

CONTROL OF LASER RADIATION PARAMETERS

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Influence of the phase difference of coupling coefficients of counterpropagating waves on relaxation oscillations in the self-modulation regime of a solid-state ring laser

I.I. Zolotoverkh

Abstract**. The effect of the phase difference of the coupling coefficients on relaxation frequencies in the emission spectrum of a solid-state ring laser operating in the self-modulation regime of the first kind is studied theoretically. A strong dependence of one of the frequencies of relaxation oscillations on the phase difference of coupling coefficients is found. The stability of the self-modulation regime is studied analytically.**

Keywords: *relaxation oscillations, solid-state ring laser, self-modulation lasing regime.*

1. Introduction

Quantum and technical fluctuations can result in the appearance of resonance peaks in the emission spectra of lasers [1]. Khandokhin and Khanin were the first to formulate and study the role of relaxation frequencies in the emission spectrum a solid-state ring laser. They found three relaxation oscillations in the spectrum of the solid-state ring laser generating two counterpropagating waves with considerably unequal constant intensities in the stationary regime [2 – 5].

Later, relaxation oscillations were studied in the emission spectrum for one of the most widely used lasing regimes of the solid-state ring laser, i.e. the self-modulation regime of the first kind, which is characterised by the out-of-phase sinusoidal self-modulation of the intensities of counterpropagating waves. The analysis of the relaxation oscillations is very important to solve the problems of nonlinear dynamics, where the parametric interaction of self-modulation and relaxation oscillations takes place, and to study the stability of the self-modulation regime in problems related to the increase in the stability of output parameters of radiation (see, for example, review [6] and references therein). Previously, relaxation oscillations in the self-modulation regime were studied theoretically and experimentally in papers [7 – 10]. As is known, the spectrum in the self-modulation regime contains two relaxation frequencies (the fundamental ω_{r} and additional $\omega_{\mathrm{r}1}$ frequencies). In the absence of the frequency nonreciprocity of the resonator, expressions for the relaxation frequencies ω_{r} and $\omega_{\mathrm{r}1}$ have the form [7]

$$\omega_{\mathrm{r}} = \left(\frac{\omega_{\mathrm{e}}}{Q}\frac{\eta}{T_1}\right)^{1/2}, \tag{1}$$

$$\omega_{\mathrm{r}1} = \frac{1}{\sqrt{2}}\left(\frac{\omega_{\mathrm{e}}}{Q}\frac{\eta}{T_1}\right)^{1/2}, \tag{2}$$

where ω_{e} is the eigenfrequency; Q is the resonator Q factor; η is the pump excess over the threshold; T_1 is the relaxation time of the inverse population.

The frequency ratio $\omega_{\mathrm{r}1}/\omega_{\mathrm{r}}$ measured in experiment [7] is in agreement with expressions (1) and (2), i.e. is equal to $1/\sqrt{2}$. However, the ratio $\omega_{\mathrm{r}1}/\omega_{\mathrm{r}}$ measured in [8] differed from $1/\sqrt{2}$. One of the possible reasons is the different polarisations of counterpropagating waves [8, 9]. We consider here another factor affecting the frequency ratio $\omega_{\mathrm{r}1}/\omega_{\mathrm{r}}$, namely, the phase difference of coupling coefficients.

2. Theoretical analysis of relaxation frequencies

The numerical and analytic investigations were performed based on the standard model of a solid-state ring laser [6] by using a system of equations

$$\begin{aligned}
&\frac{\mathrm{d}E_{1,2}}{\mathrm{d}t} = \frac{1}{2}\left(\frac{\sigma l}{T}N_0 - \frac{\omega_{\mathrm{e}}}{Q}\right)E_{1,2} + \frac{1}{2}\left(\mathrm{i}\,\tilde{m}_{1,2} + \frac{\sigma l}{T}N_{\pm}\right)E_{2,1},\\
&T_1\frac{\mathrm{d}N_0}{\mathrm{d}t} = N_{\mathrm{th}}(1+\eta) - N_0\left[1 + a\left(|E_1|^2 + |E_2|^2\right)\right]\\
&\qquad - N_{+}aE_1E_2^{*} - N_{-}aE_1^{*}E_2,\\
&T_1\frac{\mathrm{d}N_{+}}{\mathrm{d}t} = -N_{+}\left[1 + a\left(|E_1|^2 + |E_2|^2\right)\right] - N_0 aE_1^{*}E_2,\\
&N_{-} = N_{+}^{*}
\end{aligned} \tag{3}$$

for the complex amplitudes $E_{1,2}$ of the fields of counterpropagating waves and the spatial harmonics N_0 and $N_{\pm}$ of the inverse population. Here, a is the saturation parameter; T is the round-trip time of light in the resonator; σ is the

I.I. Zolotoverkh D.V. Skobeltsyn Institute of Nuclear Physics, M.V. Lomonosov Moscow State University, Vorob'evy Gory, 119992 Moscow, Russia; e-mail: Zolotoverkh@yandex.ru

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stimulated emission cross section for the laser transition; l is the resonator perimeter; $\tilde{m}_{1,2} = m \exp(\pm \mathrm{i}\theta_{1,2})$ are complex coupling coefficients; m and $\theta_{1,2}$ are the modulus and phases of coupling coefficients;

$$N_{\mathrm{th}} = \frac{T}{\sigma l}\left(\frac{\omega_{\mathrm{e}}}{Q} - m\left|\sin\frac{\theta_1 - \theta_2}{2}\right|\right)$$

is the threshold inverse population. It is assumed in Eqns (3) that the resonator Q factors and moduli of the coupling coefficients for counterpropagating waves are equal, the frequency nonreciprocity of the resonator is absent and the relative frequency detuning from the gain line centre is small and can be neglected.

The relaxation frequencies in the self-modulation regime are studied in this paper by the method similar to that used in [7]. Let us briefly describe the method. First, we pass in the initial system of equations to new variables:

$$I_{1,2} = a|E_{1,2}|^2, \;\; U = \mathrm{Re}\,(aE_1E_2^*), \;\; V = \mathrm{Im}\,(aE_1E_2^*),$$

$$N_{\mathrm{r}} = \mathrm{Re}\,N_+, N_{\mathrm{i}} = \mathrm{Im}\,N_+.$$

Then, we consider small perturbations with respect to the periodic solution:

$$i_{1,2} = I_{1,2} - I_{1,2}^0, \;\; u = U - U^0, \;\; v = V - V^0,$$

$$n_0 = N_0 - N_0^0, \;\; n_{\mathrm{r}} = N_{\mathrm{r}} - N_{\mathrm{r}}^0, \;\; n_{\mathrm{i}} = N_{\mathrm{i}} - N_{\mathrm{i}}^0.$$

Here, the superscript 0 refers to the solution describing the self-modulation regime, and small letters denote perturbations. A linear system of differential equations with coefficients periodically depending on time is obtained for these perturbations. As in [7], we neglect the time-dependent coefficients, which is valid if the inequality

$$\frac{\omega_{\mathrm{r}}^2}{\omega_{\mathrm{m}}^2} \ll 1$$

is fulfilled, where ω_{m} is the self-modulation frequency. We used the following analytic expression for ω_{m} borrowed from paper [11]:

$$\omega_{\mathrm{m}} = m\left|\cos\frac{\theta_1 - \theta_2}{2}\right|. \tag{4}$$

As a result, we obtain a system of equations with time-independent coefficients. Its solution in the general case can be written in the form

$$i_{1,2} = i_{1,2}^0 \exp(\mathrm{i}\omega t), \;\; u = u^0 \exp(\mathrm{i}\omega t), \;\; v = v^0 \exp(\mathrm{i}\omega t),$$

$$n_0 = n_0^0 \exp(\mathrm{i}\omega t), \;\; n_{\mathrm{r}} = n_{\mathrm{r}}^0 \exp(\mathrm{i}\omega t), \;\; n_{\mathrm{i}} = n_{\mathrm{i}}^0 \exp(\mathrm{i}\omega t), \tag{5}$$

where $i_{1,2}^0$, u^0, v^0, n_0^0, n_{r}^0, and n_{i}^0 are time-independent coefficients. By substituting (5) into a system of linear differential equations, we obtain a homogeneous system of linear algebraic equations. This system is rather cumbersome to solve it directly. Therefore, let us make further simplifications. In [7] all the terms proportional to $m(\omega_{\mathrm{e}}/Q)^{-1}$ were neglected, i.e. it was assumed that

$$m\left(\frac{\omega_{\mathrm{e}}}{Q}\right)^{-1} \ll 1. \tag{6}$$

We neglect only the terms of the second-order smallness, i.e. assume that the inequality

$$\left[m\left(\frac{\omega_{\mathrm{e}}}{Q}\right)^{-1}\right]^2 \ll 1 \tag{7}$$

is valid. The derived system will have a solution if its determinant is equal to zero:

$$D = D_0 + D_1 = 0, \tag{8}$$

where

$$D_0 = \omega_{\mathrm{m}}^2\left[\omega - \frac{(\omega_{\mathrm{r1}}^0)^2}{\omega}\right]\left[\omega - \frac{(\omega_{\mathrm{r}}^0)^2}{\omega}\right]; \tag{9}$$

$$D_1 = -2\left(1 + \frac{\eta_{\mathrm{eff}}}{2}\right)\frac{\omega_{\mathrm{m}}^4}{\omega^2 T_1^2}\tan^2\frac{\theta_1 - \theta_2}{2}; \tag{10}$$

$$\eta_{\mathrm{eff}} = \eta - \frac{m}{\omega_{\mathrm{e}}/Q}\left|\sin\frac{\theta_1 - \theta_2}{2}\right|; \tag{11}$$

$$\omega_{\mathrm{r}}^0 = \left(\frac{\omega_{\mathrm{e}}}{Q}\frac{\eta_{\mathrm{eff}}}{T_1}\right)^{1/2}, \tag{12}$$

$$\omega_{\mathrm{r1}}^0 = \frac{1}{\sqrt{2}}\left(\frac{\omega_{\mathrm{e}}}{Q}\frac{\eta_{\mathrm{eff}}}{T_1}\right)^{1/2}. \tag{13}$$

Determinant (8) differs from that obtained in [7] by the presence of an additional term in (10).

Let us solve Eqn (8), which determines the relaxation frequencies, by two ways. In the first case, we neglect the term in (10). Then, the solution of Eqn (8) will be expressed by Eqns (12), (13) similar to equations (1), (2) from [7]. The difference is that instead of the pump excess η over the threshold, the effective excess η_{eff}, which is determined by (11), enters relations (12), (13). This can substantially affect the relaxation frequencies in the case of large phase differences of coupling coefficients. For the small phase difference ($\theta_1 - \theta_2 \to 0$) and $\eta_{\mathrm{eff}} \to \eta$, expressions (1) and (2) are valid. For the large phase differences ($|\theta_1 - \theta_2| \to \pi$) of coupling coefficients, expressions for relaxation frequencies differ significantly from expressions (1) and (2).

In the second case, we find the exact analytic solution of (8) without neglecting the term in (10). Equation (8) is reduced to a biquadratic equation of the fourth power. Its solution has the form:

$$\omega_{\mathrm{r}}^{\mathrm{r}} = \left[\frac{b + (b^2 - 4c)^{1/2}}{2}\right]^{1/2}, \tag{14}$$

$$\omega_{\mathrm{r1}}^{\mathrm{r}} = \left[\frac{b - (b^2 - 4c)^{1/2}}{2}\right]^{1/2}, \tag{15}$$

where

$$b = \frac{3}{2}(\omega_{\rm r}^0)^2, \tag{16}$$

$$c = \frac{1}{2}(\omega_{\rm r}^0)^4 - 2\left(1 + \frac{\eta_{\rm eff}}{2}\right)\frac{\omega_{\rm m}2}{T_1^2}\tan^2\frac{\theta_1 - \theta_2}{2}. \tag{17}$$

The obtained expressions describe the influence of the phase difference of the coupling coefficients on the relaxation frequencies.

We solved numerically the initial system of equations (3) and compared the results obtained by using Eqns (14) – (17) with the results of the numerical simulation for the parameters typical of ring chip Nd : YAG lasers [1]. Figure 1 shows the dependences of the fundamental and additional relaxation frequencies on the pump excess over the threshold η for $m/(2\pi) = 1000$ kHz, $\theta_1 = 2.74$ rad, $\theta_2 = 0$, $T_1 = 240$ μs, $\omega_{\rm e}/Q = 1.1 \times 10^8$ s^{-1}. For these parameters expression (4) yields $m/(2\pi) = 200$ kHz.

Figure 1 shows the results of the numerical solution of equations (3) and calculations based on the derived analytic expressions. One can see that expressions (14), (15) well approximate the solution of Eqns (3) for the large phase differences of the coupling coefficients (for any η), while expressions (12), (13) yield low accuracy for small pump excesses over the threshold ($\eta \to 0$).

It follows from (14), (15) that the ratio $\omega_{\rm r1}^{\rm r}/\omega_{\rm r}^{\rm r}$ is determined by the expression

$$\frac{\omega_{\rm r1}^{\rm r}}{\omega_{\rm r}^{\rm r}} = \left[\frac{b - (b^2 - 4c)^{1/2}}{b + (b^2 - 4c)^{1/2}}\right]^{1/2}. \tag{18}$$

Figure 2 presents the dependence of the quantity $1 - \sqrt{2}\omega_{\rm r1}^{\rm r}/\omega_{\rm r}^{\rm r}$ calculated by using (18), which characterises the relative deviation of $\omega_{\rm r1}^{\rm r}/\omega_{\rm r}^{\rm r}$ from $1/\sqrt{2}$ on the phase difference of coupling coefficients for $\omega_{\rm m}/(2\pi) = 200$ kHz, $\omega_{\rm r}^0/(2\pi) = 60$ kHz, $T_1 = 240$ μs, $\eta_{\rm eff} = 0.3$. For $\theta_1 - \theta_2 \to \pi$, the ratio of relaxation frequencies significantly differs from $1/\sqrt{2}$. Thus, the difference of the ratio of the relaxation frequencies from $1/\sqrt{2}$ can be caused by the influence of the phase difference of the coupling coefficients. As was shown in [9], another reason for a change in the ratio $\omega_{\rm r1}^{\rm r}/\omega_{\rm r}^{\rm r}$ is the difference of radiation polarisations of counterpropagating waves. In a real laser the combined action of both factors can take place.

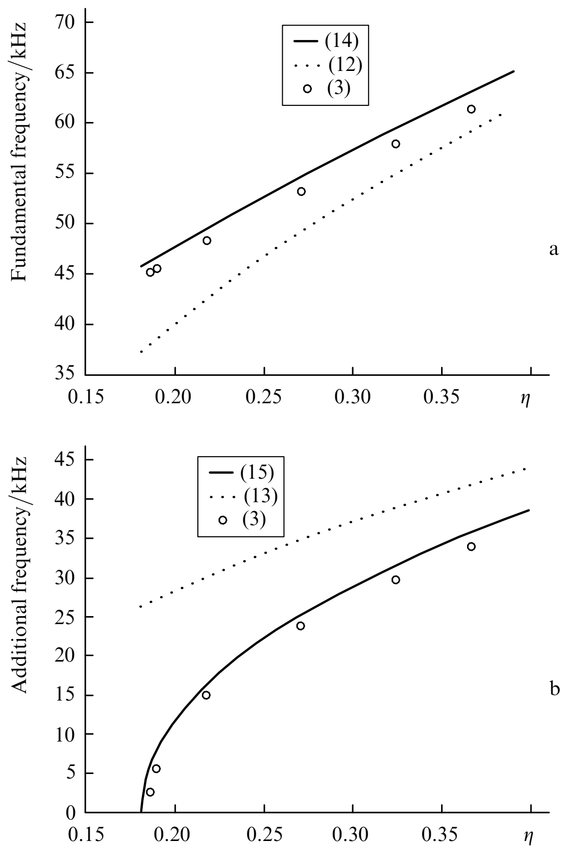

Figure 1. Dependences of the fundamental (a) and additional (b) relaxation frequencies on the pump excess over the threshold η. Solid curves are calculated by using expressions (14), (15), dashed curved are calculated by using expressions (12), (13), and points are the results of the numerical solution of equations (13).

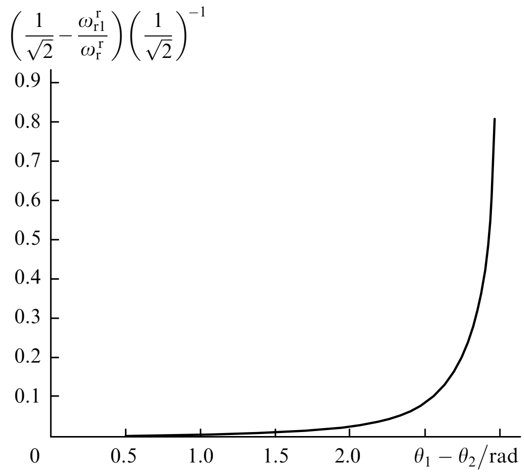

Figure 2. Dependences of the quantity $(1/\sqrt{2} - \omega_{\rm r1}^{\rm r}/\omega_{\rm r}^{\rm r})(1/\sqrt{2})^{-1}$ on the phase difference $\theta_1 - \theta_2$ of the coupling coefficients.

3. Stability of the self-modulation regime

The analysis of expression (15) shows that for low enough pump excesses over the threshold the additional relaxation frequency $\omega_{\rm r1}^{\rm r}$ vanishes. One can see from Fig. 1b that for the above parameters of the laser this occurs when $\eta \leqslant 0.18$. In this case, the quantity $(\omega_{\rm r1}^{\rm r})^2$ changes its sign from positive to negative. Physically this means that the self-modulation regime becomes unstable. Indeed, for negative values of the square of the relaxation frequency, two imaginary roots of characteristic equation (8) with opposite signs appear. The negative imaginary root corresponds to the growth increment of perturbations (5), which leads to the instability of the self-modulation regime.

Let us find the boundary of the stability region of the self-modulation regime. It appears when the relaxation frequency in (15) is equal to zero: $\omega_{\rm r1}^{\rm r} = 0$. It follows from here that $c = 0$. By solving this equation with respect to the pump excess over the threshold η and taking (7) into account, we determine the boundary of the stability region of the self-modulation regime:

$$\eta_{\rm b} = \frac{3\omega_{\rm m}}{\omega_{\rm e}/Q}\left|\tan\frac{\theta_1 - \theta_2}{2}\right|. \tag{19}$$

When the condition $\eta > \eta_{\rm b}$ is fulfilled, the self-modulation regime is stable. Moreover, as follows from the parameters of the self-modulation regime found in [11], this condition coincides with the condition of existence of the self-modulation regime with account for inequality (7). Thus, the condition $\eta = \eta_b$ determines the boundary of existence and stability of the self-modulation regime. When the opposite inequality is fulfilled ($\eta < \eta_b$), there exists the stationary regime of a standing wave with equal intensities of the counterpropagating waves. The inequality $\eta < \eta_b$ coincides with the condition of the stability of the standing wave obtained in [12]. Figure 3 shows the dependence of the boundary of the stability region of the self-modulation regime on the phase difference of the coupling coefficients.

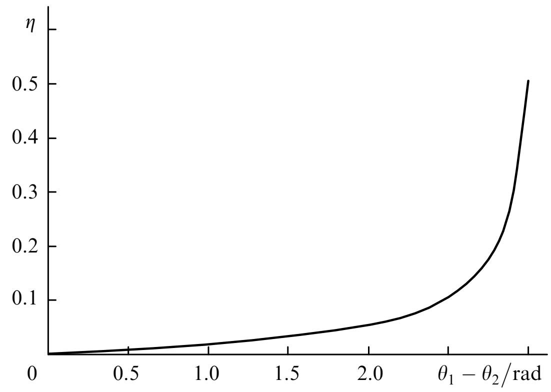

Figure 3. The boundary of the stability and existence region of the self-modulation regime in the coordinate system of η and $\theta_1 - \theta_2$ for $\omega_m/(2\pi) = 200$ kHz, $\omega_e/Q = 1.1 \times 10^8$ s^{-1}.

4. Conclusions

Thus, relaxation oscillations in the emission spectrum of a ring laser operating in the self-modulation regime have been theoretically studied. The derived expressions for the relaxation frequencies generalise the known expressions for the case of arbitrary phases of the coupling coefficients. The effect of the phase difference of the coupling coefficients on the relaxation frequencies has been analysed. The stability of the self-modulation regime has been studied. The influence of the phase difference of the coupling coefficients on the stability of the self-modulation regime has been considered. The derived relations are valid if the self-modulation frequency is significantly larger than the fundamental relaxation frequency. The error of the results does not exceed a few percent when the self-modulation frequency exceeds the fundamental relaxation frequency by more than three times.

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