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Influence of the resonator parameters and spatially nonuniform amplification on the spatial structure of the fundamental mode of stable resonator lasers

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Abstract. **The combined influence of the resonator parameters and spatially nonuniform axially symmetric amplification distribution in a four-level active medium on the spatial structure of the fundamental mode of laser radiation is studied numerically by the Fox – Li method. The model of two beams interacting due to the spatial amplification nonuniformity is used to obtain simple analytic estimates. The transition of the spatial structure of the fundamental mode from the Gaussian one to the structure formed by a set of Laguerre – Gaussian beams is studied. It is shown that near the degenerate configurations of the resonator a decrease in the diameter of the pump beam leads to an increase in the number of Laguerre – Gaussian beams forming the fundamental mode. It is found that the range of detunings from the strong degeneracy within which substantial differences of the spatial structure of the fundamental mode from the Gaussian structure are realised, increases with increasing the gain.**

Keywords: *longitudinal pumping, degenerate resonators, multibeam modes.*

1. Introduction

At present longitudinal diode pumping of solid-state lasers has found wide application [1 – 4]. It is known that the lasing efficiency increases in a four-level active medium when the pump beam radius w_p is smaller than the radius w_{am} of the zero mode of an empty resonator ($\xi = w_{am}/w_p > 1$), i.e. upon spatially nonuniform amplification [5, 6]. It was shown in [7] that when pump saturation is taken into account, there exist optimal values ξ_{opt} to obtain the maximal efficiency and the minimal lasing threshold. For the most wide-spread active media with Nd ions, $\xi_{opt} > 2$. It was demonstrated experimentally in [8] that for $\xi > 1$ the spatial structure of radiation noticeably differs from the structure of individual Laguerre – Gaussian (LG) beams in the case of the so called critical configurations of the resonator with $g_1g_2 = 0.25$, 0.5, 0.75, where $g_{1,2} = 1 - L/R_{1,2}$ (L is the resonator length and $R_{1,2}$ are the radii of curvature of mirrors). Authors of [8] explained this fact by the appearance of the frequency degeneracy of empty resonator modes, which takes place if the condition

$$\arccos\sqrt{g_1g_2} = \pi\frac{q}{s} \qquad (1)$$

is fulfilled, where q/s is the irreducible fraction.

It was pointed out in [9] that the reason for the existence of critical configurations is the resonance coupling of the zero mode of an empty resonator with a high-order mode, which appears under conditions of degeneracy. Based on the assumptions on the interaction of only two modes, it was shown within the phenomenological model in [9] that the resonance width depends both on the ratio of losses of high-order modes and the zero mode and on the coupling coefficient between them. However, this paper did not study the relation of the spatial structure of the fundamental mode with the pump and resonator parameters.

The effect of the nonuniform gain distribution, in particular, upon axially symmetric pumping on the fundamental mode of a semiconfocal resonator was studied in [10]. It was shown by using the Fox – Li method [11] that for a small gain ($G_0 = 1.2$) and $\xi > 0.67$ the intensity distribution noticeably differs from the Gaussian one. When ξ is decreased, the parameters of the laser beam approach the parameters of the eigenbeam of the empty resonator and when G_0 increases (up to 64) – to the parameters determined by the size of the equivalent Gaussian aperture. Papers [12 – 15] were devoted to the study of the spatial structure of radiation in a number of plane-spherical resonators of critical configurations.

In [16] the influence of the resonator configuration (g_1, g_2) on the spatial structure of the fundamental mode was considered by using the Fox – Li method. Lasers with stable resonators and a thin active medium with a Gaussian transverse profile of the field gain of the type

$$K(r) = 1 + (K_0 - 1)\exp\left(-\xi^2\frac{r^2}{w_{am}^2}\right) \qquad (2)$$

were studied. The parameters of the nonuniform pumping were fixed: $K_0 = 1.5$, which corresponds to amplification realised in a picosecond diode-pumped Nd : YAG generator controlled by negative and positive feedbacks [4] and $\xi = 3$, which is close to the assumed optimal value. It was shown that the number of critical configurations, i.e. the regions in the stability diagram in which multibeam (formed by a set of LG beams) fundamental mode is present, decreases with

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decreasing the Fresnel number $N_F = a^2/(L\lambda)$ (a is the radius of the resonator mirrors, λ is the radiation wavelength). For $N_F = 9$, the configurations with $q/s = 1/2$, 2/5, 3/8, 1/3, 3/10, 1/4, 1/5, 1/6, 1/8, 1/10 were distinguished. The amplitude and phase distributions in several reference planes were presented for characteristic configurations of the resonator.

However, the data on the effect of the amplification parameters K_0 and ξ on the fundamental mode in the entire stability region, which are necessary for developing longitudinally-diode-pumped lasers, are absent in the literature. In this paper, which is the continuation of [16], the combined influence of the resonator parameters and stationary nonuniform axially symmetric gain distribution (amplification K_0 on the resonator axis and relative nonuniformity ξ) on the parameters of the fundamental mode is studied. In particular, the range of detunings from the degeneracy and parameters of the gain distribution, which lead to the formation of a multibeam mode, are determined.

2. Results of numerical calculations by the Fox – Li method

The Fox – Li method was used to determine the amplitude distribution $u(r)$ of the fundamental mode. The calculation procedure in the case of cylindrical symmetry is described in detail in [16]. Calcualtions were performed for stable resonators ($0 < g_1 g_2 < 1$). The radii a of mirrors were selected equal. A thin active medium was placed on one of the mirrors. The amplitude distribution was calculated in two planes: at the input and output of the active medium. The gain profile per round trip of the active medium was specified in form (2). The difference of the spatial structure of the fundamental mode from the structure of the zero mode of the empty resonator $u_0^{\mathrm{LG}}(r,\mu)$, where μ is the complex parameter of LG beams, whose value in some plane is determined by the resonator length and parameters g_1, g_2, was characterised as in [16] by the parameter

$$|\beta_0|^2 = \left|2\pi \int u(r)(u_0^{\mathrm{LG}})^*(r,\mu) r \mathrm{d}r\right|^2. \quad (3)$$

This parameter represents a fraction of energy contained in the zero mode of the empty resonator for the normalised amplitude distribution ($2\pi \int |u(r)|^2 r\mathrm{d}r = 1$).

Quite illustrative are the dependences of $|\beta_0|^2$ in the input plane of the active medium on g_1 (or g_2) for symmetric resonators ($g_2 = g_1$). For $N_F = 9$, $\xi = 3$ and $K_0 = 1.05$, 2.0, 3.0 such dependences are presented in Fig. 1a. A significant decrease in $|\beta_0|^2$ and, hence, the multibeam mode are realised in the vicinity of a set of degenerate configurations of the resonator. In the case of strong degeneracy, the multibeam mode is realised even at small K_0. One can see from Fig. 1a that the regions within which a multibeam mode is formed, decrease with decreasing K_0. The values of $|\beta_0|^2$ between them approach unity with decreasing K_0, i.e. the mode approaches the Gaussian mode of the empty resonator.

Figure 1a also shows the dependence of $|\beta_0|^2$ on g_1 for $\xi = 3$ и $N_F = 9$ [curve (*4*)] in the case when a Gaussian aperture with the transmission function $K_{\mathrm{GA}}(r) = \exp(-\xi^2 r^2/w_{\mathrm{am}}^2)$ is mounted near the mirror instead of the active medium. The dependence of $|\beta_0|^2$ on g_1 has a qualitatively different character for the Gaussian aperture: critical configurations do not appear.

The effect of ξ on the dependence of $|\beta_0|^2$ on g_1 for symmetric resonators ($g_2 = g_1$) is illustrated in Fig. 1b. A rise in ξ leads to an increase in the number of critical configurations and a decrease in $|\beta_0|^2$ for each of them.

3. Matrix representation of interaction of beam modes upon spatial nonuniform amplification

To interpret the dependences obtained in numerical calculations, it is expedient to represent the fundamental mode in the form of superposition of LG beams (see, for example, [17]). The complex amplitude distribution $u(r)$ is represented in this case as a sum:

$$u(r) = \sum_p \beta_p u_p^{\mathrm{LG}}(r,\mu), \quad (4)$$

the coefficients β_p being determined by the expression

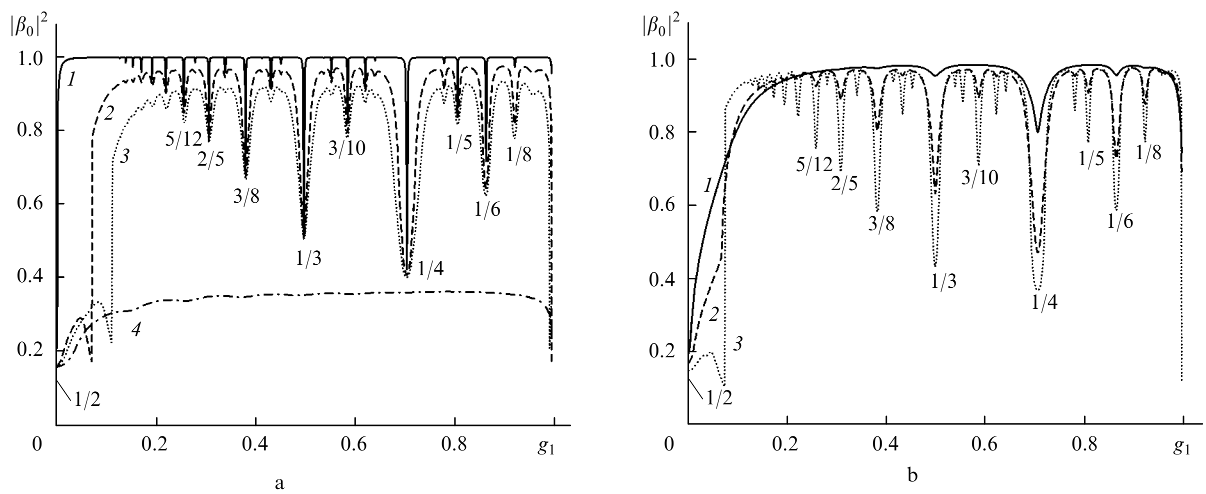

Figure 1. Dependences of $|\beta_0|^2$ on g_1 for symmetric ($g_1 = g_2$) resonators for $N_F = 9$, $\xi = 3$, $K_0 = 1.05$ (*1*), 2.0 (*2*), 3.0 (*3*) and intracavity Gaussian aperture (*4*) (a) as well as for $N_F = 9$, $K_0 = 2.0$, $\xi = 1.5$ (*1*), 2.5 (*2*), 3.5 (*3*) (b). The values of q/s are presented, which correspond to the most strongly pronounced degenerate configurations of the resonator.

$$\beta_p = 2\pi \int u(r)(u_p^{\mathrm{LG}})^*(r,\mu) r\mathrm{d}r, \quad (5)$$

where $u_p^{\mathrm{LG}}(r,\mu)$ is the complex amplitude distribution of the LG beam with the radial index p. The quantities $|\beta_p|^2$ represent a fraction of energy concentrated in the LG beam with the radial index p.

The active medium with the spatial gain profile is an element redistributing amplitudes in the system of LG beams. After a passage through the active medium, the expression for the field $u'(r)$ has the form

$$u'(r) = K(r)u(r) = \sum_{p_1} \beta'_{p_1} u_{p_1}^{\mathrm{LG}}(r,\mu), \quad (6)$$

where

$$\beta'_{p_1} = \sum_{p_2} t_{p_1 p_2} \beta_{p_2}, \quad (7)$$

and quantities $t_{p_1 p_2}$ are determined by the relation [17, 18]

$$t_{p_1 p_2} = 2\pi \int (u_{p_2}^{\mathrm{LG}})^*(r,\mu) K(r) u_{p_1}^{\mathrm{LG}}(r,\mu) r\mathrm{d}r \quad (8)$$

and represent matrix elements characterising the active medium with the profiled gain. For the gain profile described by expression (2), expression (8) has the form

$$t_{p_1 p_2} = \delta_{p_1 p_2} + (K_0 - 1) \int (u_p^{\mathrm{LG}})^*(r) \exp(-\xi^2 r^2/w_{\mathrm{am}}^2) u_{p_2}^{\mathrm{LG}}(r) r\mathrm{d}r. (9)$$

If the amplitude distribution in the input plane of the active medium is normalised to the unit power, i.e.

$$2\pi \int |(u(r)|^2 r\mathrm{d}r = \sum_{p_1} |\beta_{p_1}|^2 = 1,$$

then after the passage through the active medium the emission power is increased by

$$\sum_{p_1} |\beta'_{p_1}|^2 = \sum_{p_1} \left| \sum_{p_2} t_{p_1 p_2} \beta_{p_2} \right|^2$$

times. The complex quantity $t_{p_1 p_2}\beta_{p_2}$ is an addition to the amplitude β_{p_1} of the LG beam with the radial index p_1 from a beam with the index p_2, which is related to the amplitude redistribution during the passage through the active medium. For the gain profile described by real transmission function (2), the matrix $\|t_{p_1 p_2}\|$ is symmetric and real and for moderate amplifications ($K_0 \leqslant 2$) has the form close to the diagonal one, i.e. $|t_{p_1 p_2}| \ll |t_{p_1 p_1}|$ for $p_1 \neq p_2$.

Figure 2 shows t_{p0}, t_{p5}, and t_{pp} corresponding to the gain profiles with $\xi = 3$ and different K_0. One can see that the relative values of the nondiagonal elements increase with increasing K_0 but the set of interacting beams remains constant. Figures 2a, b show t_{p0} и t_{p5} in the case of a Gaussian aperture with the same $\xi = 3$. The expression for the elements of the matrix corresponding to the Gaussian aperture can be derived if we set K_0 equal to 2 and exclude the Kronecker symbol from Eqn (9).

Figure 3 demonstrates the effect of ξ on matrix elements t_{p0}, t_{p5}, and t_{pp} for $K_0 = 1.5$ and different ξ. For each LG beam, the number of LG beams with which it efficiently interacts increases with increasing ξ.

The matrix elements $t_{p_1 p_2}$ determine the interaction of beam modes due to the spatially nonuniform amplification. The amplitude redistribution leads in this case to the additional phase shift ψ_p of individual LG beams upon their passage through the active medium:

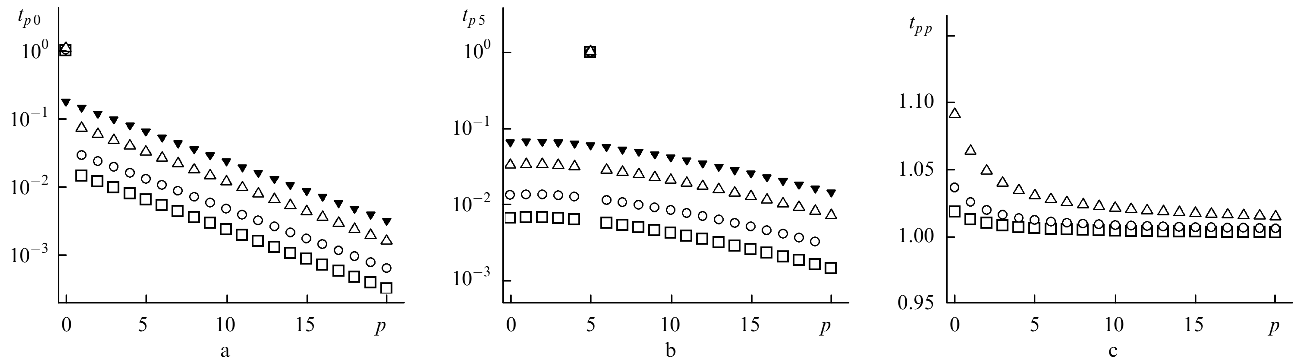

Figure 2. Matrix elements t_{p0} (a), t_{p5} (b) and t_{pp} (c) for the gain profile with $\xi = 3$, $K_0 = 1.1$ (□), 1.2 (○), 1.5 (△) and the Gaussian aperture (▼).

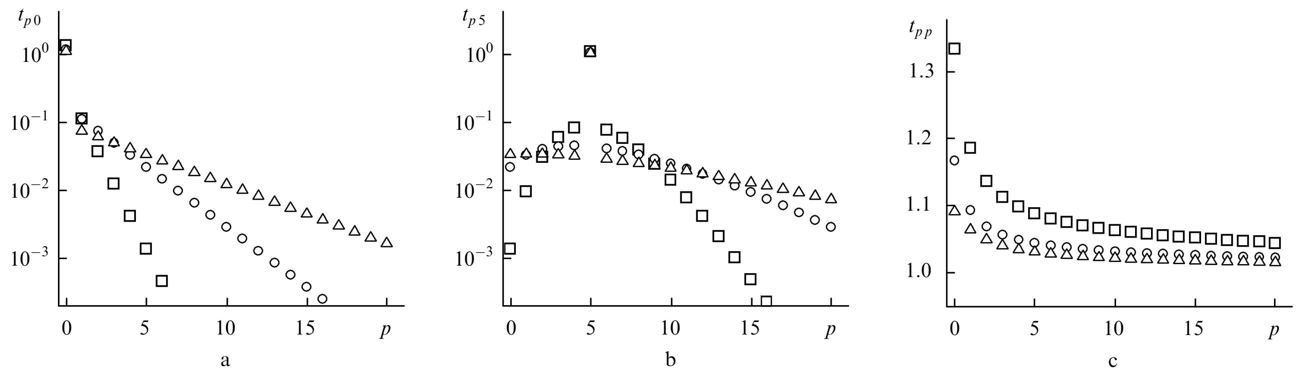

Figure 3. Matrix elements t_{p0} (a), t_{p5} (b) and t_{pp} (c) for $K_0 = 1.5$ и $\xi = 1$ (□), 2 (○), 3 (△).

$$\psi_{p_1} = \arg\sum_{p_2} t_{p_1 p_2}\beta_{p_2} - \arg\beta_{p_1}. \tag{10}$$

Note that the quantity ψ_{p_1} depends on β_{p_2}, i.e. on the amplitude distribution $u(r)$ at the input to the active medium.

Along with a change in the amplitudes and phases of LG beams upon their passage through the active medium, an additional Guoy phase shift takes place upon propagation of radiation from the active medium to the opposite mirror and back to the input to the active medium [19]:

$$\theta_p = 2(2p+1)\arccos(\pm\sqrt{g_1 g_2}). \tag{11}$$

The presence of critical configurations in the stability diagram is caused by their property to retain phase relations after the resonator round trip for arbitrary superpositions of beams with certain values of p. The conservation of the phase equality of LG beams forming the fundamental mode from one resonator round trip to another leads to the efficient energy accumulation in beams with large p during the process of multiple amplitude redistribution of beams upon amplification. The equilibrium energy distribution over the LG beams, which corresponds to the fundamental mode, is defined by the redistribution coefficients $t_{p_1 p_2}$ and an increase in the diffraction losses on limiting apertures with increasing p.

4. A two-beam model and formation of multibeam modes

Let us estimate the parameters of the gain distribution at which the Gaussian fundamental mode undergoes a transition to the multibeam mode. In this case we will use the approach similar to that proposed in [9]. We will consider the fundamental mode as a superposition of two LG beams with $p = 0$ and $p = \Delta p$, where $\Delta p = s/2$ or s for even or odd s, respectively. In this case, $\beta_0 \gg \beta_{\Delta p}$. Then, after a round trip in the resonator, β_0 and $\beta_{\Delta p}$ change as:

$$\beta_0' = \gamma_0[t_{00}\beta_0 + \exp(-\mathrm{i}\varphi)t_{0\Delta p}\beta_{\Delta p}],$$
$$\beta_{\Delta p}' = \gamma_{\Delta p}[t_{0\Delta p}\beta_0 + \exp(\mathrm{i}\varphi)t_{\Delta p\,\Delta p}\beta_{\Delta p}], \tag{12}$$

where the matrix elements $\|t_{p_1 p_2}\|$ are the functions of the parameters K_0 and ξ; $1-\gamma_0^2$ and $1-\gamma_{\Delta p}^2$ are diffraction losses of the LG beams; $\varphi = \theta_{\Delta p} - \theta_0$ is the phase shift of a beam with the radial index $p = \Delta p$ with respect to the beam with $p = 0$ upon radiation propagation from the active medium to the opposite mirror and back to the input to the active medium. If the combination of two beams under study is a resonator mode, the relative energy distribution over the beams per round trip is preserved: $\beta'/\beta_{\Delta p}' = \beta/\beta_{\Delta p}$. Taking into account that $\beta_0 \gg \beta_{\Delta p}$ and $t_{00} \gg t_{0\Delta p}$, the term $t_{0\Delta p}\beta_{\Delta p}$ in (12) can be neglected and we obtain an expression relating the beam amplitudes β_0 and $\beta_{\Delta p}$:

$$\beta_{\Delta p} = \frac{a}{1 - b\exp(\mathrm{i}\varphi)}\beta_0, \tag{13}$$

where

$$a = \frac{t_{0\Delta p}}{t_{00}}\frac{\gamma_{\Delta p}}{\gamma_0};\quad b = \frac{t_{\Delta p\Delta p}}{t_{00}}\frac{\gamma_{\Delta p}}{\gamma_0}.$$

The phase difference of the beams in the input plane of the active medium is specified by the expression

$$\vartheta = \arcsin\frac{b\sin\varphi}{(1 - 2b\cos\varphi + b^2)^{1/2}}. \tag{14}$$

Under the condition of normalisation $\beta_0\beta_0^* + \beta_{\Delta p}\beta_{\Delta p}^* = 1$ from (13), we obtain also the fraction of energy contained in the zero beam:

$$|\beta_0|^2 = \frac{1}{1+\delta^2}, \tag{15}$$

where

$$\delta^2 = \frac{a^2}{(1-b)^2}\Phi(\varphi);$$

$$\Phi(\varphi) = \left[1 + \frac{4b}{(1-b)^2}\sin^2(\varphi/2)\right]^{-1}$$

is the Airy contour whose FWHM $\Delta\varphi$, as is known, depends only on the parameter b:

$$\Delta\varphi = \frac{2(1-b)}{\sqrt{b}}. \tag{16}$$

The results obtained within the two-beam model cannot claim to give an exhaustive description of the configurations in which a substantial decrease in $|\beta_0|^2$ is observed and the fundamental mode consists of a large number of LG beams (see, for example, [16]). However, this model is useful to determine the threshold values of the resonator parameters and the gain distribution for which a multibeam mode is produced. In addition, the model allows one to estimate the fraction of background beams whose existence was observed in [16].

Let the condition $|\beta_0|^2 = 0.9$ be the criterion for the formation of the multibeam mode. Then, expression (15) is simplified:

$$|\beta_0|^2 \approx 1 - \delta^2. \tag{17}$$

Let us determine, first, the sensitivity of the fundamental mode to the amplification parameters ξ and K_0 in the case of exact degeneracy ($\varphi = 0$). In this case, it is convenient to rewrite expression (17) in the form

$$|\beta_0|^2 = 1 - \left(\frac{1}{\delta_\infty} + \frac{\gamma_0 - \gamma_{\Delta p}}{\gamma_{\Delta p}}\frac{t_{00}}{t_{0\,\Delta p}}\right)^{-2}, \tag{18}$$

where $\delta_\infty^2 = t_{0\Delta p}^2/(t_{00} - t_{\Delta p\,\Delta p})^2$. Let us estimate the value of ξ for which $|\beta_0|^2 = 0.9$. In the case of mirrors of a large enough diameter, $\gamma_0 \approx \gamma_{\Delta p}$ and the second term in brackets in (18) can be neglected. Taking into account the selected criterion we derive an equation

$$\frac{t_{00} - t_{\Delta p\,\Delta p}}{t_{0\,\Delta p}} = \sqrt{\frac{1}{1-0.9}} = \sqrt{10} \tag{19}$$

with respect to ξ, the root of it being the value $\xi_{0.9}$, so that $|\beta_0|^2 = 0.9$ for $\xi = \xi_{0.9}$. It follows from (9) that $t_{0\,\Delta p} \sim K_0 - 1$, $t_{00} - 1 \sim K_0 - 1$ and $t_{\Delta p\,\Delta p} - 1 \sim K_0 - 1$. One can see here that equation (19) does not contain K_0. The numerical calculations demonstrate a weak dependence of $\xi_{0.9}$ on K_0. The parameters $\xi_{0.9}$ for a number of degenerate configurations obtained by the Fox–Li method for $N_{\rm F} = 30$ and $K_0 = 2.0$ and calculated by using (19) are presented in Table 1.

The presented values of $\xi_{0.9}$ allow one to determine how

Table 1. Parameters of $\xi_{0.9}$ obtained by the Fox–Li method and calculated by using expression (19) for different q/s.

Δp	q/s	$\xi_{0.9}$ (Fox–Li method)	$\xi_{0.9}$ [expression (19)]
2	1/4	1.26	1.26
3	1/3, 1/6	1.75, 1.75	1.73
4	1/8, 3/8	2.11, 2.12	2.09
5	1/5, 1/10, 3/10	2.42, 2.43, 2.42	2.40
6	1/12, 5/12	2.75, 2.70	2.67
7	1/7, 2/7	2.97, 2.93	2.92
8	3/16, 5/16	3.17, 3.17	3.15
9	1/9, 2/9	3.53, 3.37	3.36

large the diameter of the pump beam should be to exclude ($\xi < \xi_{0.9}$) or provide ($\xi > \xi_{0.9}$) the appearance of some critical configuration. One can see from Table 1 that $\xi_{0.9}$ increases with increasing Δp, i.e. a decrease in the diameter of the pump beam results in the appearance of critical configurations with large s.

Figure 4 presents the composition of the fundamental mode for $\xi > \xi_{0.9}$. It also shows the values of $|\beta_p|^2$ and $\arg\beta_p$ in the input plane of the active medium and intensity distributions on the opposite mirror for a degenerate resonator with $q/s = 1/4$ ($g_1 g_2 = 0.5$) for $N_F = 9$, $K_0 = 2.0$ and different ξ. When ξ is increased, the number of LG beams forming the fundamental mode increases, remaining in this case limited by a value determined by N_F. A similar effect of ξ on the composition of the fundamental mode is observed for other degenerate configurations.

In the case of weak gains and/or finite dimensions of the mirrors the second term in brackets in (18) cannot be neglected. Indeed, for

$$\frac{\gamma_0 - \gamma_{\Delta p}}{\gamma_{\Delta p}} \frac{t_{00}}{t_{0\,\Delta p}} > \frac{0.1}{\delta_\infty}$$

we can expect a decrease in δ^2 by more than 20 % due to the dependence of δ^2 on K_0 and a corresponding increase in $|\beta_0|^2$. Because for small $K_0 - 1$ the estimate $t_{00} \approx 1$ is valid and taking into account the expression $t_{0\,\Delta p} = 2(K_0 - 1)\xi^{2\Delta p}/(2+\xi)^{\Delta p+1}$ we obtain that $|\beta_0|^2 = 0.9$ for

$$(K_0 - 1)_{0.9} = \sqrt{10}\,\frac{(2+\xi)^{\Delta p+1}}{2\xi^{2\Delta p}}\,\frac{\gamma_0 - \gamma_{\Delta p}}{\gamma_{\Delta p}}. \quad (20)$$

γ_0 and $\gamma_{\Delta p}$ can be estimated as follows:

$$\gamma_{0\,(\Delta p)} = 2\pi \int_0^a \left|u^{\rm LG}_{0\,(\Delta p)}(r,\mu)\right|^2 r\,{\rm d}r. \quad (21)$$

Table 2 presents the values of $(K_0 - 1)_{0.9}$ calculated by (20) at $\xi = 3$ and different N_F for configurations with $q/s = 1/4$, 1/3, 3/8 and 3/10. One can see that in the case of exact degeneracy the effect of K_0 on the formation of the multibeam mode takes place only for strongly diaphragmed resonators (small N_F).

Table 2. Values of $(K_0 - 1)_{0.9}$ for $\xi = 3$ and different N_F calculated by using expression (20) for some q/s.

q/s	N_F			
	2	3	4	6
1/4	0.40	0.023	9.3×10^{-4}	7.0×10^{-7}
1/3	0.48	0.027	7.9×10^{-4}	2.0×10^{-7}
3/8	0.61	0.068	0.029	1.0×10^{-6}
3/10	0.43	0.29	0.055	2.6×10^{-4}

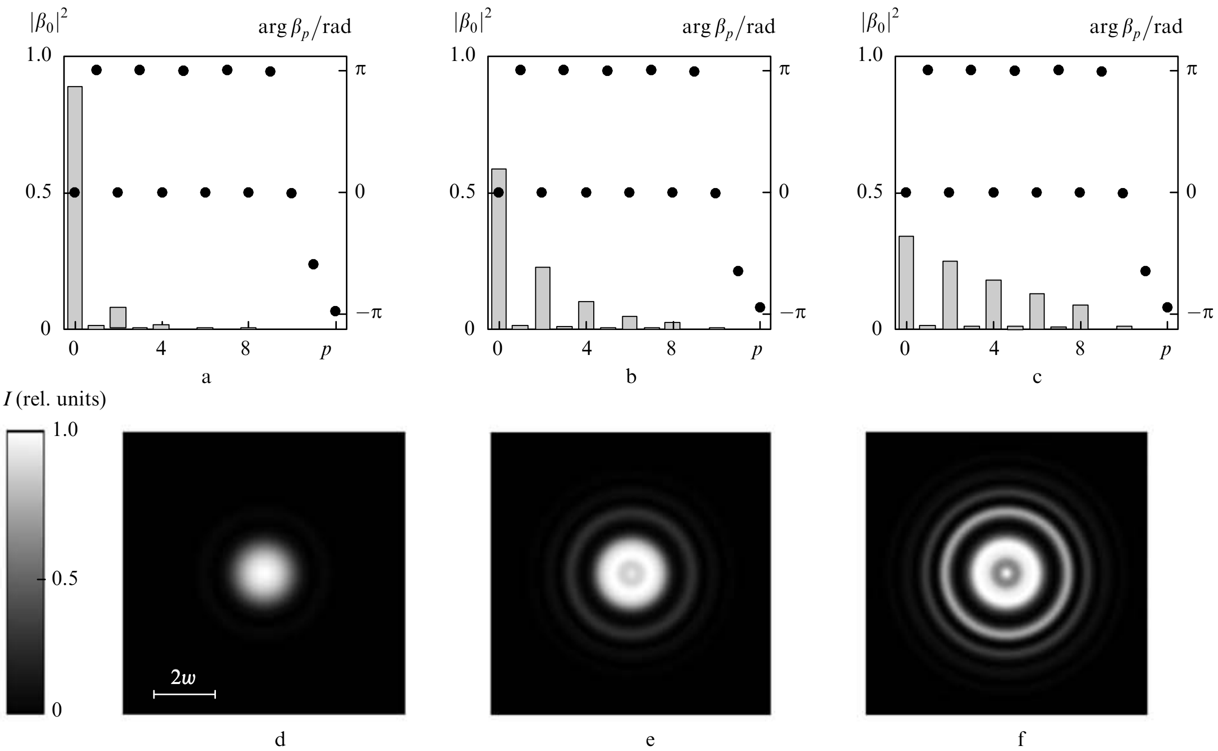

Figure 4. The values of $|\beta_p|^2$ (columns) and $\arg\beta_p$ (dots) calculated by the Fox–Li method in expansion (4) of the fundamental mode in the input plane of the active medium (a–c) and intensity distributions I on the opposite mirror (d–f) for a degenerate resonator with $q/s = 1/4$ ($g_1 g_2 = 0.5$) for $N_F = 9$, $K_0 = 2.0$, $\xi = 1.3$ (a, d), 2 (b, e), 4 (c, f).

Let us estimate the range of detunings from the degeneracy within which the multibeam mode is realised. If the degeneracy turns weak (for example, for ξ close to $\xi_{0.9}$), the FWHM $\Delta\varphi$ of the contour (16) can be treated as the resonance width. Taking into account the relation between the relative phase shift of two beams per resonator round trip (without the active medium) and detuning from the degeneracy in coordinates g_1, g_2 for critical configurations inside the stability region ($0 < g_1g_2 < 1$), we have

$$\Delta\varphi \approx \frac{\partial}{\partial(g_1g_2)}\left(\Delta p \arccos\sqrt{g_1g_2}\right)\Delta(g_1g_2)$$

$$= \frac{2\Delta p\Delta(g_1g_2)}{\{[1-(g_1g_2)_{\rm deg}](g_1g_2)_{\rm deg}\}^{1/2}}, \qquad (22)$$

where $(g_1g_2)_{\rm deg} = \cos^2(\pi q/s)$ is the value corresponding to the degeneracy. Taking into account (22), we obtain from (16) the expression for the width of the region in which the multibeam mode appears on the stability diagram:

$$\Delta(g_1g_2) = \frac{\sin(2\pi r/s)}{2\Delta p(t_{00}\gamma_0 t_{\Delta p}\gamma_{\Delta p})^{1/2}}\left(1 - \frac{\gamma_{\Delta p}}{\gamma_0}\frac{t_{\Delta p\Delta p}}{t_{00}}\right). \qquad (23)$$

Expression (23) gives a quantitative description of the dependence of the resonance width on the parameters of the resonator and nonuniform amplification mentioned in [9].

In pronounced critical configurations the two-beam model can be used only for a sufficient detuning from exact degeneracy, when the fundamental mode is mainly formed by two beams. In this case, it is expedient to determine by the level $|\beta_0|^2 = 0.9$ the width of the region where the multibeam mode appears. The expression for this width is obtained from (15) by taking (22) into account:

$$\Delta_{0.9}(g_1g_2) = \frac{\sqrt{10}\sin(2\pi q/s)}{2\Delta p}\left[\frac{t_{0\Delta p}^2}{t_{00}t_{\Delta p\Delta p}}\frac{\gamma_p}{\gamma_0}\right.$$

$$\left. - \frac{(t_{00}\gamma_0 - t_{\Delta p\Delta p}\gamma_p)^2}{10t_{00}t_{\Delta p\Delta p}\gamma_0\gamma_p}\right]^{1/2}. \qquad (24)$$

The dependences of $\Delta_{0.9}(g_1g_2)$ on K_0 for different configurations obtained by the Fox–Li method and calculated by expression (24) are presented in Fig. 5. One can see that $\Delta_{0.9}(g_1g_2)$ increases with increasing K_0.

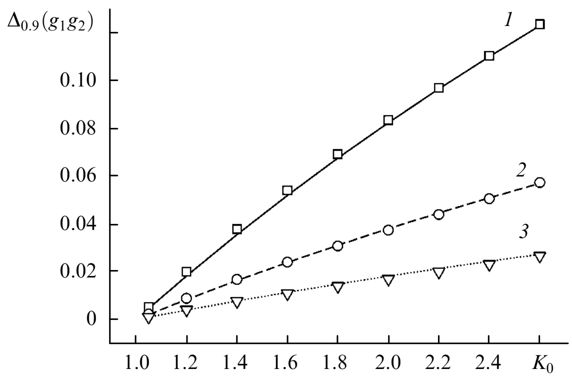

Figure 5. Dependences of $\Delta_{0.9}(g_1g_2)$ on K_0 for configurations with $q/s = 1/4$ (1), 1/3 (2), 3/8 (3) calculated by the Fox–Li method (dots) and by expression (24) (curves).

5. Conclusions

The combined influence of the parameters of a spatially nonuniform axially-symmetric intensity distribution in a four-level active medium and resonator parameters on the composition of the fundamental mode has been studied. Analytic estimates obtained within the framework of the model of two beams interacting due to the spatially nonuniform amplification agree well with the results of calculations by the Fox–Li method. Under the conditions of exact degeneracy, a decrease in the pump beam diameter leads to an increase in the number of beams forming the mode and the effect of amplification on the mode composition is noticeable only at small Fresnel numbers and/or at large enough values of K_0. A decrease in the pump beam diameter at the fixed Fresnel number results in the appearance of critical configurations of the resonator with large values of the determinator in the fraction characterising the degeneracy. On the stability diagram the region of detuning from the strong degeneracy, within which a multibeam mode is realised, expands with increasing the gain.

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