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Statistical approach in planning experiments with a copper bromide vapor laser

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Abstract. **To improve the planning of experiments, the statistical analysis of a large amount of experimental data obtained for a copper bromide laser emitting at 510.6 nm and 578.2 nm was performed. Various statistical methods such as factor analysis, method of principal components, multiple regression and others were applied for studying the influence of the ten basic input laser parameters on the output laser power. It was found that the most important parameters are the inside diameter of the laser tube, the diameter of internal rings, the length of the active area and the input electrical power.**

Keywords: *copper bromide laser, factor analysis, method of principal components, multiple regression.*

1. Introduction

Although the copper vapour lasers are well studied, their experimental investigations still continue to attract a large interest because of various applications of these lasers. The recent research has been focused on lasers with active media based on copper halides mixed with hydrogen, HCl, HBr and other additives [1 – 6], which can provide the increase in the output pulse energy, the pulse repetition rate and the improvement in the laser beam quality.

A large amount of experimental data obtained in this field have not been analysed so far by statistical methods. In this connection, a brief work [7] should be mentioned in which the influence of the basic input parameters on the efficiency of a copper bromide vapour laser was estimated.

The output parameters of a laser (output laser power, efficiency etc.) depend on many independent parameters. Conditionally they may be divided into four groups: (i) geometric parameters (length and diameter of the laser tube, the interelectrode distance); (ii) energy characteristics (input electric power, pulse repetition rate, pulse duration, pulse front steepness); (iii) thermodynamic characteristics (buffer gas pressure, pressure of additives, temperature of the reservoir with copper halide, thermal insulation of the active volume) and (iv) optical characteristics (resonator type, manufacturing technology of mirrors). In practice it is impossible to determine experimentally the complex influence of all the factors on the output characteristics of the laser. For this reason, a major task in the study of such lasers is the planning of experiments. It is necessary to find conditions and rules for conducting the experiments that would provide reliable information on the object in a compact and suitable form with a qualitative estimate of the accuracy [8 – 10].

Some of the basic planning methods applied at different stages of this research are [8 – 10]: (i) planning of a sampling experiment, which involves the choice of a group of factors among all the cumulative factors, that are most important for further detailed studies; (ii) planning of an extreme experiment, whose main goal is the experimental optimisation of the object under study; and (iii) planning of the regression experiment for obtaining regression models (polynomial, etc.).

Most often two basic methods are applied to solve these problems: structural and phenomenological. The structural approach involves the construction of a model structure of the medium consisting of a huge amount of particles – electrons, positive and negative ions, and neutral atoms. The initial lasers characteristics are a summary result of the inner movement and interaction of these particles. Computer and analytic methods used in practice for solving particular problems related to experiment planning are considered in [11 – 15]. The phenomenological approach neglects the laser medium structure and uses only experiment data for planning experiments. The results are processed by statistical methods such as factor analysis, regression analysis, cluster analysis, etc. We are not aware of studies based on the statistical analysis planning experiments with copper vapour lasers.

We studied a copper bromide vapour laser emitting at $\lambda_1 = 510.6$ and $\lambda_2 = 578.2$ nm.

We present the basic results of the extensive statistical investigation based on a large amount of experimental results obtained at the Metal Vapour Lasers Department, Georgi Nadjakov Institute of Solid State Physics, Bulgarian Academy of Sciences, Sofia in the recent two decades and reported in [16 – 25]. By using the methods of mathematical

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statistics, we solved the following problems: (i) the determination of the input laser parameters affecting the output laser power; (ii) the classification of the parameters according to their correlation with other parameters; (iii) the determination of the extent of influence of grouped variables on the output power; (iv) the construction of a linear model of this influence; (v) the interpretation of the most important physical processes determining the output laser power; and (vi) planning of the experiment according to the data obtained.

The investigation was performed by using the SPSS statistical software package [26].

2. Factor analysis

The statistical method of factor analysis is used to transfer a set of interrelated variables into a new set of uncorrelated components or factors responsible for most of variations in the data set. This approach allows one to group the input lasing characteristics in independent factors and classify their mutual dependence within the groups. Once the new independent variables (factors) are obtained, the extent of their influence on the output laser characteristics can be determined by other statistical methods such as multiple regression and analysis of variance.

In this paper, we study eleven parameters describing the CuBr laser operation. The ten independent initial variable parameters are: D is the inside diameter of the laser tube; d_r is the inside diameter of the rings; L is the length of the active area (electrode separation); P_{in} is the input electrical power; P_L is the input electrical power per unit length; f is the pulse repetition rate; p_{Ne} is the neon gas pressure; p_{H_2} is the hydrogen gas pressure; C is the equivalent capacity of the capacitor bank; and T_r is the temperature of the CuBr reservoirs. The main dependent variable is the output laser power P_{out}. These variables will be also denoted by x_i ($i = 1, 2, ..., 11$).

We used in our study the results obtained in more than 300 experiments [16–25]. Many carefully randomised samples were examined to satisfy strong requirements of the adequacy of the applied statistical method. We will present some of the most representative results obtained from 157 experiments for the above mentioned eleven variables.

Let x_{ij} be the values of the variable x_i in the j experiment ($j = 1, 2, ..., 157$). Before any statistical analysis, we can calculate standardised values $\hat{x}_i$ for any j from the expression $\hat{x}_{ij} = (x_{ij} - \bar{x}_i)/\sigma_i$, where $\bar{x}_i$ and σ_i are the mean value and the standard deviation of x_i, respectively.

In factor analysis a set of p variables should be reduced to a set of m elements ($m < p$), where m is the number of factors subject to further selection. Each factor is represented as a sum of p variables. Thus, the ith factor has the form

$$F_i = w_{i1}\hat{x}_1 + w_{i2}\hat{x}_2 + ... + w_{ip}\hat{x}_p, \quad i = 1, 2, ..., m. \qquad (1)$$

One may also express each p variable as a linear combination of m factors,

$$\hat{x}_k = a_{1k}F_1 + a_{2k}F_2 + ... + a_{mk}F_m + E_k, \;\; k = 1, 2, ..., p, \qquad (2)$$

where E_k is the variance that cannot be expressed in terms of factors and can be treated as an error term.

To perform the factor or principal component analysis, it is necessary to obtain the correlation matrix to study the interrelation between the laser parameters in our data set. The upper part of Table 1 shows the correlation coefficients and the lower part presents corresponding significance levels. We found that only the first six variables D, d_r, L, P_{in}, P_L and p_{H_2} strongly correlated with the output power P_{out}. The absolute values of the correlation coefficients for other four parameters are lower than 0.3 and the significance level of T_r is unacceptable ($0.352 > \alpha = 0.05$). This means that these four variables should not be used in the factor analysis.

Table 1. Correlation matrix of all observed initial laser parameters (determinant is equal to 4.96E-006).

		D/mm	d_r/mm	L/cm	P_{in}/kW	P_L/kW m^{-1}	p_{H_2}/Torr	f/kHz	p_{Ne}/Torr	C/pF	T_r/°C	P_{out}/W
Correlation	D	1.00	.852	.688	.633	–.559	.257	–.056	–.244	.396	.082	.655
	d_r	.852	1.00	.904	.849	–.543	.350	–.134	–.194	.341	.181	.881
	L	.688	.904	1.00	.858	–.713	.510	–.168	–.131	.217	.077	.913
	P_{in}	.633	.849	.858	1.00	–.330	.362	–.143	–.099	.302	.072	.954
	P_L	–.559	–.543	–.713	–.330	1.00	–.444	.142	.320	–.190	.004	–.452
	p_{H_2}	.257	.350	.510	.362	–.444	1.00	–.155	–.062	–.081	–.281	.451
	f	–.056	–.134	–.168	–.143	.142	–.155	1.00	.491	–.083	.061	–.184
	p_{Ne}	–.244	–.194	–.131	–.099	.320	–.062	.491	1.00	–.315	.023	–.137
	C	.396	.341	.217	.302	–.190	–.081	–.083	–.315	1.00	.224	.235
	T_r	.082	.181	.077	.072	.004	–.281	.061	.023	.224	1.00	.031
	P_{out}	.655	.881	.913	.954	–.452	.451	–.184	–.137	.235	.031	1.00
Sig.	D		.000	.000	.000	.000	.001	.247	.001	.000	.159	.000
(1-tailed)	d_r	.000		.000	.000	.000	.000	.051	.009	.000	.013	.000
	L	.000	.000		.000	.000	.000	.020	.055	.004	.174	.000
	P_{in}	.000	.000	.000		.000	.000	.041	.115	.000	.192	.000
	P_L	.000	.000	.000	.000		.000	.041	.000	.010	.481	.000
	p_{H_2}	.001	.000	.000	.000	.000		.029	.224	.161	.000	.000
	f	.247	.051	.020	.041	.041	.029		.000	.155	.228	.012
	p_{Ne}	.001	.009	.055	.115	.000	.224	.000		.000	.389	.047
	C	.000	.000	.004	.000	.010	.161	.155	.000		.003	.002
	T_r	.159	.013	.174	.192	.481	.000	.228	.389	.003		.352
	P_{out}	.000	.000	.000	.000	.000	.000	.012	.047	.002	.352	

Table 2. Total Variance Explained.

Component	Initial eigenvalues			Rotation Sums of Squared Loadings		
	Total	Variance (%)	Cumulative (%)	Total	Variance (%)	Cumulative (%)
1	4.06	67.65	67.65	2.97	49.49	49.49
2	0.89	14.78	82.43	1.45	24.09	73.58
3	0.63	10.56	92.99	1.16	19.41	92.99

Note. Method of principal components.

First, we need to check whether our data for the first six parameters satisfy the common requirements of applicability of the factor analysis. The Kaiser – Meyer – Olkin measure of sampling adequacy was calculated to be 0.681 (this index must be between 0.5 and 1) and Bartlett's test of sphericity significance level (Sig.) was equal to 0.000. Note that the corresponding values of these standard statistical indices for all the data were 0.600 and 0, respectively, which means that the factor analysis is always applicable.

The next important step is to decide how many components or factors should be retained and repackaged from p correlated variables into m uncorrelated components. By using the method of principal components, we extracted all components and chose three factors, i.e. three first eigenvalues, which account for 92.99 % of the total variance in the data set. The details are shown in Table 2.

Then, the factors were extracted by using the method of principal component and the method of Varimax rotation. The resulting rotated component matrix, also known as the loading matrix, is given in Table 3. For clearness, the correlation coefficients less than 0.3 were omitted. Because the three components are orthogonal, these correlation coefficients are also beta weights, which follows from (2)

$$
\begin{aligned}
\hat{P}_{\mathrm{in}} &\approx 0.942F_1, \\
\hat{d}_{\mathrm{r}} &\approx 0.905F_1 + 0.343F_2, \\
\hat{L} &\approx 0.789F_1 + 0.431F_2 + 0.349F_3, \\
\hat{D} &\approx 0.744F_1 + 0.489F_2, \\
\hat{P}_L &\approx -0.913F_2, \\
\hat{p}_{\mathrm{H}_2} &\approx 0.943F_3.
\end{aligned} \tag{3}
$$

From these representations becomes clear that the variables P_{in}, d_{r}, L, and D load highly on the first factor F_1, with the relevant weights 0.942, 0.905, 0.789 and 0.744. This main factor accounts for 49.49 % of data variance (see Table 2). The variable P_L with a weight -0.913 loads highly and negatively on the second factor F_2, which contributed 24.09 % of the total variance. The last variable p_{H_2} loads on F_3 with 0.943 and gives 19.41 % of the variance.

Table 3. Rotated Component Matrix.

Parameter	Component		
	1	2	3
P_{in}	.942		
d_{r}	.905	.343	
L	.789	.431	.349
D	.744	.487	
P_L		–.913	
p_{H_2}			.943

Note. Method of principal components. Rotation Method – Varimax with Kaiser Normalisation. Rotation is converged in 5 iterations.

Note here, that we used different extraction and rotation methods to obtain the component matrix and they all gave the results similar to those in Table 3. The subsequent analysis by reproduced, residual correlation matrices and other relevant statistics confirmed the adequacy and admissible error evaluation of the performed factor analysis. Also, the factor scores were obtained, which could be used as independent orthogonal variables in further statistical analyses.

The obtained results allows us to classify the observed laser parameters, as is shown in Fig. 1. By its physical meaning the second factor P_L (the input power per unit length) cannot be separated from variables L and P_{in}. Thus it should be considered only together with them, although formally it is orthogonal to the F_1 and plays its own role. The last four laser parameters, which were preliminarily rejected from the factor analysis, are not included in this scheme because of their negligible correlation with the selected variables and output power.

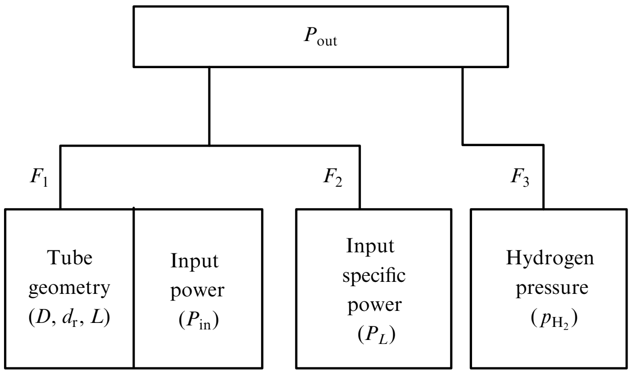

Figure 1. Structure of factors affecting the output laser power.

3. Multiple regression

In multiple regression, a relation between several independent variables $(z_1, z_2, \ldots, z_p)$ and one dependent variable (y) should be obtained. The basic linear model has the form $y = b_0 + b_1z_1 + b_2z_2 + \ldots + b_pz_p$ or, by using standard notation, $\hat{y} = \beta_1\hat{z}_1 + \beta_2\hat{z}_2 + \ldots + \beta_p\hat{z}_p$. Here the regression coefficients b_i (or β_i) are chosen so that the Pearson coefficient r between the dependent variable and the linear combination is maximal.

To apply multiple regression analysis to our data, we used three independent variables F_1, F_2, F_3. The obtained regression coefficients are presented in Table 4. The corresponding multiple regression models have the form

Table 4. Multiple regression analysis (dependent variable P_{out}).

Mode	Unstandardised coefficients		Standardised coefficients	t	Sig.	Collinearity statistics	
	B	Std. error	β			Tolerance	VIF
(Constant)	27.529	.558		11.830	.000		
F_1	25.611	.560	.894	38.791	.000	1.000	1.000
F_2	3.924	.560	.137	5.944	.000	1.000	1.000
F_3	9.080	.560	.317	13.752	.000	1.000	1.000

$$P_{out} = 27.529 + 25.611F_1 + 3.924F_2 + 9.080F_3, \quad (4)$$

$$\hat{P}_{out} = 0.894F_1 + 0.137F_2 + 0.317 + F_3. \quad (5)$$

The same results were obtained by the methods of stepwise and backward multiple regression.

To confirm the adequacy of regression models (4) and (5), we analysed the variance ANOVA. The obtained significance level is Sig. = 0, which is very satisfactory. The value of the multiple correlation coefficient is $R = 0.985$, thus the independent regression variables F_1, F_2, F_3 correlate very well with the output laser power. The other important characteristic of the model is the squared multiple correlation coefficient, which in our case is $R^2 = 0.919$. This means that the regression model describes 91.9 % of the all observed experimental data (% of the total dispersion). The Durbin–Watson autocorrelation coefficient is 1.563, which means that there are no strong systematical relations between the theoretical and empirical residuals. Finally, the multicollinearity index VIF is equal to 1 (it must be less than 10), the collinearity effect is absent and the obtained regression model is suitable for interpretation.

4. Results and discussion

In this section we present the physical interpretation of the obtained results that might be useful in practice for design and planning of the experiment.

The presence of inner rings increases the inner surface of the tube, thereby improving the cooling and thermal balance of the tube. This produces conditions for rapid and efficient relaxation of a plasma, i.e. for the diffusion of metastable atoms, electrons and ions and their relaxation and recombination in the volume and on the walls of the tube. This leads to the efficient reduction of copper bromide from copper and bromine in the tube.

The lengthening of the laser tube provides a greater amplification and, hence, a greater output power. The quantities D, L affect the volume power density and the temperature profile in the cross section of the tube. An increase in the input electric power may be related to the increase in the energy of electrons and the greater excitation of the upper laser level. In the third group of parameters, the hydrogen pressure is the most important. Its role for increasing the output laser power and efficiency is well known [16, 17].

Our study has shown that the results obtained in the paper correspond to experimental data [16–24] and correctly describe well known physical processes and their influence on the output laser power. Thus, these results can be used for further analysis.

Concerning particular model problems of planning experiments, we can draw the following conclusions:

(i) Among the independent parameters studied in the paper, five material parameters and one relative parameter affect considerably the output power.

(ii) The output laser power can be increased by varying the geometrical parameters of the laser and the input electric power while sustaining the decrease in P_L. The value of p_{H_2} strongly affects the output power. Even small deviations from the specified optimal pressure may reduce considerably the output power and, therefore, in the case of new experimental conditions the optimal hydrogen pressure should be determined again.

(iii) The conducted regression analysis, the obtained polynomial relations and the structure of factors (Fig. 1) allow us to determine the groups of independent quantities according to the degree of their influence on the output power. The geometrical dimensions and electric power are most important. The hydrogen pressure, as a thermodynamic quantity, independently affects the operation of the laser. The influence of parameters C and T_r is weak. This means that these parameters may not be investigated in further experiments and maintained within the already specified optimal intervals.

5. Conclusions

We have proposed formal statistical methods for solving particular model problems related to planning of experiments. Specific problems that enable planning of sampling and optimisation experiments have been considered. Linear relations have been obtained between independent parameters and the output laser power which allow one to estimate the influence of physical processes on the output laser characteristics. Among the nine material physical variables, four parameters only weakly affect laser parameters, which allows cutting short the investigation expenses. The analysis of our results show that the output laser power can be increased by combined variation of the geometric design of the laser tube and the electric power and by using hydrogen as an addition to the buffer neon gas.

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