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Correlation processing of signals of a single-fibre multimode interferometer upon excitation of few modes during measurements of deformation effects

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***Abstract*. The features of the correlation processing of a speckle pattern obtained after transmission of radiation from a single-fibre multimode interferometer (SMI) with few modes through a diffusion transparency are studied theoretically and experimentally. It is shown that this approach eliminates the influence of polarisation mode beats and of higher-order modes typical for the two-mode excitation of a SMI and allows the measurement of the elongation of a sensitive fibre piece by exciting few modes in a multimode interferometer.**

Keywords: *correlation speckle-pattern processing, diffusion transparency, intermode interference, deformation measurement.*

A single-fibre multimode interferometer (SMI) [1] based on the interference of modes of the same optical fibre is one of the promising types of interferometers for applications in fibreoptic measurement transducers (FOMTs). In the case of excitation of two linearly polarised lower-order modes (LP modes), FOMTs combine a very simple optical scheme with a high sensitivity to deformation perturbations and stability to uncontrollable temperature variations [1]. However, broad application of such transducers is prevented by weakly determined distortions of the intermode interference pattern due to polarisation variations in the LP_{11} mode and the influence of higher-order modes, which is difficult to eliminate. As shown in [2], stochastic optical signals can be efficiently processed by correlation methods. At the same time, statistical methods are inefficient for few-mode optical fibres because such fibres do not form a random speckle pattern. However, a random speckle pattern can be produced by transmitting radiation from a fibre through a diffusion transparency [3]. The aim of our paper is to study the features of processing a speckle pattern obtained after transmission of the few-mode SMI radiation through a diffusion transparency and to analyse the possibility of obtaining quantitative information on the deformation perturbations of the interferometer by using the correlation processing of SMI signals.

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In the method proposed, a video-camera CCD array records the reference image of a speckle pattern, which is formed after the transmission of radiation from an undistorted SMI through a diffusion transparency and corresponds to the initial state of an optical fibre in the interferometer. The deformation of the fibre caused by external perturbations leads to a change in the spatial arrangement of speckles in the recorded speckle pattern. The intensity distributions of speckle patterns before (I_1) and after (I_2) deformation action on the fibre are compared by measuring the correlation coefficient

$$\rho_{12} = \frac{\langle I_1 I_2 \rangle - \langle I_1 \rangle \langle I_2 \rangle}{(\langle I_1^2 \rangle - \langle I_1 \rangle^2)^{1/2} (\langle I_2^2 \rangle - \langle I_2 \rangle^2)^{1/2}} \quad (1)$$

for these signals, where the angle brackets mean averaging over spatial coordinates

$$\langle I_j \rangle = \frac{1}{S} \int I_j(x, y) \mathrm{d}x \mathrm{d}y, \ \ \langle I_1 I_2 \rangle = \frac{1}{S} \int I_1(x, y) I_2(x, y) \mathrm{d}x \mathrm{d}y;$$

x, y are the Cartesian coordinates in the CCD array plane; S is the area of the CCD array; and $j = 1, 2$.

The inset in Fig. 1 shows the mutual arrangement of the SMI end, diffusion transparency, and radiation recording plane. In the case when the SMI radiation is scattered by a diffusion transparency, we assume that each of the elements $\Delta\sigma = \Delta\xi\Delta\eta$ of the transparency area does not change the amplitude of a light wave incident on it but introduces a random addition to the light-wave phase. The linear dimensions $\Delta\xi$ and $\Delta\eta$ of the elements are comparable

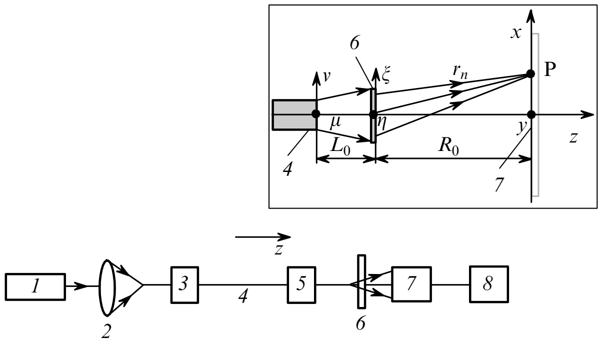

Figure 1. Scheme of the experimental setup: (*1*) He – Ne laser; (*2*) microobjective; (*3, 5*) supports; (*4*) SMI; (*5*) diffusion transparency; (*7*) video-camera CCD array (320 × 320 pixels); (*8*) PC; the inset shows the propagation of radiation through the diffusion transparency.

with the wavelength, and there is no relation between random phase additions introduced by two different elements of the diffusion transparency. By assuming that a medium between the diffusion transparency and the detection plane is homogeneous, the complex perturbation of the electromagnetic field at the point P in the detection plane, which is caused by all the elements of the transparency in the absence ($j = 1$) of external perturbations of the SMI and in their presence ($j = 2$), can be written in the form

$$V_j(x, y) \approx \sum_n \frac{1}{r_n} E_j(\xi_n, \eta_n) \exp[\mathrm{i}\varphi(\xi_n, \eta_n) + \mathrm{i}kr_n]\Delta\sigma_n,$$

where $E_j(\xi_n, \eta_n$ characterises the strength of the produced SMI wave incident on the nth element of the diffusion transparency; $\varphi(\xi_n, \eta_n)$ is the random phase addition introduced by this element of the transparency; $k = 2\pi/\lambda$; λ is the radiation wavelength; and r_n is the distance between the nth element and point P with coordinates (x, y) (Fig. 1). We will assume below that the linear dimensions of the diffusion transparency are small compared to the distance R_0 between them and the observation plane and angles between the z axis and any beams r_n are also small. Taking these assumptions into account, the product of light intensities before and after the deformation perturbation can be calculated from the expression

$$I_1(x, y)I_2(x, y) = V_1(x, y)V_1^*(x, y)V_2(x, y)V_2^*(x, y)$$

$$= \sum_l \sum_n \sum_p \sum_q E_1(\xi_l, \eta_l)E_1^*(\xi_n, \eta_n)E_2(\xi_p, \eta_p)E_2^*(\xi_q, \eta_q)$$

$$\times \Delta\sigma_l\Delta\sigma_n\Delta\sigma_p\Delta\sigma_q \exp\Big\{\mathrm{i}[\varphi(\xi_l, \eta_l) - \varphi(\xi_n, \eta_n)$$

$$+ \varphi(\xi_p, \eta_p) - \varphi(\xi_q, \eta_q)]$$

$$+ \mathrm{i}k\frac{x(\xi_l - \xi_n + \xi_\mathrm{p} - \xi_q) + y(\eta_l - \eta_n + \eta_p - \eta_q)}{R_0}\Big\}, \quad (2)$$

accurate to an insignificant factor, where l, n, p, q are the summation subscripts and '$*$' is the sign of complex conjugation. We assume here that the first- and second-order statistical moments of the intensity distribution I_1 and I_2, including mixed moments calculated for one realisation of the speckle field, remain invariable for other realisations as well obtained, for example, by replacing the diffusion transparency. From this point of view, the average of the production of intensities I_1 and I_2 should not depend on the form of the function $\varphi(\xi, \eta)$. Therefore, the average will contain only contributions of the terms in the sum in (2) for which the argument of the imaginary exponential is zero. This is possible only in cases when $l = n$ and $p = q$ or $l = q$ and $n = p$. Taking both these possibilities into account and passing in (2) from summation to integration, we obtain

$$\langle I_1 I_2 \rangle = \int_\sigma E_1 E_1^* \mathrm{d}\xi\mathrm{d}\eta \int_\sigma E_2 E_2^* \mathrm{d}\xi\mathrm{d}\eta + \left| \int_\sigma E_1 E_2^* \mathrm{d}\xi\mathrm{d}\eta \right|^2 . \quad (3)$$

Integration is expression (3) is performed over the entire surface σ of the diffusion transparency and, in addition, the notation of the coordinate dependence of the electric field strength E_1 and E_2 is omitted for simplicity.

The field of an electromagnetic wave incident on the surface of the diffusion transparency is a superposition of perturbations A_{jm} introduced by all the modes of a single-fibre interferometer.

$$E_j(x, y) = \sum_{m=1}^{N} A_{jm},$$

where N is the number of modes of a single-fibre interferometer.

The divergence of a light beam at the SMI output is determined by diffraction and the numerical aperture NA of the fibre. Taking into account only diffraction due to the NA smallness, we can write A_{jm} in the form

$$A_{jm}(\xi, \eta) = C_0 \int_\zeta F_m(\nu, \mu)$$

$$\times \exp\left\{\mathrm{i}\left[\varphi_m + (j-1)\delta\varphi_m\right] + \mathrm{i}k\frac{\xi\nu + \eta\mu}{L_0}\right\}\mathrm{d}\nu\mathrm{d}\mu, \quad (4)$$

where ν and μ are the Cartesian coordinates in the plane of the SMI end (Fig. 1); C_0 is a constant amplitude–phase coefficient depending on the distance L between the output end of the SMI and the diffusion transparency plane; $F_m(\nu, \mu)$ is the mode field amplitude distribution in the SMI cross section; φ_m is the phase of the mth mode at the SMI output; $\delta\varphi_m$ is the mode phase shift, which in the case of a change in the SMI length caused by external perturbations of the interferometer can be calculated from the expression [4]

$$\delta\varphi_m = k\left(\frac{\mathrm{d}n_m}{\mathrm{d}L} + n_m\right)\mathrm{d}L; \quad (5)$$

L is the fibre length; n_m is the effective refractive index of the mth mode. Integration in expression (4) is performed over the entire surface ζ of the SMI output end.

Taking into account expression (4) and the orthogonality of mode fields in optical waveguides [5], integrals in the second term in (3) can be written in the form

$$\int_\sigma E_1(\xi, \eta)E_2^*(\xi, \eta)\mathrm{d}\xi\mathrm{d}\eta = \frac{(C_0 L_0 \lambda)^2}{4} \sum_{m=1}^{N} \exp(\mathrm{i}\delta\varphi_m)$$

$$\times \int_\zeta F_m^2(\nu, \mu)\mathrm{d}\nu\mathrm{d}\mu. \quad (6)$$

By combining (3) and (6), we obtain

$$\langle I_1 I_2 \rangle = \left(\sum_{m=1}^{N} P_m\right)^2 + \left|\sum_{m=1}^{N} P_m \exp(\mathrm{i}\delta\varphi_m)\right|^2, \quad (7)$$

where $P_m = |\frac{1}{2}C_0 L_0 \lambda|^2 \int_\zeta F_m^2(\nu, \mu)\mathrm{d}\nu\mathrm{d}\mu$ is the parameter characterising the power carried by the mth mode. Based on a similar reasoning, we can easily obtain that $\langle I_1 \rangle = \langle I_2 \rangle = \sum_{m=1}^{N} P_m$ and $\langle I_1^2 \rangle = \langle I_2^2 \rangle = 2(\sum_{m=1}^{N} P_m)^2$. By substituting these moments into (1), we obtain the expression

$$\rho_{12} = \left|\sum_{m=1}^{N} P_m \exp(\mathrm{i}\delta\varphi_m)\right|^2 \left(\sum_{m=1}^{N} P_m\right)^{-2} \quad (8)$$

for the correlation coefficient of light-field intensities in the detection plane.

According to expression (8), the correlation coefficient is determined by the phase difference between the SMI modes and its value characterises the external perturbation producing this variation.

Curve (*1*) in Fig. 2 presents the dependence $\rho_{12}(\Delta L)$ calculated by using expressions (5), (8) and results of paper [4] for the case when external perturbations cause a change in the fibre length. The calculations were performed for a silica fibre with the numerical aperture NA = 0.11 and the core diameter 10 μm. It was assumed that the power in each of the guided 0.63-μm LP modes was the same.

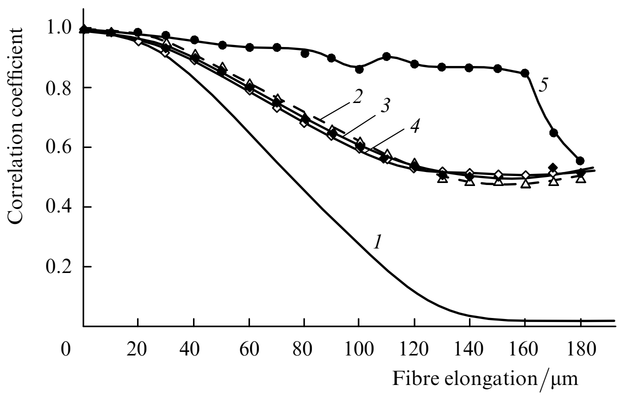

Figure 2. Calculated (*1*) and experimental (*2–5*) dependences of the correlation coefficient on the fibre elongation; curves (*2–4*) are obtained by using a diffusion transparency upon the axial stretching of the fibre, and curve (*5*) – in the absence of a diffusion transparency.

It follows from the presented results that there exists the characteristic elongation ΔL_c of the SMI fibre ($\Delta L_c = 160$ μm for the fibre used here) at which a complete decorrelation of the intermode interference patterns and the reference image is observed. In this case, a further correlation processing is impossible. If $\Delta L < \Delta L_c$c, the correlation coefficient can be quite high for calculating the fibre elongation from its measured value.

The conclusions made above were verified experimentally by using the setup shown in Fig. 1. The axial deformation of the sensitive part of the fibre in SMI (*4*) is obtained by moving support (*5*) along the z axis. A light beam emerging from the SMI is directed into diffusion transparency (*6*) and then to CCD array (*7*) of a video camera coupled with PC (*8*).

The radius of a light spot on the transparency was ~200 μm, the average size of a grain of the etched surface of a glass plate used as a diffusion transparency was ~1 μm. The average speckle size in the CCD array plane was ~60 μm, greatly exceeding the array pixel size (12 μm) and remaining much smaller than the array linear size (~4 mm), which, according to the results of paper [6], satisfies the optimal conditions of recording a speckle pattern. To exclude the influence of mutual transverse and longitudinal displacements of the emitting fibre end, diffusion transparency and video camera on the result of measurements, all these elements were mounted in a common rigid housing.

The dependence $\rho_{12}(\Delta L)$ was measured by using different reference images of the speckle pattern. Curves (*2–4*) in Fig. 2 present the results obtained. One can see that dependences $\rho_{12}(\Delta L)$ coincide as a whole. The observed scatter is explained by the statistic nature of the parameter being measured and by the noise appearing upon the detection of a light beam of a limited aperture transmitted through a random phase plate [7]. This scatter in our study is a source of errors in the measurement of ΔL. Figure 2 shows that this error depends on the fibre elongation ΔL and can achieve ±10 μm in the linear region of the dependence $\rho_{12}(\Delta L)$. The range of measurements is limited by the value ΔL_c; in our case, it is ~160 μm, exceeding twice that in the case of substantially multimode fibres [2]. The measurement range can be increased by rewriting reference images [2]. The results presented in Fig. 2 also show that, when the SMI fibre elongation exceeds 160 μm, small oscillations of the correlation coefficient are observed, in accordance with the behaviour of the calculated curve.

It follows from Fig. 2 that the main difference of the calculated curve from experimental curves (*2–4*) is that the latter have a constant component. This can be explained by the fact that the divergence of a light beam due to the nonzero NA was neglected in calculations. Finally, this approximation allowed us to use the orthogonality of mode fields in the derivation of expression (6). It seems that the orthogonality condition in the far-field radiation zone of the SMI is fulfilled not completely, and as a result, the correlation coefficient for the reference and signal intensity distributions does not vanish.

For comparison, we present in Fig. 2 [curve (*5*)] the dependence $\rho_{12}(\Delta L)$ measured without the diffusion transparency. One can see that in this case the result of correlation processing exhibits strong fluctuations.

Thus, the results obtained in the study demonstrate the possibility of processing the interference pattern for several (more that two) spatially inhomogeneous coherent light beams transmitted simultaneously through a diffusion transparency. This approach eliminates the influence of polarisation mode beats and higher-order modes, which are typical for two-mode SMIs, and allows measurements of the elongation of the sensitive part of a fibre in single-fibre multimode interferometers by exciting several modes in them. The measurement error of fibre elongation for the SMI with five guided modes is ±10 μm in the linear region of the operation characteristic.

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