 

*BOSE – EINSTEIN CONDENSATE*

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# On the experimental verification of a hypothesis about the 'megaatom' state in a Bose – Einstein condensate

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***Abstract*. A scenario of the experiment on the observation of the specific narrowing of nuclear radiative gamma lines in a Bose – Einstein condensate (BEC) due to the quantum coherence of atoms is proposed which could be used to confirm the appearance of the so-called megaatom state in the BEC. The expected narrowing of the gamma lines can occur due to the motion suppression of individual atoms and partial elimination of the inhomogeneous (Doppler) broadening. Quantitative estimates are performed, in particular, for the case of Mössbauer spectroscopy.**

***Keywords***: *Bose – Einstein condensate, quantum coherence of a BEC, inhomogeneous broadening of radiative lines, Mössbauer spectroscopy, quantum nucleonics.*

**1**. A Bose – Einstein condensate (BEC) [1 – 3] contains a finite number of boson atoms in the lowest energy state with zero momenta and overlapped wave functions. It can be expected that individual motions of atoms in this state will be substantially restricted due to the quantum coherence appearing in the BEC, if not excluded at all [4, 5]. This state of an atomic ensemble with the minimal dispersion of velocities over kinetic degrees of freedom can be called the megaatom state. The inhomogeneous broadening of atomic and nuclear radiative transitions involving this state, in particular, Doppler broadening should be considerably suppressed [6]. The theoretical consideration and quantitative estimates of this hypothesis [6, 7] are questionable [8]. However, its confirmation (or refutation) could be important for studying ultranarrow lines [7] in quantum nucleonics, in particular, in attempts to observe stimulated gamma radiation of nuclei [8]. Therefore, the experimental verification of the hypothesis about the appearance of the magaatom state in a BEC is of current interest.

The proposed experiment [9] involves the measurement of the spectral width $\Delta\omega_{\rm obs}$ of radiative transitions in a BEC at temperature $T$, which is compared then with the Doppler width $\Delta\omega_{\rm D}$ calculated for this temperature. If the ratio of these linewidths is less than unity,

$$\Delta\omega_{\rm obs}/\Delta\omega_{\rm D} < 1, \qquad (1)$$

this fact demonstrates the specific narrowing of spectral lines in the BEC, thereby confirming the hypothesis about the suppression of 'atomic individualities' and the appearance of the megaatom state. It is convenient to characterise this state by the effective temperature of the megaatom

$$T_{\rm eff} = T\left(\frac{\Delta\omega_{\rm obs}}{\Delta\omega_{\rm D}}\right)^2, \qquad (2)$$

which is not, of course, the thermodynamic temperature.

**2.** A simple estimate shows that this experiment cannot be performed for atomic transitions in the optical range because the natural radiative linewidth of these transitions is $\sim 10^8$ s$^{-1}$, i.e. greatly exceeds the tentative inhomogeneous broadening, which is assumed to be smaller than the Doppler linewidth

$$\frac{\Delta\omega_{\rm D}}{2\pi} = 2\frac{\omega}{2\pi}\left(2\ln 2\frac{k_{\rm B}T}{Mc^2}\right)^{1/2} \approx 0.7\times10^{-6}\left(\frac{T}{A}\right)^{1/2}\frac{\omega}{2\pi}, \qquad (3)$$

which is only $\sim 10^4$ s$^{-1}$ at typical temperatures $\sim 10^{-6}$ K of the BEC existence. Here, $\omega$ is the transition frequency; $M$ is the atom mass; $A$ is atomic mass number; $k_{\rm B}$ is the Boltzmann constant; and $c$ is the speed of light.

This problem does not exist, however, in the gamma ray range. Indeed, the natural width of gamma transitions, for example, in metastable Mössbauer isomers can be $\sim 10^5$ s$^{-1}$, while the Doppler width is $10^{-10}$ s$^{-1}$. In this case, the relevant experiment consists in the recording of gamma-ray absorption line of nuclei in a BEC excited by a narrowband radiation source whose linewidth is considerably narrower than the Doppler width of absorbing nuclei. Such experiments can be performed by using different schemes.

In the case of experiments based on classical Mössbauer spectroscopy, both nuclides (emitting and absorbing) should be the same boson, which is convenient both for experiments with a BEC and Mössbauer measurements. The search for such a nuclide can be a challenging problem. It seems that the $^{133}_{55}$Cs Mössbauer nuclide, which is often used in laser cooling experiments, is an exclusive example [10].

The use of a narrowband tunable X-ray source on relativistic electrons can simplify the choice of an appropriate absorber for producing a BEC; however, the experimental setup will be very complicated in this case.

Finally, if it were possible to find a pair of different nuclides with close gamma transition energies, of which one

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could be used as a Mössbauer emitter and the other – as a resonance absorber in a BEC, this would made experiments more flexible. In this case, the former nuclide should not be necessarily a boson convenient for entering into the BEC, while the latter one should not be necessarily a Mössbauer isomer.

**3.** In all experimental schemes proposed above, the concentration $n$ of gas atoms is restricted from above by several conditions. It is obvious that gas should be rarefied enough to avoid the collision line broadening ($n \ll 10^{19}$ cm$^{-3}$ is a quite acceptable condition).

In addition, the frequency $(\Delta t_{\rm col})^{-1}$ of collisions of BEC atoms with other gas atoms should be noticeably smaller than the natural radiative width $\Delta\omega_{\rm rad}/2\pi = \tau^{-1}$ of the gamma transition (where $\tau$ is the spontaneous decay time of the metastable nuclear state; it is also assumed that the internal electronic conversion coefficient is $\alpha \ll 1$). This restricts [9] both the gas concentration,

$$n < 0.43\left(\frac{M}{\hbar\sigma_{\rm col}\tau}\right)^{3/4}(2J_{\rm a}+1)^{1/4}\left(\frac{T_{\rm c}}{T}\right)^{3/2}$$

$$\approx 107\left(\frac{A}{\sigma_{\rm col}\tau}\right)^{3/4}(2J_{\rm a}+1)^{1/4}\left(\frac{T_{\rm c}}{T}\right)^{3/2}, \qquad (4)$$

and its temperatue

$$T < (1.9/k_{\rm B})\hbar^{3/2}[(2J_{\rm a}+1)M\sigma_{\rm col}\tau]^{-1/2}$$

$$\approx 3.6\times10^{-13}[(2J_{\rm a}+1)A\sigma_{\rm col}\tau]^{-1/2}, \qquad (5)$$

where $J_{\rm a}$ is the angular momentum of the atom; $\sigma_{\rm col}$ is the collision cross section ($\tau$ is measured in nanoseconds);

$$T_{\rm c} = \frac{3.3\hbar^2 n^{2/3}}{k_{\rm B}M(2J_{\rm a}+1)^{2/3}} \approx \frac{1.6\times10^{-14}}{A}\left(\frac{n}{2J_{\rm a}+1}\right)^{2/3} \qquad (6)$$

is the critical temperature of the phase transition to the BEC for a simple model of gas in the infinite volume [11]. The concentration $n_{\rm BEC}$ of BEC atoms is a part of the total concentration of atoms in gas:

$$n_{\rm BEC} = n\left[1-\left(\frac{T}{T_{\rm c}}\right)^{3/2}\right]. \qquad (7)$$

Under these conditions, the scheme of the experiment is as follows. A quantum trap contains a preliminarily prepared mixture of BEC atoms and usual cooled gas, which plays the role of a resonance absorber. The resonance absorption spectrum of the gas target is recorded by varying the frequency of a radiation source, in particular, by varying its velocity in the case of classical Mössbauer spectroscopy. The expected absorption spectrum should contain two components: the narrow absorption line of width $\Delta\omega_{\rm obs}$ of BEC nuclei and a pedestal of a lower intensity with the Doppler width corresponding to absorption of radiation by nuclei in usual gas at temperature $T$.

Taking into account that the atomic concentration $n_{\rm BEC}$ in the gas target is restricted, the gas target length $L$ should be sufficient for providing resonance absorption with the acceptable probability

$$w = \sigma_\gamma n_{\rm BEC} L, \qquad (8)$$

where $\sigma_\gamma$ is the total resonance absorption cross section. In addition, the blue shift $\Delta\omega_{\rm rec}$ of the nuclear absorption line with respect to the nuclear level energy $E$ (in kiloelectronvolts) due to the free nucleus recoil

$$\hbar\Delta\omega_{\rm rec} = \frac{E^2}{2Mc^2} \approx 0.535\frac{E^2}{A}\,(\rm meV) \qquad (9)$$

should be taken into account.

To observe resonance absorption in experiments with identical nuclides (emitters and absorbers), the spectral shift $\Delta\omega_{\rm rec}$ should be compensated by moving a Mössbauer emitter towards the gas target (or the target towards the emitter) at the velocity $V$ satisfying the relation

$$\frac{V}{c} = \frac{E^2}{2Mc^2} \approx 0.535\times10^{-6}\frac{E}{A}. \qquad (10)$$

If the required velocity is too high for a simple mechanical displacement of the emitter, it can be achieved in the atomic beam of the gas target [12, 13].

In addition, to avoid the excess time-of-flight broadening, the condition

$$L/V > \tau \qquad (11)$$

should be fulfilled. Finally, it is assumed that the technological confinement time $\Delta t_{\rm tr}$ of the BEC in a quantum trap not only exceeds all the characteristic times of the experiment but also is sufficient for performing all experimental procedures.

**4.** Quantitative estimates of the possible parameters for the classical Mössbauer scheme are presented below for the $^{133}_{55}$Cs nuclide with the gamma transition energy $E = 81$ keV, the lifetime $\tau \approx 6.3$ ns, the total resonance absorption cross section $\sigma_\gamma \approx 1.03\times10^{-19}$ cm$^2$, the coefficient of internal electronic conversion $\alpha = 1.72$ and the angular atomic momentum $J_{\rm a} = 7/2$. Excited $^{133}_{55}$Cs isomers are produced in Mössbauer experiments during the $\beta$ decay of the $^{133}_{54}$Xe nuclei with the lifetime 5.29 days [10].

The Bose condensation of $^{133}_{55}$Cs can be observed, although it is somewhat complicated due to the reduced rate of evaporation cooling [12]. Therefore, it is necessary to consider the isothermal formation of a BEC with increasing the gas concentration $n$, which does not require rapid cooling, when the critical conditions of the phase transition $T < T_{\rm c}$ are fulfilled by increasing $T_{\rm c}$ (6) for $T = {\rm const}$ [8].

By assuming that the absorption probability $W = 0.01$ is sufficient for reliable measurements, we obtain that, according to (8), for $L = 10^3$ cm the required concentration of atoms in the BEC is $n_{\rm BEC} = 10^{14}$cm$^{-3}$. By assuming that $T_{\rm c}/T = 1.25$, we obtain the total concentration $n = 3.6\times10^{14}$ cm$^{-3}$. This value does not exceed the restriction on the total concentration (4) $n < 1.9\times\ 10^{15}$ cm$^{-3}$, where it is assumed that $\sigma_{\rm col} = 10^{-16}$ cm$^2$. According to (6), the critical temperature of the phase transition is $T_{\rm c} = 0.15\times10^{-6}$ K and, correspondingly, $T = 0.12\times10^{-6}$ K, which is consistent with the restriction on temperature (5) $T < 0.45\times10^{-6}$ K. Finally, the compensating velocity (10) $V \approx 100$ m s$^{-1}$ can be more conveniently produced with the help of an atomic beam [12, 13] rather than by using a mechanical device of the Mössbauer emitter. In this case, the excess time-of-flight broadening is absent because the value

$L/V = 0.1$ s exceeds $\tau = 6.3$ ns by many orders of magnitude.

These estimates, which were made without any attempts of optimisation (in particular, of the choice of a nuclide), show that, although apparent quantitative contradictions are absent, the experiment proposed in the paper requires the use of quite sophisticated approaches based on various methods of experimental physics.

**5.** The quantum coherence of the BEC is not, of course, absolute, and this limits the degree of possible elimination of the inhomogeneous broadening of nuclear gamma lines in the BEC. The fundamental limitation of the quantum coherence (in the absence of other perturbing factors) is characterised by the natural lifetime $\Theta_{\rm BEC}$ of BEC atoms in dynamic equilibrium with the rest of gas atoms [8]. The natural lifetime $\Theta_{\rm BEC}$ determined by the rate of continuous dynamic exchange between atoms in these two fractions has the same meaning as the natural lifetime of excited states of an atom in the thermodynamic equilibrium with respect to spontaneous decay. Unfortunately, neither theoretical nor experimental estimates of $\Theta_{\rm BEC}$ are available at present. At the same time, it seems that the fundamental value of $\Theta_{\rm BEC}$ can be estimated from the experiment proposed in the paper.

The positive result of the experiment demonstrating the fulfilment of inequality (1) would confirm the hypothesis of a megaatom and open up new experimental possibilities for solving problems in quantum nucleonics [8].

If inequality (1) is not observed under all the restricting conditions listed above, the three conclusions are possible:

(i) The hypothesis about the appearance of the magaatom state has not been confirmed because of the imperfection of the experiment or violation of the restricting conditions listed above;

(ii) the fundamental lifetime $\Theta_{\rm BEC}$ of BEC atoms proved to be shorter than other characteristics times of the experiment ($\Theta_{\rm BEC} < (\Delta\omega_{\rm D}/2\pi)^{-1} < \tau$);

(iii) the hypothesis about the megaatom state in the BEC is incorrect.

To exclude the two of these conclusions, further studies are required.

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