

On the possibility of using aerosol backscattering in the adaptive loop of atmospheric optical systems

V.A. Banakh, V.V. Zhmylevskii, A.B. Ignat'ev, V.V. Morozov, D.S. Rychkov

Abstract. The mean received power of partially coherent cw laser radiation scattered in the atmosphere is calculated depending on the spatial coherence of the initial field and the angular divergence of a laser beam formed by a circular output aperture. It is shown that due to diffraction on the circular aperture, the dependence of the mean radiation power on the divergence angle is nonmonotonic and has a maximum. The divergence angle corresponding to this maximum is determined by the spatial coherence of the initial field. The possibility of controlling the adaptive loop of the laser system by means of signals backscattered in the atmosphere is discussed.

Keywords: laser beam, backward scattering, partially coherent radiation, adaptive loop.

1. Introduction

High-power laser radiation experiences phase fluctuations and uncontrollable distortions in the optical path propagated by a laser beam from the laser to the output aperture. This deteriorates the spatial coherence of radiation, leads to the increase in the divergence of laser beams and reduces the efficiency of light energy transfer over the specified distance. During the propagation of radiation in the atmosphere along inclined and high-altitude paths, the influence of turbulence is small, and these intrinsic fluctuations of the initial field become the main factor preventing the achievement of the required quality of a laser beam. The parameters of output radiation in modern optical systems can be improved by using the adaptive control of the initial wave front of the beam with a feedback loop containing a reference source. As reference sources, artificial reflectors are commonly used. Radiation scattered from the reflector contains information on the distortions of the incident wave field and is used to form a signal to control a wave-front corrector. In [1], it was proposed to use an atmospheric aerosol as a natural

reflector and the received power of scattered radiation – as a control signal.

In this paper, we analysed numerically the possibility of using radiation backscattered by an aerosol to close the feedback loop. The mean received power of partially coherent cw laser radiation scattered in the atmosphere is calculated as a function of the angular divergence of the laser beam and the spatial coherence of the initial field.

2. Formulation of the problem

Figure 1 shows the scheme for receiving radiation scattered in the atmosphere. The laser system has a combined optical scheme of a receiving–transmitting channel and a cw radiation source [1]. A laser beam formed on the output aperture propagates in the atmosphere and is scattered by aerosol particles. We will assume that the control signal correcting distortions of the initial field of the propagating beam is formed based on the measured mean power P_s of radiation backscattered in the atmospheric region of longitudinal size L . The output aperture is a ring with radii a and $b = a/M$, where $M > 1$ is a numerical coefficient. A beam formed on the output aperture has the wave front with the radius of curvature F , so that the angular divergence of the beam is determined by the ratio $\alpha = a/F$. Scattered radiation is received through a circular lens of radius a_0 with a focal distance f and is incident on a photodetector of radius a_d located in the focal plane of the lens.

We will describe random distortions of the initial field by using the phase screen model. In this case, the field U of a partially coherent beam on the output aperture can be written in the form [2, 3]

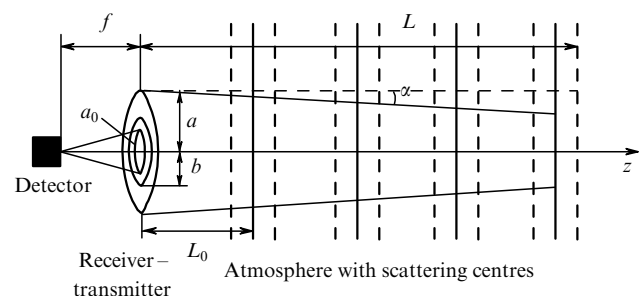


Figure 1. Scheme for receiving radiation scattered in the atmosphere.

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$$U(\boldsymbol{\rho}) = A \exp \left[i \frac{k\rho^2}{2F} + i\varphi(\boldsymbol{\rho}) \right], \quad (1)$$

where A is the field amplitude; $\exp[i\varphi(\boldsymbol{\rho})]$ is the phase screen; $\varphi(\boldsymbol{\rho})$ is a random phase with the zero mean value and the Gaussian correlation function $B_s(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2) = \sigma^2 \exp[-(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2)^2 / (2l^2)]$; σ^2 is the variance of phase fluctuations; l is the phase correlation scale; $k = 2\pi/\lambda$; and λ is the radiation wavelength. Model (1) uniquely defines the radius ρ_c of spatial coherence of the source field [4]. Below, we will omit the factor A for simplicity.

The field of radiation at a distance of z in the parabolic approximation is described by the integral over the output aperture surface S_{ab} [5]:

$$U(\mathbf{R}, z) = \frac{\exp[ikz + ikR^2/(2z)]}{i\lambda z} \times \int_{S_{ab}} U(\boldsymbol{\rho}) \exp \left[\frac{ik}{2z} (\rho^2 - 2\mathbf{R}\boldsymbol{\rho}) \right] d\boldsymbol{\rho}, \quad (2)$$

where \mathbf{R} is the transverse radius vector in the scattering region. The phase fluctuations of the initial field affect the mean received power due to variations in the average intensity of incident and scattered radiation. According to (1) and (2), the average intensity is described by the expression

$$\langle I_i(\mathbf{R}, z) \rangle = \langle U(\mathbf{R}, z) U^*(\mathbf{R}, z) \rangle = \frac{1}{\lambda^2 z^2} \int_{S_{ab}} \int_{S_{ab}} \langle U(\boldsymbol{\rho}_1) U^*(\boldsymbol{\rho}_2) \rangle \times \exp \left\{ \frac{ik}{2z} [\rho_1^2 - \rho_2^2 - 2\mathbf{R}(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2)] \right\} d\boldsymbol{\rho}_1 d\boldsymbol{\rho}_2, \quad (3)$$

$$\langle U(\boldsymbol{\rho}_1) U^*(\boldsymbol{\rho}_2) \rangle = U_0(\boldsymbol{\rho}_1) U_0^*(\boldsymbol{\rho}_2) \exp \left[-\frac{1}{2} D_s(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2) \right], \quad (4)$$

where $D_s(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2) = 2[B_s(0) - B_s(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2)]$ is the structural function of the initial field phase and $U_0(\boldsymbol{\rho})$ is the non-fluctuating part of field (1).

Under the condition that the radius of a scattering particle is $r_s \ll \lambda$, the average intensity of laser radiation backscattered in the atmosphere and received in the focal plane of the lens $z = -f$ can be written in the form [2, 3]

$$\langle I_s(\boldsymbol{\rho}_0, f) \rangle = \frac{\sigma_s}{\lambda^2 f^2} \int_V d\mathbf{R} dz \langle I_i(\mathbf{R}, z) \rangle I_r(\mathbf{R}, \boldsymbol{\rho}_0, z), \quad (5)$$

where V is the volume of the scattering region; $\sigma_s = \alpha_s^2/4\pi$ is the cross section for aerosol backscattering; α_s is the scattering amplitude;

$$I_r(\mathbf{R}, \boldsymbol{\rho}_0, z) = \frac{1}{z^2} \left| \int_{S_{a_0}} d\boldsymbol{\rho}' \exp \left[ik \left(\frac{\rho'^2}{2z} - \frac{\boldsymbol{\rho}_0 \boldsymbol{\rho}'}{f} - \frac{\boldsymbol{\rho}' \mathbf{R}}{z} \right) \right] \right|^2; \quad (6)$$

S_{a_0} is the receiving lens surface; and $\boldsymbol{\rho}_0$ is the radius vector in the plane of the receiving lens aperture. By comparing expression (6) and (3), we see that the function $I_r(\mathbf{R}, \boldsymbol{\rho}_0, z)$ can be interpreted as the intensity of a collimated beam of radius a_0 propagating from the receiver-transmitter plane at an angle of $\boldsymbol{\rho}_0/f$ to the axis.

The mean received power of scattered radiation is the integral of intensity (5) over the photodetector surface S_d

$$P_s(f) = \int_{S_d} d\boldsymbol{\rho}_0 \langle I_s(\boldsymbol{\rho}_0, f) \rangle. \quad (7)$$

Below, we will omit the variable f in the function $P_s(f)$ by assuming that the photodetector is always located in the focal plane.

Note here that expression (7) neglects the attenuation of the intensity of propagating and scattered beams. This approximation restricts the size of the scattering region in the longitudinal direction so that its optical thickness is $\tau \lesssim 1$.

3. Estimate of the mean received power in the approximation of Gaussian transmission functions of apertures

The Gaussian approximation of the transmission functions of the output and receiving apertures allows one to obtain the analytic expression for the average intensity of scattered radiation and to analyse the mean received power as a function of the divergence angle α and the spatial coherence of the initial field.

Let the regular component of the field incident on the output aperture be a plane wave. Then, the initial distribution of the field for the Gaussian transmission function of the circular aperture has the form

$$U_0(\boldsymbol{\rho}) = \left[\exp \left(-\frac{\rho^2}{2a^2} \right) - \exp \left(-\frac{\rho^2 M^2}{2a^2} \right) \right] \exp \left(-\frac{ik\rho^2}{2F} \right). \quad (8)$$

The circular receiving aperture also has the Gaussian transmission function $\exp[-\rho^2/(2a_0^2)]$ of radius a_0 .

In this case, integration in (3) and (6) can be performed in the infinite limits:

$$\langle I_i(\mathbf{R}, z) \rangle = \frac{1}{\lambda^2 z^2} \int_{-\infty}^{\infty} d\boldsymbol{\rho}_1 \int_{-\infty}^{\infty} d\boldsymbol{\rho}_2 \exp \left[-\frac{ik}{2F} (\rho_1^2 - \rho_2^2) \right] \times \exp \left\{ -\left[\frac{\rho_1^2}{2a^2} - \frac{\rho_2^2}{2a^2} - 2\mathbf{R}(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2) \right] \frac{ik}{2z} - \frac{1}{2} D_s(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2) \right\} \left[\exp \left(-\frac{\rho_1^2}{2a^2} \right) - \exp \left(-\frac{M^2 \rho_1^2}{2a^2} \right) \right] \times \left[\exp \left(-\frac{\rho_2^2}{2a^2} \right) - \exp \left(-\frac{M^2 \rho_2^2}{2a^2} \right) \right], \quad (9)$$

$$\langle I_i(\mathbf{R}, \boldsymbol{\rho}_0, z) \rangle = \frac{1}{z^2} \int_{-\infty}^{\infty} d\boldsymbol{\rho}_1 \int_{-\infty}^{\infty} d\boldsymbol{\rho}_2 \left[-\frac{\rho_1^2}{2a_0^2} - \frac{\rho_2^2}{2a_0^2} + ik \left(-\frac{\boldsymbol{\rho}_0 \boldsymbol{\rho}_1}{f} + \frac{\boldsymbol{\rho}_0 \boldsymbol{\rho}_2}{f} + \frac{\mathbf{R} \boldsymbol{\rho}_2}{z} - \frac{\mathbf{R} \boldsymbol{\rho}_1}{z} + \frac{\rho_1^2 - \rho_2^2}{2z} \right) \right]. \quad (10)$$

As a result, the received mean power of scattered radiation in the plane $z = -f$ can be written in the form

$$P_s = Q \sum_{j=1}^4 P_j, \quad (11)$$

$$P_j = q_j \int_0^L \frac{dz}{z^2} \left\{ 1 - \exp \left[-\frac{k^2 \alpha_0^2 a_0^2}{1 + (ka_0^2/z)^2 - (a_0/a)^2 F_j(z)} \right] \right\},$$

where

$$Q = \lambda \sigma_s P_0 \left(\frac{a_0}{a} \right)^2 \frac{M^2}{2} \frac{M^2 + 1}{(M^2 - 1)^2};$$

$$q_j = \begin{cases} 1, & j = 1, \\ -2/(M^2 + 1), & j = 2, \\ 2/(M^2 + 1), & j = 3, \\ M^{-2}, & j = 4; \end{cases}$$

$$F_j(z) = 4\chi + q_j + q_j \left[ka^2 \left(\frac{1}{z} - \frac{1}{F} \right) \right]^2 + iv_j ka^2 \left(\frac{1}{z} - \frac{1}{F} \right);$$

$$\chi = \frac{a^2 \sigma^2}{l^2}; \quad \alpha_0 = a_d/f;$$

$$v_j = \begin{cases} 0, & j = 1, 4, \\ -2(M^2 - 1)/(M^2 + 1), & j = 2, \\ 2(M^2 - 1)/(M^2 + 1), & j = 3; \end{cases}$$

P_0 is the coherent source power and $P_0 M^2 (M^2 + 1) \times [2(M^2 - 1)^2]^{-1}$ is the output power of a coherent collimated beam for the Gaussian transmission function of the circular aperture. It can be shown that integral (11) converges for finite a_d .

Figure 2 shows average intensity (9) of a collimated beam for $\rho = 0$ as a function of the coordinate z for different degrees of the spatial coherence of the initial field. One can see that, as the initial coherence radius is decreased, the maximum of average intensity (9) shifts to the detector. Correspondingly, the average power of scattered radiation depends nonmonotonically on the spatial coherence radius ρ_c if the Fresnel number $N_F = ka^2/z$ of the transmitting aperture is large enough. This follows from the results of calculations of the mean power presented in Fig. 3. As the longitudinal size of the scattering region is increased, the dependence of the mean power on the initial coherence radius of the field becomes monotonic (Fig. 4).

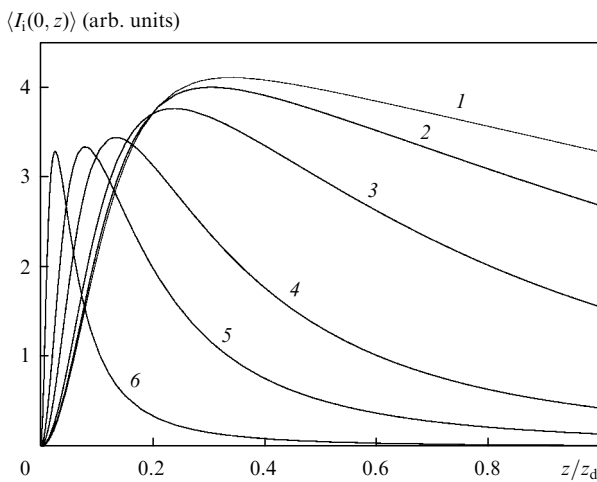


Figure 2. Mean intensity $\langle I_i(0, z) \rangle$ on the axis of a collimated illuminating beam in the approximation of the Gaussian transmission function of the output aperture ($a = 0.3$ m, $M = 3$) in the absence of phase fluctuations of the initial field (1) and in their presence for $\sigma^2 = 10^{-2}$ (2), 5×10^{-2} (3), 3×10^{-1} (4), 1 (5), 10 rad^2 (6), $l = 4 \times 10^{-2}$ m; z_d is the effective diffraction length.

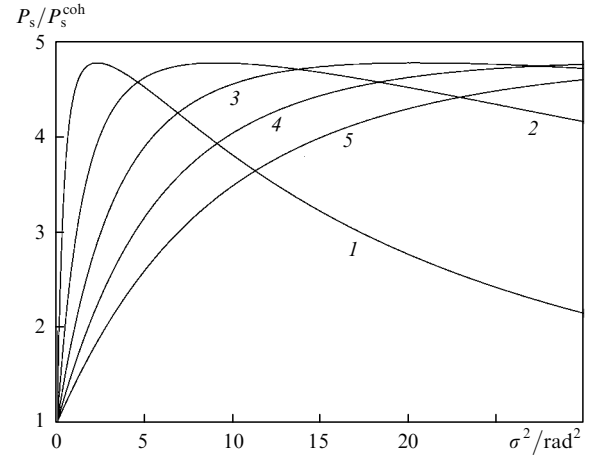


Figure 3. Mean power P_s of a scattered collimated beam ($a = 0.3$ m, $M = 3$) as a function of the phase variance for the receiving lens radius $a_0 = 0.1$ m, the longitudinal size of the scattering region $L \ll z_d$, the Fresnel number $N_F(L) \approx 110$, $l = 2$ (1), 4 (2), 6 (3), 8 (4), and 10 mm (5); P_s^{coh} is the mean received power in the absence of phase fluctuations of the initial field.

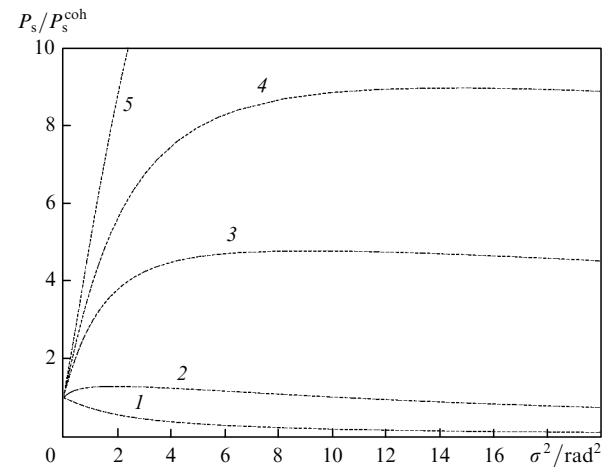


Figure 4. Mean power P_s of a scattered collimated beam ($a = 0.3$ m, $M = 3$) as a function of the phase variance for $a_0 = 0.1$ m, $l = 4$ mm, $N_F(L) \approx 20$ (1), 50 (2), 110 (3), 150 (4), and 200 (5); P_s^{coh} is the mean received power in the absence of phase fluctuations of the initial field.

4. Effect of diffraction on the edges of the transmitting and receiving apertures on the mean received power

We will assume that the apertures of the receiving-transmitting channel of the optical system in Fig. 1 have the rectangular transmission function, i.e. their boundary is sharp. In this case, integrals (3), (5), and (6) have finite integration regions S_{ab} and S_{a_0} . Taking the sharp boundaries into account, mean power (7) can be calculated by using the fact that integral (5) is the convolution of functions (3) and (6) and applying the fast Fourier transform.

Let us write expressions for the average intensity of scattered radiation (5) and received power (7) in the form

$$\langle I_s(\rho_0, f) \rangle = \frac{\sigma_s}{\lambda^2 f^2} \int_0^L dz \times$$

$$\times \int_{-\infty}^{\infty} \Phi_i(s, z) \Phi_r(s, z) \exp\left(2\pi i s \rho_0 \frac{z}{f}\right), \quad (12)$$

$$P_s = \frac{\sigma_s}{\lambda^2} \int_0^L \frac{dz}{z^2} \int_{S'_d(z)} d\mathbf{r}_0 \int_{-\infty}^{\infty} \Phi_i(s, z) \Phi_r(s, z) \exp\left(2\pi i s r_0\right), \quad (13)$$

where $\Phi_i(s, z)$ and $\Phi_r(s, z)$ are the spatial spectra of intensities (3) and (6); $r_0 = \rho_0 z/f$; $S'_d(z) = S_d z^2/f^2$ is the integration surface determined by the angular projection of the photodetector on a plane placed at a distance z perpendicular to the beam axis.

Let us calculate the spectrum $\Phi_i(s, z)$ [spectrum $\Phi_r(s, z)$ is calculated similarly]. By definition, the spectrum of the average intensity $\langle I_i(\mathbf{R}, z) \rangle$ is

$$\Phi_i(s_1, z) = \int_{-\infty}^{\infty} d\mathbf{R} \langle I_i(\mathbf{R}, z) \rangle \exp(2\pi i \mathbf{R} s). \quad (14)$$

The substitution of expression (3) for $\langle I_i(\mathbf{R}, z) \rangle$ into (14) gives the Dirac delta function $\delta(\lambda z s + \rho_1 - \rho_2)$ for the integral over the variable \mathbf{R} and (14) takes the form

$$\Phi_i(s, z) = \int_{S_{ab}} \int_{S_{ab}} d\rho_1 d\rho_2 \delta(\lambda z s + \rho_1 - \rho_2) \times \exp\left\{\frac{ik\mu}{2z}(\rho_1^2 - \rho_2^2) + \sigma^2 \left\{\exp\left[-\frac{(\rho_1 - \rho_2)^2}{2l^2}\right] - 1\right\}\right\}, \quad (15)$$

where $\mu = 1 - z/F$.

Let us represent the integral over S_{ab} in (15) as the difference of two integrals over S_a and S_b :

$$\int_{S_{ab}} \int_{S_{ab}} = \int_{S_a} \int_{S_a} - \int_{S_b} \int_{S_b} + 2\text{Re}\left(\int_{S_a} \int_{S_b}\right). \quad (16)$$

Now, by making the substitution $\rho_{1,2} = r \pm \lambda z s/2$, we obtain for any of the three double integrals in (16)

$$\Phi_{i_{ab}}(s, z) = \exp\left\{\sigma^2 \left\{\exp\left[-\frac{1}{2}\left(\frac{\lambda z s}{l}\right)^2\right] - 1\right\}\right\} \times \int_{-\infty}^{\infty} d\mathbf{r} f_a\left(\mathbf{r} - \frac{\lambda z s}{2}\right) f_b\left(\mathbf{r} + \frac{\lambda z s}{2}\right) \exp(2\pi i \mu \mathbf{r} s), \quad (17)$$

where

$$f_x(\rho_j) = \begin{cases} 0, & |\rho_j| > x, \\ 1, & |\rho_j| \leq x \end{cases} \quad (18)$$

is the form factor of a circle; $j = 1, 2$ and $x = a, b$. Integral (17) is reduced to a one-dimensional integral in finite limits.

The calculation of spectra $\Phi_i(s, z)$ and $\Phi_r(s, z)$ for large Fresnel numbers of the transmitting aperture is complicated because the spectrum strongly broadens for small z , and to perform calculations with the required spatial resolution, a calculation grid with a very large number N^2 of nodes is required. In the limit $z \rightarrow 0$, the number N^2 of nodes increases infinitely. The maximum possible N for the computer used automatically specifies the minimum distance $z = L_0$ determining the near boundary of the scattering region. The far boundary $z = L$ is determined, as mentioned above, by the condition $\tau < 1$ and the diffraction broadening of the beam.

Thus, the expression for the calculation of the mean received power takes the form

$$P_s = P_s(\Delta L = L - L_0) = \frac{\sigma_s}{\lambda^2} \int_{L_0}^L \frac{dz}{z^2} \int_{S'_d(z)} d\mathbf{r}_0 \times \int_{-\infty}^{\infty} \Phi_i(s, z) \Phi_r(s, z) \exp(2\pi i s r_0) = \frac{\sigma_s}{\lambda^2} \sum_{j=1}^{M_{NF}} \sum_{z=1}^8 \frac{\Delta z_j}{8z^2} \sum_{n'=0}^{N-1} N-1 \sum_{m'=0}^{N-1} S'_d(z_{jz, n' \Delta r_0, m' \Delta r_0}) \times \sum_{m=0}^{N-1} \sum_{n=0}^{N-1} \Phi_i(m \Delta s, n \Delta s, z_{jz}) \Phi_r(m \Delta s, n \Delta s, z_{jz}) \times \exp\left[2\pi i \left(\frac{mm'}{N} + \frac{nn'}{N}\right)\right] (\Delta s \Delta r_0)^2, \quad (19)$$

where Δr_0 and Δs are the spatial resolution and spatial frequency resolution, respectively and $S'_d(z_j, n' \Delta r_0, m' \Delta r_0)$ is the form factor of the photodetector projection on a plane located at a distance of z_j from the detector. The integral along the longitudinal direction in expression (19) was replaced in calculations by the sum of radiation powers scattered by thin layers between points on the path $z_j = z_d/j$; j are the ordinal numbers of the average intensity extrema along the beam propagation direction in the absence of fluctuations; Δz_j is the distance between extrema at points z_j and z_{j+1} ; $z_d = a^2(1 - M^{-2})/\lambda$ is the effective diffraction length for the circular output aperture; and \varkappa is the number of layers between adjacent extrema. The reading is performed from the extremum located at the point $z_1 = z_d$ to the extremum whose position is specified by the Fresnel number $N_F = 10^3$. The number \varkappa of layers determines the calculation error of the integral along the path. For $\varkappa \geq 8$, the relative error did not exceed 10%, and therefore we used $\varkappa = 8$ in calculations.

Figure 5 presents the dependences of the average power of scattered radiation on the angular divergence α of the laser beam calculated by expressions (11) and (19) for different spatial coherences of the field on the output aperture. Each of the curves is normalised to the maximum average power $P_{s \max}^{\text{coh}}$ obtained in calculations for the completely coherent beam. One can see that the scattered radiation power is maximal for a certain angular divergence. The position of the maximum on the abscissa and its value are determined by the spatial coherence of the initial field. The worse the initial coherence, the greater the angular divergence of the beam at which the mean power becomes maximal and the lower its maximum value. By comparing the solid and dashed curves in Fig. 5, we see that diffraction on the edges of the receiving and transmitting apertures causes the shift of maxima along the abscissa and a small change in their value. Note that to compare correctly the results obtained from expression (11) for the Gaussian approximation with more rigorous calculations by expression (19), the radii of the output and receiving apertures were set equal to $a\sqrt{2}$ and $a_0\sqrt{2}$, respectively [6]. Expression (11) can be used to estimate the mean detected power at large enough coherence radii of the initial field $a/\rho_c < 10$.

It follows from the results presented in Fig. 5 that the additional divergence of the laser beam due to worsening of spatial coherence of the initial field is manifested in the shift

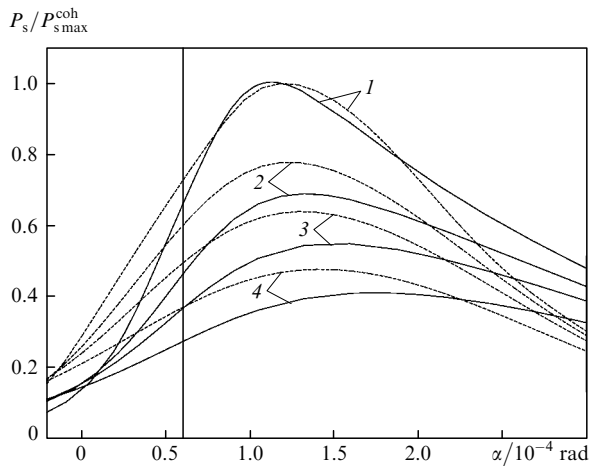


Figure 5. Mean received power P_s of a scattered beam ($a = 0.3$ m, $M = 3$) as a function of the angular divergence α in the absence (1) and presence of fluctuations for $\rho_c = 2$ (2), 1 (3), 0.5 mm (4), $a_0 = 0.1$ m. The solid curves are calculated by (11), the dashed curves are calculated by (19) taking into account diffraction on aperture edges. The vertical straight line indicates the value $\alpha = 6.1 \times 10^{-3}$ rad corresponding to focusing at the far boundary of the scattering region.

of the maximum along the abscissa and in the decrease in the mean detected power of scattered radiation. Thus, by varying the angular divergence of the beam, it is possible to find the value of α at which the mean power of scattered radiation achieves the maximum and determine the angular divergence of a partially coherent beam. The difference of angular divergences corresponding to the maxima of the mean power of scattered radiation calculated for coherent and partially coherent beams can be used as a control signal for the wave-front corrector.

5. Conclusions

The possibility of using backscattered radiation in the atmosphere to control the adaptive loop of an optical system with a circular output aperture has been considered. The expression for the mean received power of scattered radiation has been derived in the approximation of the Gaussian transmission apertures of a receiving–transmitting channel. The effect of diffraction on the edges of the transmitting and receiving apertures on the received power has been studied. The dependence of the mean received power on the divergence angle of a laser beam and the spatial coherence of the initial field has been analysed.

It has been shown that the mean power of backscattered radiation depends on the spatial coherence of the initial field, i.e. contains information on the distortions of the beam wave front at the output aperture. This allows the use of backscattered radiation of a beam propagating in the feedback loop of the adaptive control of the initial wave front to compensate distortions appearing in the optical path between the laser source and transmitting aperture.

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