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On the gravitational-deceleration initiation of the phase transition of gas to a Bose condensate

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***Abstract*. A scenario of the experiment on the observation of the isothermal Bose condensation of cooled gas with increasing the concentration of atoms caused by the deceleration of a vertical atomic beam in the gravitational field resulting in a decrease in the phase transition critical temperature below the gas temperature is considered. Coherent phenomena accompanying the evolution of the Bose condensate during further beam deceleration are pointed out.**

Keywords: *Bose condensate, Mössbauer spectroscopy, radiative linewidth in a Bose condensate, quantum nucleonics.*

1. The emission of coherent atomic ensembles by the so-called atomic lasers occurs due to extraction of a fragment of the Bose – Einstein fraction from a quantum trap, this fraction being preliminarily prepared by decreasing the gas temperature T below the critical temperature T_c ($T < T_c$) [1 – 3]. The preliminarily prepared Bose fraction is sometimes accelerated in the gravitational field [3].

In this paper, a variant of an atomic laser is considered which does not require the preliminary preparation of a Bose condensate. Unlike the previous case, the phase transition of cooled gas to the Bose condensate occurs isothermally at a constant temperature T due to the deceleration of a cold atomic beam in the gravitational field.

A similar isothermal phase transition to a Bose – Einstein condensate (BEC), but in the absence of gravitational forces, can occur during the propagation of a cooled atomic beam in a quantum channel with the transverse characteristic energy increasing along the channel length [4].

2. Consider a source of deeply cooled atoms with temperature T emitting an atomic beam with a circular cross section and the transport velocity V directed vertically upward, the thermodynamic velocity of atoms

$$V_T = \left(\frac{3k_B T}{M}\right)^{1/2}$$

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(where M is the atom mass and k_B is the Boltzmann constant) being considerably smaller than the transport velocity V. This condition imposes restrictions

$$T \ll \frac{MV^2}{3k_B}, \quad E_a = \frac{MV^2}{2} \gg \frac{3k_B T}{2} \tag{1}$$

on the temperature and kinetic energy E_a of atoms in the beam.

Such atomic sources have long been used in experiments; for example, a sodium atomic beam of intensity 10^9 at. s^{-1} and transport velocity up to 500 m s^{-1} was observed in [5].

An atomic beam elevated in the gravitational field through a height z is decelerated to the velocity and kinetic energy

$$V(z) = V_a - (2gz)^{1/2}, \quad E(z) = E_a - gMz \tag{2}$$

(where V_a is the initial velocity of atoms), and the relative increase in its cross section due to the thermal motion of atoms is described by the expression

$$\frac{S + \Delta S}{S} = \left(1 + \frac{\Delta\rho}{\rho}\right)^2 = \left[1 + \frac{V_T}{\rho}\left(\frac{2z}{g}\right)^{1/2}\right]^2$$
$$= \left[1 + a\left(\frac{z}{z_{\max}}\right)^{1/2}\right]^2 \leqslant (1 + a)^2, \tag{3}$$

where $z_{\max} \geqslant z$ is the coordinate of the atomic beam stop for $E(z_{\max}) = 0$; 2ρ is the initial diameter of the atomic beam for $z = 0$;

$$a \equiv \frac{z_{\max}}{\rho}\left(\frac{6k_B T}{E_a}\right)^{1/2};$$

and $g = 9.81$ m s^{-2}. The concentration $n(z)$ of atoms in the beam, which changes during the elevation of atoms, can be estimated as the ratio of a constant total atomic flux $\Phi = \text{const}$ to the varying cross section area $S(z)$ and velocity $V(z)$:

$$n(z) = \frac{\Phi}{S(z)V(z)} = \frac{n_0}{(1 + \Delta S/S)\left[1 - (2gz)^{1/2}/V_a\right]}$$
$$= \frac{n_0}{\left[1 + a(z/z_{\max})^{1/2}\right]^2\left[1 - (z/z_{\max})^{1/2}\right]}, \tag{4}$$

where $n_0 = \Phi/(SV)$ is the initial gas concentration at the source output for $z = 0$.

The apparent divergence of expression (4) for $z \to z_{\max}$ and $gMz_{\max} = E_a$ is, of course, not realised physically because thermal velocities V_T spread the value of the height $z_{\max}$, which only conditionally characterises the total energy E_a. The relative spreading of the coordinate $z_{\max}$ is

$$\frac{\Delta z_{\max}}{z_{\max}} \approx \frac{k_B T}{E_a} \ll 1.$$

Leaving aside this question, the kinematics of the vertical propagation of the atomic beam in the gravitational field can be simply described as the motion upward until a complete exhaustion of the initial momentum of atoms accompanied by the increase in their concentration (4) and then the reverse motion of atoms (falling) to the initial state. This picture is violated by the gas condensation to the liquid phase if the atomic concentration achieves the critical value $n(z_{\rm liq})$ at a height $z = z_{\rm liq} < z_{\max}$ for the given temperature T.

Due to competition between the increase in the beam cross section and its deceleration, the gas concentration increases $[n(z) > n_0]$ if the denominator in expression (4) is smaller than unity, i.e. at a height $z \geqslant z^*$, which for $a \gg 1$ is only slightly smaller than the maximum height

$$z^* \approx z_{\max}(1 - a^{-1}).$$

3. The picture considered here may change considerably if atoms belong to the boson class. Then, already before the formation of the usual condensed (liquid) state at a height $z = z_{\rm BEC} < z_{\rm liq}$ when the critical concentration n_c is exceeded

$$n(z) > n_c = \frac{2J_a + 1}{6\hbar^3}(Mk_B T)^{3/2}$$

$$\approx 0.6 \times 10^{20}(2J_a + 1)(AT)^{3/2} \qquad (5)$$

(where J_a is the angular momentum of an atom and A is its mass number), the second-order phase transition is possible in which a part of atoms form a BEC with the concentration

$$n_{\rm BEC} = n(z) - n_c. \qquad (6)$$

The height $z_{\rm BEC}$, as $z_{\max}$, is spread.

This phase transition initiated by the deceleration of atoms in the gravitational field resulting in the increase in the atomic beam concentration does not differ, of course, from a standard concept of BEC [6]. A purely external difference is that usually the case in point is the phase transition occurring due to a decrease in the gas temperature below the critical value ($T < T_c$), the gas concentration $n = \text{const}$ being constant, while here we consider the isothermal transition taking place when the critical temperature $T_c(n)$ depending on the gas concentration exceeds the invariable gas temperature $T = \text{const}$.

Here it is necessary to make an important remark. Expressions (5) and (6) are simply algebraically transformed formulas for the critical temperature T_c and concentration $n_{\rm BEC}$ of the Bose fraction of homogeneous free gas of an infinite volume [6]. However, gas in the atomic beam is homogeneous in a finite volume, although it is free from external fields {unlike gas in quantum traps of different types with discrete atomic states, where expressions (5) and (6) are inapplicable at all [7]}. We will assume here that this difference from the standard approach (homogeneity in a finite volume) is not principal and can affect only some quantitative estimates.

The height $z_{\rm BEC}$ of the phase transition of gas to the Bose condensate can be determined by equating $n(z)$ (4) to n_c (5), which for $a \gg 1$ leads to the approximate relation

$$z_{\rm BEC} \approx z_{\max}\left(1 - \frac{2}{a^2}\frac{n_0}{n_c}\right)$$

$$= z_{\max}\left[1 - \frac{2\hbar^3 g^2 M^{1/2} n_0 \rho^2}{(2J_a + 1)E_a(k_B T)^{5/2}}\right]. \qquad (7)$$

4. In accomplishing the experimental scheme under study, prerequisites appear for observing several accompanying phenomena.

After the Bose condensation of atoms, i.e. when the height $z > z_{\rm BEC}$ (but before $z \leqslant z_{\rm liq}$), a further elevation of the Bose condensate in the decelerating gravitational field leads to a decrease in the vertical component of the atomic momentum and, therefore, to an increase in of the de Broglie wavelength. In this case, the degree of overlap of the wave functions of atoms in the BEC increases and the BEC approaches the state of the so-called megaatom, in which individual motions of atoms can be substantially suppressed.

This phenomenon could be observed by the decrease in radiative transition linewidths recorded in the horizontal direction compared to Doppler linewidths at temperature T. It is unlikely that such an observation is possible in the optical range because the Doppler broadening at low (micro- and submicrokelvin) temperature of the BEC (and the expected decrease in this broadening) is considerably smaller than the natural linewidths of the allowed electronic transitions in atoms.

It seems that such an observation could be performed in the gamma range (where the natural linewidth of radiative transitions in metastable nuclei is many orders of magnitude smaller than the Doppler width even at very low gas temperatures) by using Mössbauer gamma spectroscopy for recording the contour of the broadened absorption line of nuclei in the BEC taking into account the spectral line shift caused by the nuclear recoil. The success of such an experiment would demonstrate the suppression of individual motions of BEC atoms in the megaatom state, which is substantial for solving problems of quantum nucleonics [7].

If it were proved that no catastrophic degradation of the BEC and no formation of the usual condensed phase ($z < z_{\rm liq}$) occurred near $z = z_{\max}$, then after the turning of the coherent atomic beam backward, the falling of atoms in the gravitational field would be accompanied by the interference of the incident beam with the elevating coherent beam. In this case, due to the dependence of the de Broglie wavelength on the height z, the step of the interference pattern would be also variable with height. The measurement of the interference pattern would give information on the kinematics of the vertical atomic beam in the gravitational field.

5. To obtain the quantitative notion on the phenomena considered above, we present some estimates for the $^{133}_{55}Cs$ isotope. Consider a beam of atoms with temperature 10^{-6} K, the initial transport velocity $V = 50$ cm s^{-1}, the energy $E_a = 0.174 \times 10^{-6}$ eV and diameter $2\rho = 0.1$ cm propagating vertically until the maximum height $z_{max} =$ 1.41 cm with the spreading $\Delta z_{max} \approx 0.7 \times 10^{-3}$ cm. For the parameter $a = 15.2$, the height z^* for which the beam concentration begins to increase is $0.925 z_{max} \approx$ 1.3 cm. The critical concentration $n_c \approx 3 \times 10^{14}$ cm^{-3} for the initial concentration $n_0 = 3 \times 10^{13}$ cm^{-3} is achieved at a height $z_{BEC} \approx 0.987 z_{max} \approx 1.39$ cm, i.e. the BEC evolution occurs within a relatively small interval of heights $z_{max} - z_{BEC} \approx$ 0.02 cm, which, however, exceeds the spreading Δz_{max} by three times. These estimates, which are far from being optimised, suggest that the experimental scenario considered above is quite realistic, although it requires a rather sophisticated approach.

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