

LAYERED PERIODIC STRUCTURES

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Propagation of light in a one-dimensional photonic crystal: analysis by the Floquet – Bloch function method

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***Abstract*. The problem of light propagation in a layered periodic medium with a step refractive index profile is considered. The exact solution of this problem is presented in the form of a nonuniform wave, for which the field amplitude distribution is written in an analytic form and the shape of its wave surfaces is determined. The reflection coefficient is obtained for a plane wave incident from the homogeneous medium at the boundary of a semi-infinite layered periodic medium and exciting a Floquet – Bloch wave. Critical conditions are found in which the Floquet – Bloch wave is infinite in the semi-infinite layered medium and exponentially decays in the adjacent homogeneous medium. Dispersion equations and field distributions of surface waves (modes) localised near the boundary of the semi-infinite layered medium are derived and conditions of their appearance are determined. The boundaries of admissible values of the refractive index of the adjacent medium depending on the parameters of the layered periodic medium are established. Dispersion relations for the surface modes in the semi-infinite layered periodic medium (bounded by a system of coupled waveguides) are obtained upon changing the thickness of the boundary layer.**

Keywords: *layered periodic structure, coupled waveguides, photonic crystal, Floquet – Bloch waves.*

1. Introduction

A medium, whose properties are constant on each plane perpendicular to the fixed direction, is called a layered medium. The problem of light propagation in a layered medium has been long known. As an example, recall the work of Stokes [1] written more than 150 years ago and devoted to the study of light scattering in a medium consisting of N layers, as well as work of Rayleigh [2] devoted to the study of the light reflection phenomenon in crystals. The history of this issue is described in detail in review [3]. Multilayered systems consisting of thin plane – parallel films play an important role in optics. The methods for their deposition are well elaborated and the composition and thickness of some layers can be controlled with a high accuracy [4, 5]. The simplest example of practical realisation of such a system is an ordinary multilayer dielectric interference mirror, which, under certain conditions, can be used to filter certain spectral regions [6]. Another example is a multilayer structure consisting of a large number of coupled planar waveguides [7] or a grating of channel waveguides produced in a planar waveguide film lying on a substrate with a lower refractive index. Interest in the problem of light propagation, amplification and generation in a system of tunnel-coupled waveguides is explained, first of all, by the practical need for the improvement of the radiation power and quality of fibre and semiconductor lasers [8, 9].

The system of tunnel-coupled waveguides is called homogeneous if it is formed by the same equidistantly spaced waveguides, whose light propagation constants are independent of the longitudinal and transverse coordinates. The light propagation in homogeneous systems of channel waveguides was studied earlier in [10 – 16]. Note, in particular, that a homogeneous system of coupled waveguides can be considered as a one-dimensional photonic crystal where the light propagation is conveniently described by the Floquet – Bloch waves [16]. If the structure period is comparable with the wavelength of optical radiation, the so-called photonic forbidden bands (frequency regions or angles of incidence in which the propagation of radiation into the depths of the structure is forbidden) appear in a photonic crystal. In the absence of absorption the appearance of forbidden bands is caused by coherent interference of waves multiply reflected at interlayer boundaries.

It has been shown recently [17, 18] that one-dimensional periodic structures (photonic crystals) can provide total reflection of radiation in a given frequency range for all angles of incidence and polarisation states, which explains an increasing interest in the study of propagation of optical radiation in one-dimensional periodic structures [19 – 25]. To describe this propagation, the Floquet – Bloch approach, the theory of coupled waves or the transfer matrix method are usually used. Among these methods the standard theory of coupled modes [26] is the most physically illustrative and in a limited number of cases yields simple analytic results. Unfortunately, this theory contains a number of omissions [26 – 28], which often lead to incorrect results.

The rigorous theory of coupled modes [29, 30], taking into account all diffraction waves in periodic structures, leads to a system of equations consisting of the infinite number of differential equations of the second order and solvable numerically with the retention of a limited number

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of diffraction waves. In this sense, it is similar to the approach, in which Floquet – Bloch waves are used, and at present it is well elaborated to solve specific problems but not to determine laws of wave propagation in periodic structures.

The matrix method for one-dimensional periodic structures presented in [31, 32] and based on the Bloch theory [33] is exact. In particular, the use of this method allows one to study the optical surface waves in restricted multilayer structures with a periodic distribution of the refractive index [34, 35]. The investigation of surface waves in spatially-limited multilayer structures is important for determining the necessary surface properties providing the elimination of possible losses caused by emission in surface modes [24, 36, 37] as well as for their application as sensors [38 – 40].

Unfortunately, the use of complex matrices often means numerical calculations and does not demonstrate physically the processes being studied [41, 42]. This is explained by the fact that the general expressions for the field distribution and eigenvalue equations for transverse electric (TE) and magnetic (TM) modes are derived only for particular cases of a symmetric layered periodic medium [43], and total dispersion curves for the surface electromagnetic modes are known only for the case, when the tunnel coupling of waveguide layers of a structure is realised by neglecting the change in the parameters of the first layer (cell) of a periodic medium [44]. In this case, numerical methods are used. For example, in [45] dispersion relations are derived for the surface modes upon changing the parameters of the first cell calculated by the supercell method.

From this point of view, to describe the propagation of electromagnetic waves in one-dimensional photonic crystals, it is desirable to develop an analytic theory providing more exact results than the standard theory of coupled modes in the case of a high-contrast refractive index. In this connection, the theoretical description of light propagation in a one-dimensional crystal without the use of complex matrix formalism is developed in this paper. The aim of the paper is to demonstrate that in the simplest case of a periodic medium consisting of a pair of alternating layers of transparent materials with different refractive indices an exact solution of a wave equation can be derived in an explicit analytic form.

2. A Floquet – Bloch wave in an infinite layered periodic medium

To describe processes of light propagation in a homogeneous system of coupled waveguides, it can be represented (Fig. 1a) as a one-dimensional layered periodic medium composed of alternating f- and s-type layers of thickness h and s with the refractive indices n_f and n_s and the refractive index profile

$$n(x) = \begin{cases} n_f, & \Lambda m < x < \Lambda m + h, \quad m = 0, \pm 1, \pm 2, ..., \\ n_s, & \Lambda m + h < x < \Lambda(m+1), \quad \Lambda = h + s. \end{cases} \tag{1}$$

For a TE wave with the frequency ω polarised along the y axis and propagating in the positive direction of the z axis, the field distribution can be written in the form

$$E(x,z,t) = E(x,z)\exp(\mathrm{i}\omega t) = E(x)\exp[\mathrm{i}(\omega t - \beta z)], \tag{2}$$

where β is the wave propagation constant in the direction z; t is the time. Inside the system layers, the function $E(x)$ satisfies the Helmholtz equation:

$$\frac{\mathrm{d}^2 E(x)}{\mathrm{d}x^2} - E(x)[\beta^2 - k_0^2 n^2(x)] = 0, \tag{3}$$

where $k_0 = \omega/c$ is a wave vector of radiation in vacuum. Considering a layered periodic medium as a one-dimensional crystal formed by the repetition of a cell from f and s layers, the electric field distribution inside each homogeneous layer of the mth crystal cell can be represented as a sum of an incident and reflected plane wave with complex amplitudes a_m, b_m and c_m, d_m in f and s cell layers, respectively (according to the Brillouin concept these wave are called partial waves):

$$E(x,z) = \begin{cases} \{a_m \exp[-\mathrm{i}\kappa_f(x - m\Lambda)] + b_m \exp[\mathrm{i}\kappa_f(x - m\Lambda)]\} \exp(-\mathrm{i}\beta z), & \Lambda m < x < \Lambda m + h, \\ \kappa_f = [k_0^2 n_f^2 - \beta^2]^{1/2}, \\ \{c_m \exp[-\mathrm{i}\kappa_s(x - m\Lambda)] + d_m \exp[\mathrm{i}\kappa_s(x - m\Lambda)]\} \exp(-\mathrm{i}\beta z), & \Lambda m + h < x < \Lambda(m+1), \\ \kappa_s = (k_0^2 n_s^2 - \beta^2)^{1/2}, \end{cases} \tag{4}$$

where κ_f and κ_s are propagation constants of plane waves in the direction of the x axis for f and s layers of the medium, respectively. If the conditions $n_f > n_s > 0$ and $k_0 n_f > \beta > 0$ are fulfilled, the propagation constant κ_f can take only real values, while κ_s – both real (in the case of radiation coupling of f layers through s layers of the medium, when $k_0 n_s > \beta$) and imaginary values (in the case of tunnel coupling, when $k_0 n_s < \beta$). In the case of tunnel coupling it is convenient to use a decay constant γ_s so that $\kappa_s = \mathrm{i}\gamma_s$, $\gamma_s > 0$. The complex amplitudes a_m, b_m, c_m and d_m of plane waves are coupled by the continuity condition at the boundaries of layers; for $x = \Lambda m$

$$a_m + b_m = c_{m-1}\exp(-\mathrm{i}\kappa_s\Lambda) + d_{m-1}(\exp \mathrm{i}\kappa_s\Lambda),$$

$$\kappa_f(a_m - b_m) = \kappa_s[c_{m-1}\exp(-\mathrm{i}\kappa_s\Lambda) - d_{m-1}(\exp \mathrm{i}\kappa_s\Lambda)], \tag{5a}$$

for $x = \Lambda m + h$

$$a_m \exp(-\mathrm{i}\kappa_f h) + b_m \exp(\mathrm{i}\kappa_f h) = c_m \exp(-\mathrm{i}\kappa_s h) + d_m \exp(\mathrm{i}\kappa_s h),$$

$$\kappa_f[a_m \exp(-\mathrm{i}\kappa_f h) - b_m \exp(\mathrm{i}\kappa_f h)] \tag{5b}$$

$$= \kappa_s[c_m \exp(-\mathrm{i}\kappa_s h) - d_m \exp(\mathrm{i}\kappa_s h)].$$

For a periodic medium according to the Floquet (Bloch) theorem, the light propagation can be represented as a Floquet – Bloch wave (Bloch wave)

$$E(x,z,t) = E_K(x)\exp(-\mathrm{i}Kx)\exp[\mathrm{i}(\omega t - \beta z)],$$

$$E_K(x + \Lambda m) = E_K(x), \quad m = 0, \pm 1, ..., \tag{6}$$

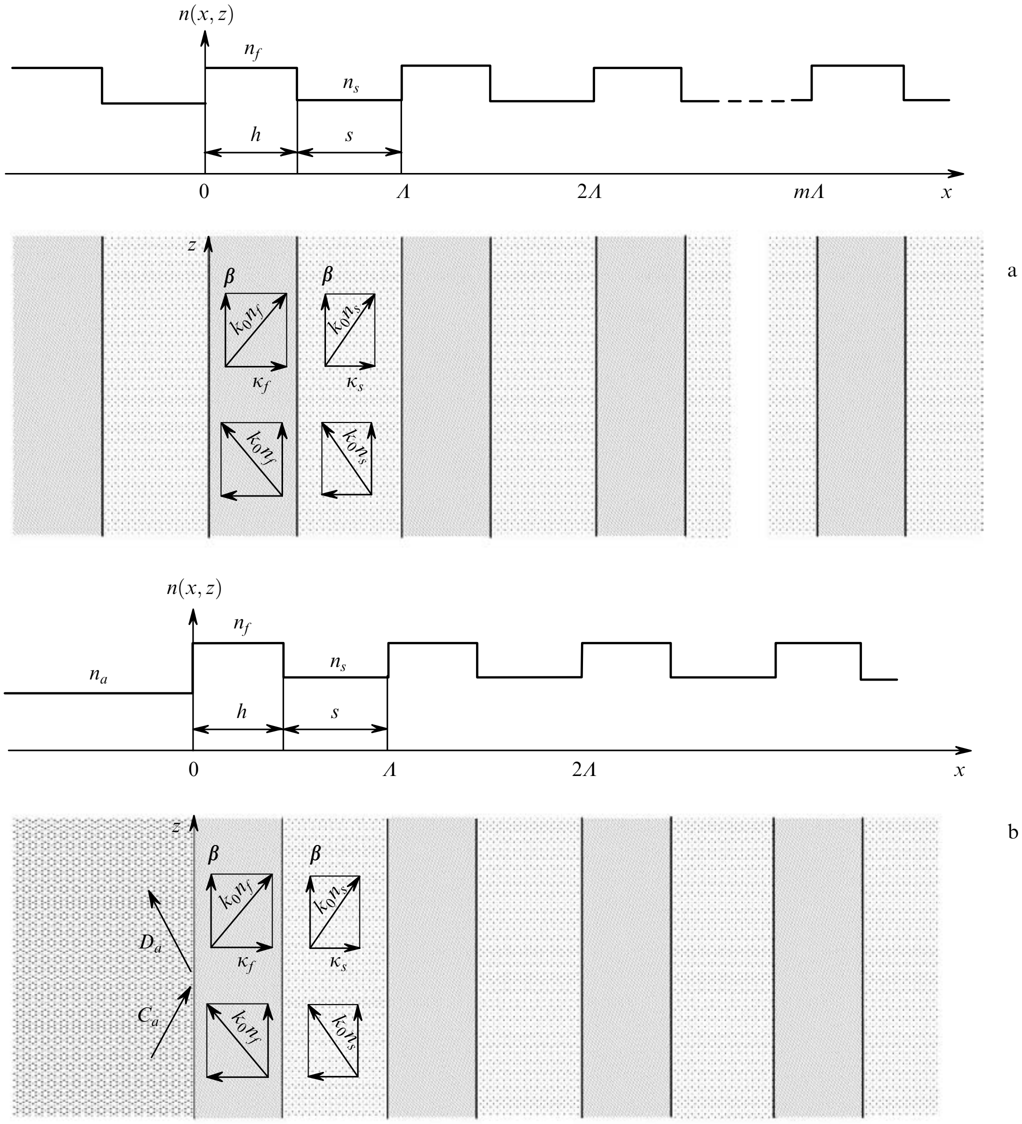

Figure 1. Schematic representation of the infinite (a) and semi-infinite (b) layered periodic medium.

where $E_K(x)$ is a periodic function depending on the K constant whose period is equal to the period Λ of the structure. The K constant known as a Bloch wave number can be derived from a dispersion relation [27, 28]

$$\cos K\Lambda = \cos(\kappa_f h)\cos(\kappa_s s) - \frac{1}{2}\left(\frac{\kappa_s}{\kappa_f} + \frac{\kappa_f}{\kappa_s}\right)$$

$$\times \sin(\kappa_f h)\sin(\kappa_s s), \quad \beta < k_0 n_s, \tag{7a}$$

$$\cos K\Lambda = \cos(\kappa_f h)\cosh(\gamma_s s) + \frac{1}{2}\left(\frac{\gamma_s}{\kappa_f} - \frac{\kappa_f}{\gamma_s}\right)$$

$$\times \sin(\kappa_f h)\sinh(\gamma_s s), \quad k_0 n_s < \beta < k_0 n_f. \tag{7b}$$

In those cases when according to (7a) and (7b) the inequality $|\cos K\Lambda| < 1$ is fulfilled, the obtained quantity of K is real and corresponds to waves propagating without a decay. If the right-hand side of Eqns (7a) and (7b) with respect to modulus exceeds unity, the quantity of K becomes complex and we deal with a decaying wave.

Note that a wave decribed by expression (6) is an inhomogeneous wave and in this case not only data on field amplitude distribution in a cell of a periodic medium but also on the shape of the wave surface are important. At the same time, authors of known papers restrict themselves to the calculation of the phase $K\Lambda$ of the wave determining the band edges of a periodic medium, while the wave front of the wave is not calculated. To calculate the wave front it is necessary to perform additional investigations.

Let us take advantage of periodicity condition (6) valid for the Floquet–Bloch waves and rewrite Eqn (5a) in the form

$$\exp(-\mathrm{i}K\Lambda)(a_m + b_m) = c_m \exp(-\mathrm{i}\kappa_s\Lambda) + d_m \exp(\mathrm{i}\kappa_s\Lambda), \tag{8}$$

$$\kappa_f \exp(-\mathrm{i}K\Lambda)(a_m - b_m) = \kappa_s[c_m \exp(-\mathrm{i}\kappa_s\Lambda) - d_m \exp(\mathrm{i}\kappa_s\Lambda)].$$

By considering Eqns (5b) and (8) together, for $\kappa_f \neq 0$ and $\kappa_s \neq 0$ we can derive two equations relating complex amplitudes a_m and b_m of partial waves inside the f layer,

$$(\kappa_s+\kappa_f)a_m\sin\frac{1}{2}(\kappa_f h+\kappa_s s-K\Lambda)$$
$$=(\kappa_s-\kappa_f)b_m\sin\frac{1}{2}(\kappa_f h-\kappa_s s+K\Lambda)\exp(\mathrm{i}\kappa_f h),$$
$$(\kappa_s-\kappa_f)a_m\sin\frac{1}{2}(\kappa_f h-\kappa_s s-K\Lambda)$$
$$=(\kappa_s+\kappa_f)b_m\sin\frac{1}{2}(\kappa_f h+\kappa_s s+K\Lambda)\exp(\mathrm{i}\kappa_f h), \tag{9}$$

and two analogous equations relating complex amplitudes c_m and d_m of partial waves inside the s layer:

$$(\kappa_s+\kappa_f)c_m\sin\frac{1}{2}(\kappa_f h+\kappa_s s-K\Lambda)$$
$$=(\kappa_s-\kappa_f)d_m\sin\frac{1}{2}(\kappa_f h-\kappa_s s-K\Lambda)\exp[\mathrm{i}\kappa_s(s+2h)],$$
$$(\kappa_s-\kappa_f)c_m\sin\frac{1}{2}(\kappa_f h-\kappa_s s+K\Lambda)$$
$$=(\kappa_s+\kappa_f)d_m\sin\frac{1}{2}(\kappa_f h+\kappa_s s+K\Lambda)\exp[\mathrm{i}\kappa_s(s+2h)]. \tag{10}$$

The case when all the sines in Eqns (9) and (10) are simultaneously equal to zero is special and requires a separate consideration. In all other cases as is expected the requirement of a simultaneous fulfillment of conditions (9) or (10) results in dispersion condition (7). Besides, one can see from (9) and (10) that the ratios of the complex amplitudes and phase shifts for waves inside each layer are independent of the number m of the medium cell. Equations (5b) and (8) allow one to express amplitudes c_m and d_m in terms of a_m and b_m and then by using coupling equation (9) to obtain the following analytic expressions for complex amplitudes of partial plane waves in f and s layers of a periodic medium:

$$a_m=A\exp(\mathrm{i}\phi_0)\exp(\mathrm{i}\kappa_f h/2)\exp(-\mathrm{i}K\Lambda m),\ \ m=0,\ \pm1,\ ...,$$
$$b_m=B\exp(\mathrm{i}\phi_0)\exp(-\mathrm{i}\kappa_f h/2)\exp(-\mathrm{i}K\Lambda m),$$
$$c_m=C\exp(\mathrm{i}\phi_0)\exp[\mathrm{i}\kappa_s(s+2h)/2]\exp[-\mathrm{i}K\Lambda(2m+1)/2],$$
$$d_m=D\exp(\mathrm{i}\phi_0)\exp[-\mathrm{i}\kappa_s(s+2h)/2]\exp[-\mathrm{i}K\Lambda(2m+1)/2], \tag{11}$$

where ϕ_0 is the initial phase of a wave. The amplitude coefficients A, B, C, D, which are real quantities in the case of real κ_f, κ_s and K, can be written in the form:

$$A=A_0\sin\frac{1}{2}(\kappa_f h-\kappa_s s+K\Lambda)\sin\frac{1}{2}(\kappa_f h+\kappa_s s+K\Lambda),$$
$$B=A_0\frac{\kappa_s-\kappa_f}{\kappa_s+\kappa_f}\sin\frac{1}{2}(\kappa_f h-\kappa_s s+K\Lambda)$$
$$\times\sin\frac{1}{2}(\kappa_f h-\kappa_s s-K\Lambda),$$
$$C=A_0\frac{\kappa_s+\kappa_f}{2\kappa_s}\sin\frac{1}{2}(\kappa_f h+\kappa_s s+K\Lambda)\sin\kappa_f h,$$
$$D=A_0\frac{\kappa_s-\kappa_f}{2\kappa_s}\sin\frac{1}{2}(\kappa_f h-\kappa_s s+K\Lambda)\sin\kappa_f h, \tag{12}$$

where A_0 is an arbitrary constant playing the role of the Floquet–Bloch amplitude.

It is appropriate to present here the relations for the squares of amplitude coefficients A, B and C, D, derived from expressions (9)–(12):

$$\frac{B^2}{A^2}=\frac{\cos\kappa_s s-\cos(\kappa_f h-K\Lambda)}{\cos\kappa_s s-\cos(\kappa_f h+K\Lambda)},$$
$$\frac{D^2}{C^2}=\frac{\cos\kappa_f h-\cos(\kappa_s s-K\Lambda)}{\cos\kappa_f h-\cos(\kappa_s s+K\Lambda)}. \tag{13}$$

If for the Floquet–Bloch wave being considered, the time averaged value of the transverse component of the energy flux (directed along the x axis) is represented as a difference of corresponding partial waves in f and s layers, from the continuity condition of this component of the flux at interlayer boundaries

$$\kappa_f A^2\left(1-\frac{B^2}{A^2}\right)=\kappa_s C^2\left(1-\frac{D^2}{C^2}\right) \tag{14}$$

and relations (13) one can readily derive a relation for the squares of amplitude coefficients A and C:

$$\frac{C^2}{A^2}=\frac{\kappa_f}{\kappa_s}\frac{\sin\kappa_f h}{\sin\kappa_s s}\frac{\cos\kappa_f h-\cos(\kappa_s s+K\Lambda)}{\cos\kappa_s s-\cos(\kappa_f h+K\Lambda)}. \tag{15}$$

It follows from (4), (9)–(12) that the amplitude coefficients of reflection r_{ab}, r_{cd} and r_{ba}, r_{dc} of partial waves at upper and lower boundaries, respectively, are related by the expressions:

$$r_{ab}=|r_{ab}|\exp(\mathrm{i}\phi_{ab})=\frac{1}{r_{ba}^*}=\frac{\exp(\mathrm{i}\phi_{ba})}{|r_{ba}|}=\frac{b_m\exp(\mathrm{i}\kappa_f h)}{a_m\exp(-\mathrm{i}\kappa_f h)}$$
$$=\frac{\kappa_s-\kappa_f}{\kappa_s+\kappa_f}\frac{\sin\frac{1}{2}(\kappa_f h-\kappa_s s-K\Lambda)}{\sin\frac{1}{2}(\kappa_f h+\kappa_s s+K\Lambda)}\exp(\mathrm{i}\kappa_f h), \tag{16a}$$

$$r_{cd}=|r_{cd}|\exp(\mathrm{i}\phi_{cd})=\frac{1}{r_{dc}^*}=\frac{\exp(\mathrm{i}\phi_{dc})}{|r_{dc}|}=\frac{d_m\exp[\mathrm{i}\kappa_s(s+h)]}{c_m\exp[-\mathrm{i}\kappa_s(s+h)]}$$
$$=\frac{\kappa_s-\kappa_f}{\kappa_s+\kappa_f}\frac{\sin\frac{1}{2}(\kappa_f h-\kappa_s s+K\Lambda)}{\sin\frac{1}{2}(\kappa_f h+\kappa_s s+K\Lambda)}\exp(\mathrm{i}\kappa_s s). \tag{16b}$$

One can see from (16) that the product of moduli of reflection coefficients at the upper and lower boundaries of each layer is equal to unity. In addition, the phase shift appearing upon reflection of the partial wave at the layer boundary is determined (with the accuracy to the term $2\pi m, m=0,\ 1, ...$) by the layer phase thickness equal to $\kappa_f h$ in the case of the f layer and to $\kappa_s s$ in the case of the s layer. By using this, we can write the conditions for the transverse resonance for partial waves in f and s layers of the medium:

$$2\kappa_f h=\phi_{ab}+\phi_{ba}+2\pi m_f=2\phi_{ab}+2\pi m_f$$
$$=2\phi_{ba}+2\pi m_f,\ \ m_f=0,\ 1,\ ...\,, \tag{17}$$

$$2\kappa_s s = \phi_{cd} + \phi_{dc} + 2\pi m_s = 2\phi_{cd} + 2\pi m_s$$
$$= 2\phi_{dc} + 2\pi m_f, \quad m_s = 0,\, 1,\, \ldots\,.$$

Taking into account the phase shift determined for the complex amplitudes of partial waves with the phase thickness of each layer, it is convenient to use local coordinates $\xi_f = x - h/2 - \Lambda m$ and $\xi_s = x - s/2 - h - \Lambda m$ measured from the middles of the corresponding layers when representing the field distribution in the mth cell of the structure under study:

where

$$\bar{C} = \frac{A_0\sin(\kappa_f h)}{2\gamma_s}\left\{\frac{1}{2}(\kappa_f^2+\gamma_s^2)[\cosh(\gamma_s s) - \cos(\kappa_f h + K\Lambda)]\right\}^{1/2};$$

$$\phi_c = \arctan\left[\left(\gamma_s\tanh\frac{\gamma_s s}{2} - \kappa_f\tan\frac{\kappa_f h + K\Lambda}{2}\right)\right.$$
$$\left.\times\left(\kappa_f\tanh\frac{\gamma_s s}{2} + \gamma_s\tan\frac{\kappa_f h + K\Lambda}{2}\right)^{-1}\right].$$

$$E(x,z) = \begin{cases} [A\exp(-\mathrm{i}\kappa_f\xi_f) + B\exp(\mathrm{i}\kappa_f\xi_f)]\exp[\mathrm{i}(\phi_0 - K\Lambda m - \beta z)] = E_f(\xi_f)\exp[-\mathrm{i}\phi_f(\xi_f)]\exp[\mathrm{i}(\phi_0 - K\Lambda m - \beta z)], \\ E_f(\xi_f) = [A^2 + B^2 + 2AB\cos(2\kappa_f\xi_f)]^{1/2}, \qquad -h/2 < \xi_f < h/2, \\ \phi_f(\xi_f) = \arctan\left[\dfrac{A-B}{A+B}\tan(\kappa_f\xi_f)\right], \qquad -s/2 < \xi_s < s/2, \\ [C\exp(-\mathrm{i}\kappa_s\xi_s) + D\exp(\mathrm{i}\kappa_s\xi_s)]\exp\{\mathrm{i}[\phi_0 - K\Lambda(m+1/2) - \beta z]\} = E_s(\xi_s)\exp[-\mathrm{i}\phi_s(\xi_s)]\exp\{\mathrm{i}[\phi_0 - K\Lambda(m+1/2) - \beta z]\}, \\ E_s(\xi_s) = [C^2 + D^2 + 2CD\cos(2\kappa_s\xi_s)]^{1/2}, \quad \phi_s(\xi_s) = \arctan\left[\dfrac{C-D}{C+D}\tan(\kappa_s\xi_s)\right]. \end{cases} \tag{18}$$

Here, the functions $E_f(\xi_f)$ and $E_s(\xi_s)$ describe the field amplitude distribution of the Floquet–Bloch wave in f and s layers of the mth cell, while the functions $\phi_f(\xi_f)$, $\phi_s(\xi_s)$ – the shape of wave surfaces. The surfaces of the constant amplitude specified by the conditions

$$E_f(\xi_f) = \text{const}, \quad E_s(\xi_s) = \text{const}, \tag{19}$$

and surface of the constant phase

$$\beta z + \phi_f(\xi_f) + K\Lambda m = \text{const},$$
$$\beta z + \phi_s(\xi_s) + K\Lambda(m + 1/2) = \text{const} \tag{20}$$

do not coincide, i.e. the Floquet–Bloch wave is inhomogeneous.

Equations (2)–(4) together with (9)–(20) determine completely the Floquet–Bloch wave freely propagating in an infinite layered periodic medium.

To describe the light propagation in an infinite homogeneous system of tunnel-coupled waveguides, it is necessary to consider the case when κ_s is an imaginary quantity. For the imaginary $\kappa_s = \mathrm{i}\gamma_s$ and real κ_f, K quantities, the coefficients A and B, according to Eqns (12) are as before real:

$$A = \frac{A_0}{2}[\cosh(\gamma_s s) - \cos(\kappa_f h + K\Lambda)],$$
$$B = \frac{A_0}{4}\left(\frac{\kappa_f}{\gamma_s} + \frac{\gamma_s}{\kappa_f}\right)\sin(\kappa_f h)\sinh(\gamma_s s), \tag{21a}$$

while coefficients C and D become complex:

$$C = D^* = \frac{A_0\sin\kappa_f h}{2\gamma_s}\left\{\left[\kappa_f\sinh\frac{\gamma_s s}{2}\cos\frac{\kappa_f h + K\Lambda}{2}\right.\right.$$
$$\left.+\gamma_s\sin\frac{\kappa_f h + K\Lambda}{2}\cosh\frac{\gamma_s s}{2}\right] + \mathrm{i}\left[\gamma_s\sinh\frac{\gamma_s s}{2}\cos\frac{\kappa_f h + K\Lambda}{2}\right.$$
$$\left.\left.-\kappa_f\sin\frac{\kappa_f h + K\Lambda}{2}\cosh\frac{\gamma_s s}{2}\right]\right\} = \bar{C}\exp(\mathrm{i}\phi_c), \tag{21b}$$

In this case, when writing the field distribution in the mth cell of the structure, one can use local coordinates $\xi_f = x - h/2 - \Lambda m$ and $\xi_s = x - s/2 - h - \Lambda m$, measured from the middles of the corresponding layers. Thus, the field amplitude distribution in the f layer is described by the same functions as in (18). The field amplitude distribution in the s layer can be written in the form:

$$E(x,z) = \bar{C}[\exp(\gamma_s\xi_s + \mathrm{i}\phi_c) + \exp(-\gamma_s\xi_s - \mathrm{i}\phi_c)]$$
$$\times\exp\{\mathrm{i}[\phi_0 - K\Lambda(m+1/2) - \beta z]\} = E_s(\xi_s)$$
$$\times\exp[\mathrm{i}\phi_s(\xi_s)]\exp\{\mathrm{i}[\phi_0 - K\Lambda(m+1/2) - \beta z]\}, \tag{22}$$
$$E_s(\xi_s) = \bar{C}\{2[\cosh(2\gamma_s\xi_s) + \cos(2\phi_c)]\}^{1/2},$$
$$\phi_s(\xi_s) = \arctan[\tan\phi_c\tanh(\gamma_s\xi_s)].$$

3. Reflection of a plane wave at the boundary of a semi-infinite layered periodic medium

Consider a semi-infinite homogeneous medium (Fig. 1b) with the refractive index n_a adjacent with a semi-infinite layered periodic medium composed of alternating f and s layers with the refractive indices n_f and n_s (for definiteness we assume below that the condition $n_f > n_s > n_a$ is fulfilled). The refractive index profile of this medium is

$$n(x) = \begin{cases} n_f, & \Lambda m < x < \Lambda m + h, \; m = 0,\, 1,\, 2\, ,\ldots, \\ n_s, & \Lambda m + h < x < \Lambda(m+1), \; \Lambda = h + s, \\ n_a, & x < 0. \end{cases} \tag{23}$$

According to the model being considered, in a homogeneous region $x < 0$ at a propagation constant $\beta < k_0 n_a$, the solution of Eqn (3) can be represented as a sum of the incident and reflected homogeneous plane waves

$$E(x,z,t) = E(x)\exp[\mathrm{i}(\omega t - \beta z)] = [\tilde{C}_a\exp(-\mathrm{i}\kappa_a x)$$
$$+ \tilde{D}_a\exp(\mathrm{i}\kappa_a x)]\exp[\mathrm{i}(\omega t - \beta z)] \tag{24}$$

with complex amplitudes $\tilde{C}_a = C_a \exp(\mathrm{i}\phi_c), \tilde{D}_a = D_a \exp(\mathrm{i}\phi_d)$ (where C_a, D_a, ϕ_c, ϕ_d are real quanitites) and with the transverse propagation constant $\kappa_a = (k_0^2 n_a^2 - \beta^2)^{1/2}$. For $\beta > kn_a$ the solution of Eqn (3) is an inhomogeneous wave

$$E(x,z,t) = \tilde{F}_a \exp(\gamma_a x) \exp[\mathrm{i}(\omega t - \beta z)], \quad \tilde{F}_a = F_a \exp(\mathrm{i}\phi_f), \tag{25}$$

whose amplitude is maximal at the boundary $x = 0$ and exponentially decreases with the decay coefficient $\gamma_a = (\beta^2 - k_0^2 n_a^2)^{1/2}$ as it moves away from it. The relation of the coefficients C_a, D_a (or F_a) with the amplitude coefficients A, B, C, D of the Floquet–Bloch wave excited in the layered periodic medium is set from the continuity conditions at the boundary $x = 0$:

$$\begin{aligned} &A \exp(\mathrm{i}\kappa_f h/2) + B \exp(-\mathrm{i}\kappa_f h/2) \\ &\quad = C_a \exp(\mathrm{i}\phi_c) + D_a \exp(\mathrm{i}\phi_d), \\ &-\mathrm{i}\kappa_f [A \exp(\mathrm{i}\kappa_f h/2) - B \exp(-\mathrm{i}\kappa_f h/2)] \\ &\quad = -\mathrm{i}\kappa_a [C_a \exp(\mathrm{i}\phi_c) - D_a \exp(\mathrm{i}\phi_d)]. \end{aligned} \tag{26}$$

$$|r_a| = \frac{D_a}{C_a} = \left[\frac{(\kappa_a - \kappa_f)^2 A^2 + (\kappa_a + \kappa_f)^2 B^2 + 2AB(\kappa_a^2 - \kappa_f^2)\cos\kappa_f h}{(\kappa_a + \kappa_f)^2 A^2 + (\kappa_a - \kappa_f)^2 B^2 + 2AB(\kappa_a^2 - \kappa_f^2)\cos\kappa_f h} \right]^{1/2}. \tag{29}$$

For $\cos(\kappa_f h/2) \neq 0$, the phase coefficients ϕ_c, ϕ_d have the from:

$$\begin{aligned} \phi_c &= \arctan\left[\frac{(\kappa_a + \kappa_f)A - (\kappa_a - \kappa_f)B}{(\kappa_a + \kappa_f)A + (\kappa_a - \kappa_f)B} \tan(\kappa_f h/2) \right], \\ \phi_d &= \arctan\left[\frac{(\kappa_a - \kappa_f)A - (\kappa_a + \kappa_f)B}{(\kappa_a - \kappa_f)A + (\kappa_a + \kappa_f)B} \tan(\kappa_f h/2) \right]. \end{aligned} \tag{30}$$

The difference in coefficients ϕ_d and ϕ_c determines the phase shift quantity of a plane wave upon its reflection at the boundary of a semi-infinite layered periodic medium. In the case of its complete reflection for $A^2 = B^2$, $\tilde{C}_a = \tilde{D}_a^*$ according to (29), (30).

In the cases especially interesting from the point of view of practical realisation, when $\kappa_a = \kappa_f$ $(n_a = n_f)$ or $\kappa_a = \kappa_s$ $(n_a = n_s)$, the phase shift is determined by the phase thickness $\kappa_f h$ or $\kappa_s s$ respectively. In this case, taking (13) into account, the reflection coefficient has the form:

$$r_a = \begin{cases} \left(\dfrac{B^2}{A^2}\right)^{1/2} \exp(\mathrm{i}\kappa_f h) = \left[\dfrac{\cos\kappa_s s - \cos(\kappa_f h - K\Lambda)}{\cos\kappa_s s - \cos(\kappa_f h + K\Lambda)}\right]^{1/2} \exp(\mathrm{i}\kappa_f h), & n_a = n_f, \\ \left(\dfrac{D^2}{C^2}\right)^{1/2} \exp(\mathrm{i}\kappa_s s) = \left[\dfrac{\cos\kappa_f h - \cos(\kappa_s s - K\Lambda)}{\cos\kappa_f h - \cos(\kappa_s s + K\Lambda)}\right]^{1/2} \exp(\mathrm{i}\kappa_s s), & n_a = n_f. \end{cases} \tag{31}$$

Here, the initial phase of the Floquet–Bloch wave, without the loss of generality, was set equal to zero $(\phi_0 = 0)$. In the case of real κ_f, κ_s, κ_a boundary conditions (26) are readily represented in the form of a system of equations with real quantities only:

$$\begin{aligned} &(A + B)\cos(\kappa_f h/2) = C_a \cos\phi_c + D_a \cos\phi_d, \\ &\kappa_f (A + B)\sin(\kappa_f h/2) = \kappa_a (C_a \sin\phi_c - D_a \sin\phi_d), \\ &(A - B)\sin(\kappa_f h/2) = C_a \sin\phi_c + D_a \sin\phi_d, \\ &\kappa_f (A - B)\cos(\kappa_f h/2) = \kappa_a (C_a \cos\phi_c - D_a \cos\phi_d). \end{aligned} \tag{27}$$

When solving system (27) it is easy to derive the equations

$$\begin{aligned} 4\kappa_a^2 C_a^2 &= (\kappa_a + \kappa_f)^2 A^2 + (\kappa_a - \kappa_f)^2 B^2 \\ &\quad + 2AB(\kappa_a^2 - \kappa_f^2)\cos(\kappa_f h), \\ 4\kappa_a^2 D_a^2 &= (\kappa_a - \kappa_f)^2 A^2 + (\kappa_a + \kappa_f)^2 B^2 \\ &\quad + 2AB(\kappa_a^2 - \kappa_f^2)\cos(\kappa_f h) \end{aligned} \tag{28}$$

to determine the amplitude coefficients C_a and D_a of the incident and reflected waves in a homogeneous medium.

Equations (28) agree with the continuity condition at the boundary $x = 0$ of the transverse component of the optical flux $\kappa_a(C_a^2 - D_a^2) = \kappa_f(A^2 - B^2)$. Then, the modulus of the reflection coefficient of a plane wave at the boundary of a semi-infinite layered periodic structure has the from:

4. A critical Floquet–Bloch wave in a semi-infinite layered periodic medium

Consider the critical case when the wave field occupies the entire semi-infinite layered periodic medium and exponentially decays in a homogeneous semi-infinite medium. In the region $x < 0$ the field is represented by distribution (25) and the time average from the Poynting vector projection on the x axis is zero. In this case, the Floquet–Bloch wave (critical wave) transfers the energy only along the z axis and the squares of amplitude coefficients of partial waves in periodic medium layers are pairwise equal: $A^2 = B^2$, $C^2 = D^2$. According to (13) this is possible, for example, when the Bloch wave vector is located at the edge of the forbidden band:

$$K\Lambda = \pi l, \tag{32}$$

where $l = 0, 1, \ldots$ is the order of the forbidden band. For $A = B$ and real κ_f, κ_s, according to (18) the field amplitude distribution in f layers is specified by the symmetric function $2A\cos(\kappa_f \xi_f)$ and for $A = -B$ – by the antisymmetric function $2A\sin(\kappa_f \xi_f)$. Similarly, for $C = D$ $(C = -D)$ symmetric (asymmetric) field amplitude distribution in s layers is given by the symmetric function $2C\cos(\kappa_s \xi_s)$ [antisymmetric function $2C\sin(\kappa_s \xi_s)$]. In the case $\kappa_s = \mathrm{i}\gamma_s$, to determine the field amplitude distribution in f and s layers, it is enough to substitute $\sin(\kappa_s \xi_s) \to \mathrm{i}\sinh(\gamma_s \xi_s)$ and $\cos(\kappa_s \xi_s) \to \cosh(\gamma_s \xi_s)$. While choosing the possible combinations of symmetric and antisymmetric distributions in the f and s cell layers of the medium one should take into account the order of the forbidden band under study. The field in adjacent cells should be in-phase for even l and out-of-phase for odd l.

Let us write the boundary conditions on the plane $x = 0$ by using relations (4), (11), (25) and assuming that $\phi_f = 0$:

$$[A\exp(\mathrm{i}\kappa_f h/2) + B\exp(-\mathrm{i}\kappa_f h/2)]\exp(\mathrm{i}\phi_0) = F_a,$$
$$-\mathrm{i}\kappa_f[A\exp(\mathrm{i}\kappa_f h/2) - B\exp(-\mathrm{i}\kappa_f h/2)]\exp(\mathrm{i}\phi_0) = \gamma_a F_a. \tag{33}$$

In the case of symmetric ($A = B$) or antisymmetric ($A = -B$) field distribution in f layers of the periodic layered medium, by selecting $\phi_0 = 0$ or $-\pi/2$, respectively, we derive from (33)

$$\gamma_a = \kappa_f\tan(\kappa_f h/2),\ F_a = 2A\cos(\kappa_f h/2),\ A = B,$$
$$\gamma_a = -\kappa_f\cot(\kappa_f h/2),\ F_a = 2A\sin(\kappa_f h/2),\ A = -B \tag{34}$$

for $\sin(\kappa_f h) \neq 0$.

Two different eigenvalues, $\kappa_f = \kappa_{l_+}$ and $\kappa_f = \kappa_{l_-}$, determined by condition (32) correspond to two boundaries of each of the l forbidden bands; equalities (34) are respectively fulfilled for $n_a = n_{l_+}$ and $n_a = n_{l_-}$:

$$n_a = \left[n_f^2 - \frac{\kappa_f^2}{k_0^2\cos^2(\kappa_f h/2)}\right]^{1/2},\ A = B,$$
$$n_a = \left[n_f^2 - \frac{\kappa_f^2}{k_0^2\sin^2(\kappa_f h/2)}\right]^{1/2},\ A = -B, \tag{35}$$

where eigenvalues κ_{l_+} or κ_{l_-} are chosen for the transverse wave vector κ_f. In this case, the effective refractive index $n^* = \beta/k_0$ of the Floquet–Bloch wave propagating in a semi-infinite layered medium along its layers, according to expressions (34), (35), takes the form:

$$n^* = [n_f^2\sin^2(\kappa_f h/2) + n_a^2\cos^2(\kappa_f h/2)]^{1/2},\ A = B,$$
$$n^* = [n_f^2\cos^2(\kappa_f h/2) + n_a^2\sin^2(\kappa_f h/2)]^{1/2},\ A = -B. \tag{36}$$

Let us denote them by $\kappa_f = \kappa_{l_+}$ and $\kappa_f = \kappa_{l_-}$ for $n^*_{l_+}$ and $n^*_{l_-}$, respectively. Note that for the wave under study the amplitude coefficients A, B, C and D are constant in all cells of the layered periodic medium and the exponential decay of the field amplitude is observed only in the homogeneous region for $x < 0$.

5. A surface wave at the boundary of a semi-infinite layered periodic medium

Consider the range of values κ_f, for which the condition $|\cos K\Lambda| > 1$ is fulfilled and the Bloch wave vector according to (7) becomes complex:

$$K = l(\pi/\Lambda) - \mathrm{i}K_i, \quad l = 0,\ 1,\ \ldots. \tag{37}$$

Photonic forbidden bands, whose order is determined by the parameter l (it can take any integral nonnegative values), are related to these ranges of complex values of the Bloch wave vector. In an infinite periodic medium the existence of waves with the complex values of the Bloch wave vector are forbidden. If the layered periodic medium is semi-infinite, the exponentially decaying waves exist near its boundary.

For real κ_f, κ_s (the case of radiation-coupled layers of the periodic medium), by substituting (37) into (12) one can obtain the expression

$$A = \frac{A_0}{2}[\cos(\kappa_s s) - (-1)^l\cos(\kappa_f h)\cosh(K_i\Lambda) - \mathrm{i}(-1)^l\sin(\kappa_f h)\sinh(K_i\Lambda)] = |A|\exp(\mathrm{i}\phi_a),$$
$$|A| = \frac{A_0}{2}\left\{\cos^2(\kappa_s s) + \frac{1}{2}[\cosh(2K_i\Lambda) + \cos(2\kappa_f h)] - 2(-1)^l\cos(\kappa_f h)\cos(\kappa_s s)\cosh(K_i\Lambda)\right\}^{1/2},$$
$$\phi_a = \arctan\{\sin(\kappa_f h)\sinh(K_i\Lambda) \times[\cos(\kappa_f h)\cosh(K_i\Lambda) - (-1)^l\cos(\kappa_s s)]^{-1}]\},$$
$$B = \frac{A_0}{4}\left(\frac{\kappa_f}{\kappa_s} - \frac{\kappa_s}{\kappa_f}\right)\sin(\kappa_f h)\sin(\kappa_s s) = |B|\exp(\mathrm{i}\phi_b),$$
$$\phi_b = 0\ (B > 0),\ \phi_b = \pi\ (B < 0) \tag{38}$$

for amplitude coefficients A and B (hereafter, $K_i > 0$), where B, ϕ_a, ϕ_b are real quantities and A is a complex quantity, and as follows from (13) and (17), $|A| = |B|$ and $|C| = |D|$. This allows one to represent the amplitude coefficients in the form:

$$C = \bar{C}\exp(\mathrm{i}\phi_c), D = -\bar{C}\exp(\mathrm{i}\phi_d),$$
$$\bar{C} = A_0\sin(\kappa_f h)\frac{\kappa_s + \kappa_f}{2\kappa_s}\left\{\frac{1}{2}[\cosh(K_i\Lambda) - (-1)^l \times\cos(\kappa_f h + \kappa_s s)]\right\}^{1/2} = -A_0\sin(\kappa_f h)\frac{\kappa_s - \kappa_f}{2\kappa_s} \times\left\{\frac{1}{2}[\cosh(K_i\Lambda) - (-1)^l\cos(\kappa_f h - \kappa_s s)]\right\}^{1/2},$$
$$\phi_c = -\arctan\left(\tanh\frac{K_i\Lambda}{2}\cot\frac{\kappa_f h + \kappa_s s + \pi l}{2}\right),$$
$$\phi_d = -\arctan\left(\tanh\frac{K_i\Lambda}{2}\cot\frac{\kappa_f h - \kappa_s s + \pi l}{2}\right). \tag{39}$$

By applying relations (11), (38), (39) to distribution (4), we will write the field amplitude distribution $E(x)$ in the f and s layers of the mth cell of the layered medium in the form

$$E(x) = \begin{cases} 2(-1)^{ml}|A|\exp(-K_i\Lambda m)\cos[(\phi_b - \phi_a)/2 + \kappa_f\xi_f]\exp\{\mathrm{i}[\phi_0 + (\phi_a + \phi_b)/2]\}, \\ \xi_f = x - h/2 - \Lambda m, \quad \Lambda m < x < \Lambda m + h, \quad m = 0,\ 1,\ \ldots, \\ 2(-1)^{ml}\bar{C}\exp[(-K_i\Lambda)(m + 1/2)]\sin[(\phi_d - \phi_c)/2 + \kappa_s\xi_s]\exp\{\mathrm{i}[\phi_0 + (\phi_c + \phi_d - \pi l)/2]\}, \\ \xi_s = x - h - s/2 - \Lambda m, \quad \Lambda m + h < x < \Lambda(m + 1). \end{cases} \tag{40}$$

The substitution of (40) into boundary conditions (33) for $x = 0$ leads to relations

$$\gamma_a = \kappa_f \tan[(\kappa_f h + \phi_a)/2], \quad B = |B|,$$
$$\gamma_a = -\kappa_f \cot[(\kappa_f h + \phi_a)/2], \quad B = -|B|, \tag{41}$$
$$F_a = 2(-1)^{ml}|A|\cos[(\kappa_f h + \phi_a - \phi_b)/2],$$

which are fulfilled inside each of the l forbidden bands for values of κ_f in the range from κ_{l_+} to κ_{l_-}. Equalities (41) are fulfilled only for those values of the refractive index of the homogeneous medium, which satisfy the conditions from (41):

$$n_a = \left\{n_f^2 - \frac{\kappa_f^2}{k_0^2\cos^2[(\kappa_f h + \phi_a)/2)]}\right\}^{1/2}, \quad B = |B|,$$
$$n_a = \left\{n_f^2 - \frac{\kappa_f^2}{k_0^2\sin^2[(\kappa_f h + \phi_a)/2)]}\right\}^{1/2}, \quad B = -|B|. \tag{42}$$

The admissible values of the refractive index n_a at which surface waves localised near the boundary of the layered medium exist, are in the range from n_{l_+} to n_{l_-}, while the values of the effective refractive index n_a^* are in the range from $n_{l_+}^*$ to $n_{l_-}^*$:

$$n^* = \{n_f^2\sin^2[(\kappa_f h + \phi_a)/2] + n_a^2\cos^2[(\kappa_f h + \phi_a)/2]\}^{1/2}, \quad B = |B|,$$
$$n^* = [n_f^2\cos^2[(\kappa_f h + \phi_a)/2] + n_a^2\sin^2[(\kappa_f h + \phi_a)/2]\}^{1/2}, \quad B = -|B|. \tag{43}$$

By using the relation $n^{*2} = n_f^2 - \kappa_f^2/k_0^2$ fulfilled for κ_f and $n^* = \beta/k_0$ in the f layer, we will write the dispersion equations for surface waves at the boundary of the semi-infinite periodic layered medium in the form:

$$\kappa_f^2 = k_0^2(n_f^2 - n_a^2)\cos^2[(\kappa_f h + \phi_a)/2], \quad B = |B|,$$
$$\kappa_f^2 = k_0^2(n_f^2 - n_a^2)\sin^2[(\kappa_f h + \phi_a)/2]B = -|B|. \tag{44}$$

For real κ_f and imaginary $\kappa_s = \mathrm{i}\gamma_s$ (the case of tunnel-coupled waveguide layers of a periodic medium), by substituting (37) into (12) for the amplitude coefficients A and B we can obtain

$$A = \frac{A_0}{2}[\cosh(\gamma_s s) - (-1)^l\cos(\kappa_f h)\cosh(K_i\Lambda) - \mathrm{i}(-1)^l\sin(\kappa_f h)\sinh(K_i\Lambda)] = |A|\exp(\mathrm{i}\phi_a),$$

$$|A| = \frac{A_0}{2}\left\{\cosh^2(\gamma_s s) + \frac{1}{2}[\cosh(2K_i\Lambda) + \cos(2\kappa_f h)] - 2(-1)^l\cos(\kappa_f h)\cosh(\gamma_s s)\cosh(K_i\Lambda)\right\}^{1/2}, \tag{45}$$

$$\phi_a = \arctan\{\sin(\kappa_f h)\sinh(K_i\Lambda) \times[\cos(\kappa_f h)\cosh(K_i\Lambda) - (-1)^l\cosh(\gamma_s s)]^{-1}\},$$

$$B = \frac{A_0}{4}\left(\frac{\kappa_f}{\gamma_s} + \frac{\gamma_s}{\kappa_f}\right)\sin(\kappa_f h)\sinh(\gamma_s s) = |B|\exp(\mathrm{i}\phi_b),$$

$$\phi_b = 0\ (B > 0), \quad \phi_b = \pi\ (B < 0),$$

where B, ϕ_a, ϕ_b are still real quantities; A is the complex quantity; and $|A| = |B|$. The amplitude coefficients C and D are written in the form:

$$C = \bar{C}\exp(\mathrm{i}\phi_c), \ \bar{C} = \frac{A_0\sin(\kappa_f h)}{2\gamma_s} \times\{[\cosh(\gamma_s s - K_i\Lambda) - (-1)^l\cos(\kappa_f h)](\gamma_s^2 + \kappa_f^2)/2\}^{1/2},$$

$$\phi_c = \arctan\frac{\gamma_s\tanh[(\gamma_s s - K_i\Lambda)/2] - \kappa_f\tan[(\kappa_f h + \pi l)/2]}{\kappa_f\tanh[(\gamma_s s - K_i\Lambda)/2] + \gamma_s\tan[(\kappa_f h + \pi l)/2]},$$

$$D = \bar{D}\exp(\mathrm{i}\phi_d), \ \bar{D} = \frac{A_0\sin(\kappa_f h)}{2\gamma_s} \times\{[\cosh(\gamma_s s + K_i\Lambda) - (-1)^l\cos(\kappa_f h)](\gamma_s^2 + \kappa_f^2)/2\}^{1/2}, \tag{46}$$

$$\phi_d = -\arctan\frac{\gamma_s\tanh[(\gamma_s s + K_i\Lambda)/2] - \kappa_f\tan[(\kappa_f h + \pi l)/2]}{\kappa_f\tanh[(\gamma_s s + K_i\Lambda)/2] + \gamma_s\tan[(\kappa_f h +)/2]}.$$

Now we will write the field distribution in the mth cell of the structure by using local coordinates $\xi_f = x - h/2 - \Lambda m$ and $\xi_s = x - s/2 - h - \Lambda m$, measured from the middles of the corresponding layers:

$$E(x,z) = 2(-1)^{ml}|A|\exp(-K_i\Lambda m)\cos[(\phi_b - \phi_a)/2 + \kappa_f\xi_f] \times\exp\{\mathrm{i}[\phi_0 + (\phi_a + \phi_b)/2 - \beta z]\}, \quad -h/2 < \xi_f < h/2,$$

$$E(x,z) = (-1)^{ml}E_s(\xi_s)\exp[\mathrm{i}\phi_s(\xi_s)]\exp[-K_i\Lambda(m + 1/2)] \times\exp\{\mathrm{i}[\phi_0 + (\phi_c + \phi_d)/2 - \pi l/2 - \beta z]\}, \quad -s/2 < \xi_s < s/2, \tag{47}$$

$$E_s(\xi_s) = [\bar{C}^2\exp(2\gamma_s\xi_s) + \bar{D}^2\exp(-2\gamma_s\xi_s) + 2\bar{C}\bar{D}\cos(\phi_c - \phi_d)]^{1/2},$$

$$\phi_s(\xi_s) = \arctan\left[\frac{\bar{C}\exp(\gamma_s\xi_s) - \bar{D}\exp(-\gamma_s\xi_s)}{\bar{C}\exp(\gamma_s\xi_s) + \bar{D}\exp(-\gamma_s\xi_s)}\tan\frac{\phi_c - \phi_d}{2}\right].$$

By using (25), (33) and (47), it is easy to ascertain the validity of Eqns (41)–(44) and, in this case, of the tunnel-coupled waveguide layers. The only difference consists in the fact that the coefficient A and phases ϕ_a, ϕ_b are calculated not from (38) but from (45).

When δh of the f layer of thickness h in a zero cell of the layered medium changes, the range of admissible values of the refractive medium n_a of the homogeneous medium can be expanded. In this case the condition for the appearance of surface waves takes the form:

$$\gamma_a = \kappa_f \tan[\phi_a/2 + \kappa_f(h/2 + \delta h)], \quad B = |B|,$$
$$\gamma_a = -\kappa_f \cot[\phi_a/2 + \kappa_f(h/2 + \delta h)], \quad B = -|B|. \qquad (48)$$

In this case as in the case of (42), the values of the transverse wave vector corresponding to the forbidden band of the order l are selected, i.e. in the range from κ_{l_+} to κ_{l_-}. In this case the values of the effective refractive index n^* remain in the range from $n^*_{l_+}$ to $n^*_{l_-}$:

$$n^* = \{[n_f^2 \sin^2[(\kappa_f h + \phi_a)/2] + n_a^2 \cos^2[(\kappa_f h + \phi_a)/2]\}^{1/2}, \quad B = |B|,$$
$$n^* = \{[n_f^2 \cos^2[(\kappa_f h + \phi_a)/2] + n_a^2 \sin^2[(\kappa_f h + \phi_a)/2]\}^{1/2}, \quad B = -|B|. \qquad (49)$$

Nevertheless the range of admissible refractive indices values of the homogeneous medium, for which surface wave are possible due to a change in δh, changes according to the condition from equalities (48):

$$n_a = \left\{ n_f^2 - \frac{\kappa_f^2}{k_0^2 \cos^2[\phi_a/2 + \kappa_f(h/2 + \delta h)]} \right\}^{1/2}, \; B = |B|,$$
$$n_a = \left\{ n_f^2 - \frac{\kappa_f^2}{k_0^2 \sin^2[\phi_a/2 + \kappa_f(h/2 + \delta h)]} \right\}^{1/2}, \; B = -|B|. \qquad (50)$$

Upon changing the thickness of the boundary layer, the dispersion equations for surface waves at the boundary of a semi-infinite periodic layered medium have the form:

$$\kappa_f^2 = k_0^2(n_f^2 - n_a^2)\cos^2[\phi_a/2 + \kappa_f(h/2 + \delta h)], \; B = |B|,$$
$$\kappa_f^2 = k_0^2(n_f^2 - n_a^2)\sin^2[\phi_a/2 + \kappa_f(h/2 + \delta h)], \; B = -|B|. \qquad (51)$$

To illustrate the use of data obtained in this paper, we calculated for $n^* = 1.45975$ the phase front (Fig. 2a) and the transverse distribution of the field amplitude (Fig. 2b) of the Floquet–Bloch wave propagating in an infinite layered periodic medium with the parameters $h = 1.1$ μm, $s = 1.3$ μm, $n_f = 1.465$ and $n_s = 1.46$, which correspond to the parameters of the structure studied in [15]. The shape of the wave front is not plane even within the homogeneous layers of the periodic medium, i.e. not only refraction of the wave front at interlayer boundaries takes place but also distortion of the wave surfaces. The field amplitude distribution is presented in Fig. 3 for a critical Floquet–Bloch wave propagating in a semi-infinite layered periodic medium (Fig. 3a) and a surface wave (Fig. 3b). Figure 4 shows the dependence of the square of the modulus of the reflection coefficient of a plane wave incident from the homogeneous medium with $n_a = n_s$ calculated by using expression (31). One can see from Fig. 4 that in this case the periodic layered medium can be characterised as a one-dimensional photonic crystal in which the boundaries of photonic forbidden bands correspond to the values of the transverse wave vector κ_a of the incident wave, which are equal to κ_{l_-} and κ_{l_+}.

6. Conclusions

Thus, an exact solution of the wave equation for the case of light propagation in a layered periodic medium with a step profile of the refractive index has been obtained in this paper. The solution is represented in the form of an

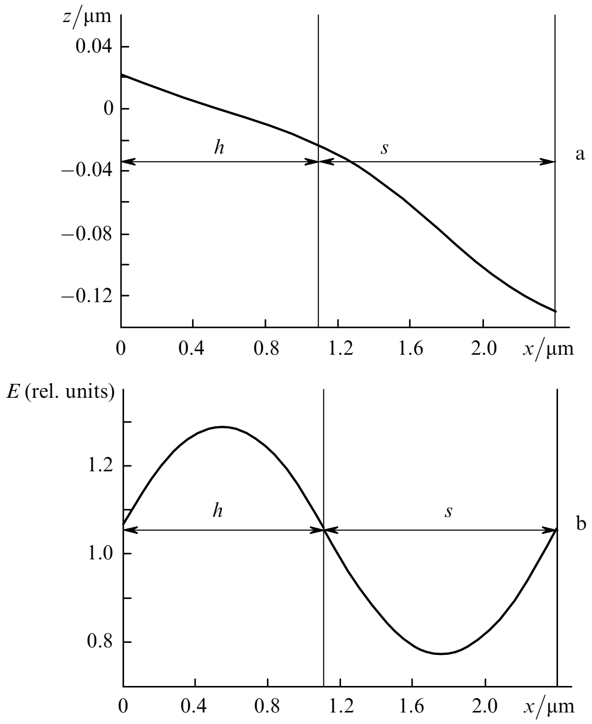

Figure 2. Phase front (a) and transverse distribution of the field amplitude (b) of the Floquet–Bloch wave propagating in the infinite layered periodic medium with the parameters $h = 1.1$ μm, $s = 1.3$ μm, $n_f = 1.465$, $n_s = 1.46$, $n^* = 1.45975$.

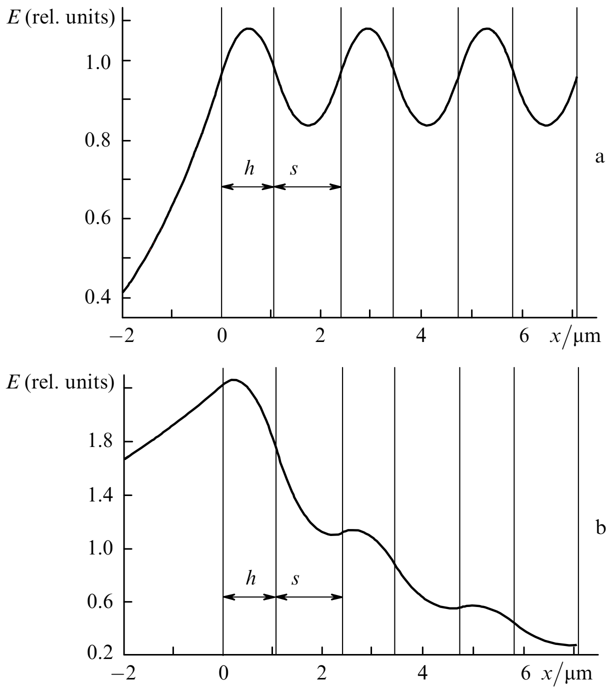

Figure 3. Transverse distributions of the field amplitude of the critical Floquet–Bloch wave in the semi-infinite layered periodic medium with the parameters $h = 1.1$ μm, $s = 1.3$ μm, $n_f = 1.465$, $n_s = 1.46$, $n_a = 1.46188$, $n^* = 1.46251$ (a) and the surface wave with the parameters $h = 1.1$ μm, $s = 1.3$ μm, $n_f = 1.465$, $n_s = 1.46$, $n_a = 1.46271$, $n^* = 1.46278$ (b).

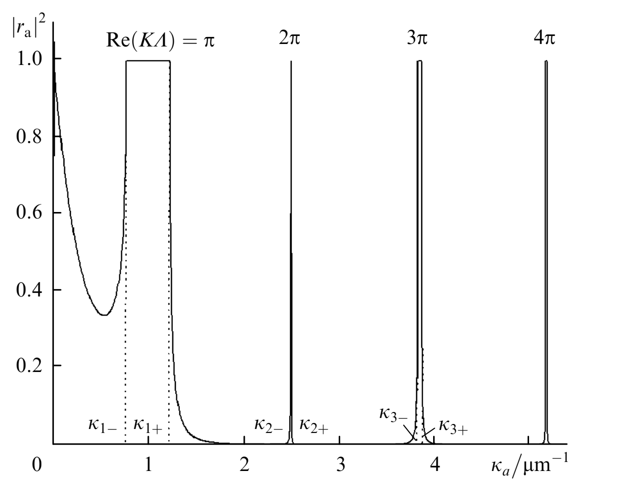

Figure 4. Dependences of the square of the modulus of the reflection coefficient r_a on the transverse component κ_a of the wave vector of the plane wave incident on the boundary of the semi-infinite layered periodic medium with the parameters $h = 1.1$ μm, $s = 1.3$ μm, $n_f = 1.465$, $n_s = 1.46$, $n_a = 1.46$.

inhomogeneous wave (Floquet – Bloch), for which the field amplitude distribution has been derived in an analytic from and the shape of its wave surfaces has been determined. The radiation and tunnel coupling between layers has been considered. The obtained solutions have allowed us to find the reflection coefficient of a plane wave incident from the homogeneous medium at the boundary of a semi-infinite layered periodic structure and exciting the Floquet – Bloch wave in it. The critical Floquet – Bloch wave, which is an intermediate case between a freely propagating and decaying Floquet – Bloch waves in the semi-infinite periodic medium, has been selected and considered. The conditions for the appearance of a critical Floquet – Bloch wave have been determined. Dispersion equations and the field distribution for surface waves (modes) localised near the boundary of the semi-infinite layered medium have been written, the conditions for their appearance have been defined and the boundaries of admissible values of the refractive index of the adjacent medium have been determined as a function of the parameters of the layered periodic medium. Dispersion relations for surface modes of the semi-infinite layered periodic medium (limited system of coupled waveguides) have been derived upon varying the thickness of the boundary layer.

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