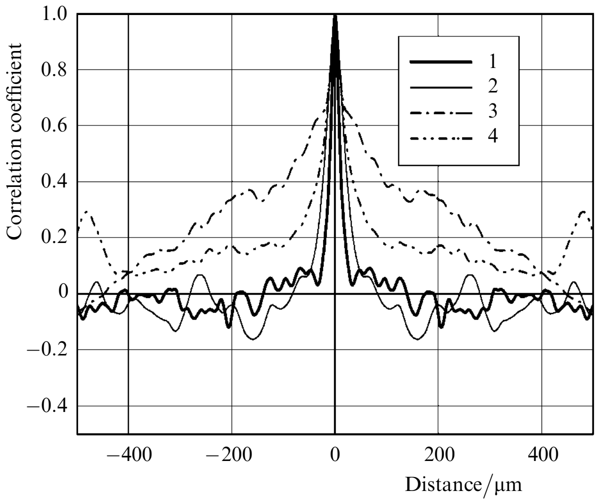

Figure 4. Autocorrelation functions of the surface roughness for samples 1–4.

Experiments were performed by the following method. The focal distance f of the telescope was varied from $0.5d$ to $1.5d$ with the step $\Delta f = 1 - 5$ cm, where $d = 1$ m is the fixed distance from the telescope to the surface under study. The speckle pattern observed for each focal distance was recorded and processed in the PC. The camera operated in the linear regime to avoid errors caused by the CCD saturation.

The number of speckles in images corresponding to different focal distances was different. To obtain the uniform statistical sampling, the number of processed frames was varied from 1 to 15 depending on the focal distance. The parameters of the speckle pattern for the specified f were calculated by averaging over an ensemble of frames. To exclude external factors such as, for example, a change in the total illumination of the surface, experiments for each of the plates were performed several times and then the data were averaged.

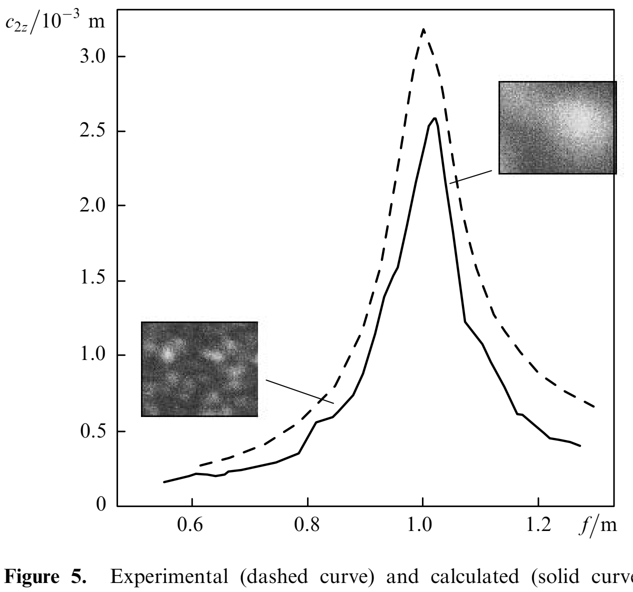

Figure 5. Experimental (dashed curve) and calculated (solid curve) dependences of the mean speckle size c_{2z} on the focal distance f for sample 1. The distance from a telescope to a target is $d = 1$ m. Inserts show typical images of the speckle pattern for two different values of f.

The dependence of the mean size of the speckle pattern on the focal distance of the telescope was obtained by the manual PC processing of frames. A set of speckles was separated in each frame and then the mean speckle size averaged over frames and over speckles separated in the frames was constructed as a function of the focal distance f. Figure 5 presents this dependence obtained for sample 1.

The dependence of the mean speckle intensity on the focal distance was obtained by the following method. Each frame contained the image of a speckle pattern in 256 grey shades. The maximum possible intensity corresponded to 255. The maximum intensity found in each frame was normalised to 255 and then the mean intensity was obtained by averaging the normalised intensities in each frames over the number of frames. The mean intensity calculated in this way was treated as the maximum mean intensity of the speckle pattern for the given f. This method provided the processing of the entire speckle pattern and excluded the influence of the human factor.

Figure 6 presents the experimental and calculated dependences of the speckle pattern intensity on the focal distance for sample 1, which are in good agreement.

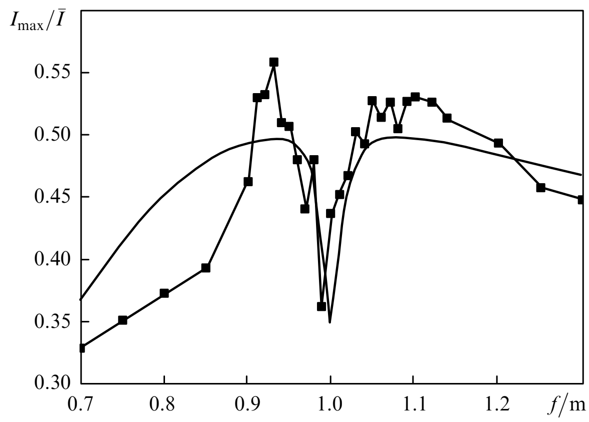

Figure 6. Experimental (points) and calculated (solid curve) dependences of the maximum intensity I_{max} of a speckle normalised to the mean radiation intensity at the transmitter output on the focal distance f. The distance from a telescope to a target is $d = 1$ m.

4. Conclusions

We have developed a new mathematical model describing the mean and maximum intensity, the number and the mean size of a speckle pattern obtained upon illumination of a remote rough surface by a focused laser beam. The model takes into account in the diffraction approximation the laser beam diameter on the object, the radiation divergence and coherence. The dependences of the parameters of speckle patterns on the focal distance of the optical system of a transmitter, laser beam characteristics and irregularities of the object surface have been studied numerically. The theoretical analysis has been confirmed by the experimental study.

It was found that the maximum intensity of speckles was achieved when the laser beam was not precisely focused on the object surface. In the case of small aberrations of the laser beam and small focal distances, the number of irregularities within the focal spot does not exceed 100 and the intensity of individual speckles exhibits considerable fluctuations, achieving 50 % – 70 %.

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