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Application of Young – Michelson and Brown – Twiss interferometers for determining geometric parameters of nonplanar rough objects

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***Abstract*. The possibility of using Young – Michelson and Brown – Twiss interferometers for measuring the angular dimensions and parameters of the surface shape of remote passively scattering and self-luminous nonplanar rough objects by optical radiation propagating from them is substantiated. The analysis is based on the properties of approximate transverse functions of field coherence B_t and B_t' and intensity coherence B_{ti} and B_{ti}' formed by the time averaging of the products of fields and intensities taken at two points of a receiving aperture (the prime denotes self-luminous objects). The averaging time is set to be much longer than the coherence time of radiation propagating from an object. It is shown that for the radiation coherence length much smaller than the depth of the visible region of the object, the functions B_t and B_t' are proportional to the Fourier transform of the intensity distribution in the image of a remote object, which is the generalisation of the Van Cittert – Zernicke theorem to the case of a nonplanar object, while functions B_{ti} and B_{ti}' are proportional to the squares of the modulus of the Fourier transform of this distribution. It is also shown that the recording of functions B_t and B_t' with a Young – Michelson interferometer gives only the angular dimensions of the visible region of objects, whereas the recording of functions B_{ti} and B_{ti}' with a Brown – Twiss interferometer allows one to find these dimensions and the radius of curvature of the object surface.**

Keywords: *coherence, Young – Michelson and Brown – Twiss interferometers, correlation properties of the fields formed by passively scattering and self-luminous objects, the Van Cittert – Zernicke theorem for nonplanar objects.*

1. Introduction

It is known that the use of Young – Michelson and Brown – Twiss interferometers to obtain information on remote rough objects is based on the analysis of the statistical characteristics of random speckles formed due to the interference of light beams coming from different sites of the object surface [1 – 5]. As a rule, this information is analysed by assuming that these objects are planar and self-luminous. As a result, the number of parameters of self-luminous objects that can be determined is limited. For example, only the angular dimensions of objects were determined, whereas the surface shape was not studied. The possibility of measuring the parameters of passively scattering objects with the help of these interferometers was not investigated as well. In this connection the necessity appears to analyse the statistical characteristics of speckle patterns in the case of passively scattering and self-luminous objects of an arbitrary, in particular, nonplanar shape. Among them are, for example, the transverse coherence function (correlation function) $B_f(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, t) = \langle E(\boldsymbol{\rho}_1, t)E^*(\boldsymbol{\rho}_2, t)\rangle_f$ of the field $E(\boldsymbol{\rho}, t)$ formed by a passively scattering rough nonplanar objects [5, 6] and the transverse coherence function $B_f'(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, t) = \langle E'(\boldsymbol{\rho}_1, t)E'^*(\boldsymbol{\rho}_2, t)\rangle_f$ of the field $E'(\boldsymbol{\rho}, t)$ formed by a self-luminous rough nonplanar object, which is often called an extended source in the literature [1 – 4]. Here, $\boldsymbol{\rho}_1$ and $\boldsymbol{\rho}_2$ are the radius vectors located in the receiving aperture plane; the prime refers to a self-luminous object and angle brackets $\langle\rangle_f$ denote the averaging over an ensemble of fields E and E'.

The transverse coherence function $B_f'(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, t)$ of the field $E'(\boldsymbol{\rho}_2, t)$ formed on average by a planar self-luminous object representing an extended planar narrowband source was analysed in papers [1 – 4]. The analysis was based on the assumption that the field $V_s'(\boldsymbol{r})$ emitted by the source is delta-correlated near the source surface (where $\boldsymbol{r}$ is the radius vector of a point on the source surface). This means that the correlation function of the emitted field is $J'(\boldsymbol{r}_1, \boldsymbol{r}_2) = \langle V_s'(\boldsymbol{r}_1)V_s'^*(\boldsymbol{r}_2)\rangle_f = \rho_\varphi' I_s'(\boldsymbol{r}_1)\delta(\boldsymbol{r}_1 - \boldsymbol{r}_2)$, where $I_s'(\boldsymbol{r}_1) = |V_s'(\boldsymbol{r})|$ is the intensity distribution on the surface and $\rho_\varphi' \approx \lambda_0$ is the correlation radius of the field V_s', which is constant over the entire surface of the object. Under this assumption, the function $B_f'(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, t)$ is proportional to the Fourier transform of the intensity distribution on the object surface $I_s'(\boldsymbol{r}) = |V_s'(\boldsymbol{r})|^2$. This is the known Van Cittert – Zernicke theorem. The function $B_f'(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, t)$ is quite often approximated by the approximate transverse coherence function $B_t'(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) = \langle E'(\boldsymbol{\rho}_1, t)E'^*(\boldsymbol{\rho}_2, t)\rangle_t$, where $\langle A(t)\rangle_t = T^{-1}\int_{t_0}^{t_0+T} A(t)\mathrm{d}t$ is the time average of the function $A(t)$; T is the processing (averaging) time of the received field $E'(\boldsymbol{\rho}, t)$ selected from the condition $T \gg \tau_c$ (τ_c is the coherence time of radiation forming the field E'); and t_0 is the initial moment of averaging [3]. It is assumed that B_t' $B_t'(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) \approx B_{fm}'(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2)$, where B_{fm}' is the correlation function of the field E' formed at one frequency ω_0. This means that $\langle B_t'(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2)\rangle_f = B_f'(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2)$ and the accuracy of this approximation is high: $\eta' = \langle|B_t' - B_f'|^2\rangle_f / |B_f'|^2 \ll 1$. However, this

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condition was not substantiated in the literature. Note also that the analysis of the function B_{t}' is based on the unsubstantiated assumption that the emission spectrum is a random delta-correlated process. This spectral approach to the analysis of the coherence function B_{t}' of the field E' formed by radiation coming from a planar extended thermal source is described in detail in [3].

In this paper, the coherence functions B_{t} and B_{t}' are analysed by using the non-spectral approach, which is based on the time correlation function $B_u(\tau) = \langle u(t)u^*(t+\tau)\rangle_{\mathrm{t}}$ introduced in papers [7, 8], where $u(t)$ is the modulation function of radiation forming the fields E and E' under the condition that the function $u(t)$ changes slower than $\exp(i\omega_0 t)$, where ω_0 is the central (carrier) radiation frequency. In this case, the averaging time proposed for the formation of the function $B_u(\tau)$ was selected according to the condition $T \gg \tau_{\mathrm{c}}$. It was assumed that $B_u(\tau) \approx B(\tau) = \langle u(t)u^*(t+\tau)\rangle_u$, where $\langle\rangle_u$ is the averaging over different realisations of the function $u(t)$. The function $B_u(\tau)$ introduced in papers [7, 8] was used to analyse the statistical characteristics of the intensity $I(\boldsymbol{\rho}, t) = |E(\boldsymbol{\rho}, t)|^2$ averaged over the time T, which considerably exceeded the coherence time $\tau_{\mathrm{c}}\bar{I}(\boldsymbol{\rho}) = B_{\mathrm{t}}(\boldsymbol{\rho}, \boldsymbol{\rho}) = T^{-1}\int_{t_0}^{t_0+T} I(\boldsymbol{\rho}, t)\mathrm{d}t$. Then, methods are considered for obtaining information on remote rough nonplanar objects with the help of Young – Michelson and Brown – Twiss interferometers, which are based on the use of functions B_{t} and B_{t}' and the non-spectral approach to their analysis. First the statistical models of the fields $E(\boldsymbol{\rho}, t)$ and $E'(\boldsymbol{\rho}, t)$ are introduced. Then, it is shown that the functions $B_{\mathrm{t}}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2)$ and $B_{\mathrm{t}}'(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2)$ for the radiation coherence length L_{c} considerably smaller than the depth L_{s} of the visible region of the object are proportional to the Fourier transform of the intensity distribution in the image of the remote object, which is the generalisation of the Van Cittert – Zernicke theorem to the case of a nonplanar object. Then, the Young – Michelson interferometer scheme is described for measuring the functions B_{t} and B_{t}' from which the angular dimensions of remote objects can be determined.

The approximate transverse coherence functions $B_{\mathrm{ti}}(\boldsymbol{\rho}_1, t, \boldsymbol{\rho}_2, t) = \langle I(\boldsymbol{\rho}_1, t)I(\boldsymbol{\rho}_2, t)\rangle_{\mathrm{t}} - \bar{I}(\boldsymbol{\rho}_1, t)\bar{I}(\boldsymbol{\rho}_2, t)$ and $B_{\mathrm{ti}}'(\boldsymbol{\rho}_1, t, \boldsymbol{\rho}_2, t) = \langle I'(\boldsymbol{\rho}_1, t)I'(\boldsymbol{\rho}_2, t)\rangle_{\mathrm{t}} - \bar{I}'(\boldsymbol{\rho}_1, t)\bar{I}'(\boldsymbol{\rho}_2, t)$ (where $I(\boldsymbol{\rho}) = |E(\boldsymbol{\rho}, t)|^2$ and $I'(\boldsymbol{\rho}) = |E'(\boldsymbol{\rho}, t)|^2$), which were not discussed earlier in the literature, are analysed in the Fresnel approximation. It is shown that for $L_{\mathrm{s}} \gg L_{\mathrm{c}} \geqslant 20\lambda_0$ ($\lambda_0 = 2\pi c/\omega_0$ is the central radiation wavelength and c is the speed of light) these functions are proportional with a high accuracy to the square of the modulus of the Fourier transform of the intensity distribution in the image of a remote nonplanar object. A variant of a Brown – Twiss interferometer is proposed for recording the functions B_{ti} and B_{ti}', which can be used for determining the angular dimensions of a remote nonplanar rough object and parameters of its surface shape. The known variants of this interferometer could be used to determine only the angular size of objects.

The accuracy of the relation $B_u(\tau) \approx B(\tau) = \langle u(t)u^*(t+\tau)\rangle_u$ is estimated in Appendix 1 by assuming that the averaging time proposed for the formation of the function $B_u(\tau)$ satisfies the condition $T \gg \tau_{\mathrm{c}}$. The accuracy of the approximation of the function B_{t} by the function B_{fm}, where B_{fm} is the correlation function of the field E formed at one frequency ω_0, is calculated in Appendix 2. It is shown that the condition $\eta_{\mathrm{t}} \approx L_{\mathrm{s}}/L_{\mathrm{c}} \leqslant 1$ is fulfilled for the depth of the visible region of objects $L_{\mathrm{s}} \gg L_{\mathrm{c}}$.

2. Statistical models of fields formed by rough passively scattering objects and self-luminous nonplanar objects

Consider the statistical models of fields formed by rough passively scattering and self-luminous nonplanar objects. The field scattered by the object probed by a source with the radiation pattern $D(\boldsymbol{r}_\Sigma)$ ($\boldsymbol{r}_\Sigma$ is the radius vector of the object surface) and the central radiation frequency ω_0 in the Kirchhoff approximation has the form [5] (Fig. 1)

$$E(\boldsymbol{\rho}, t) \approx \frac{\mathrm{i}\omega_0}{r_{\mathrm{c}}c}\int\!\!\int E_{\mathrm{s}}(\boldsymbol{r}_\Sigma)\exp\left[\mathrm{i}\omega_0\left(t - \frac{|\boldsymbol{r}_\Sigma - \boldsymbol{\rho}_{\mathrm{s}}| + |\boldsymbol{r}_\Sigma - \boldsymbol{\rho}|}{c}\right)\right] \times u\left(t - \frac{|\boldsymbol{r}_\Sigma - \boldsymbol{\rho}_{\mathrm{s}}| + |\boldsymbol{r}_\Sigma - \boldsymbol{\rho}|}{c}\right)\mathrm{d}\boldsymbol{r}_\Sigma, \quad (1)$$

where $\boldsymbol{\rho}_{\mathrm{s}}$ is the radius vector of the centre of the aperture of the probe radiation source; $u(t)$ is a modulation function; $E_{\mathrm{s}}(\boldsymbol{r}_\Sigma) = AD(\boldsymbol{r}_\Sigma)k(\boldsymbol{r}_\Sigma)$; A is the field amplitude of the probe source on the object surface; $k(\boldsymbol{r}_\Sigma)$ is the distribution of Fresnel reflection coefficients, i.e. the distribution of the reflected field along the average object surface; $r_{\mathrm{c}} = |\boldsymbol{r}_{\mathrm{c}}|$ is the distance to the object; and $\boldsymbol{r}_{\mathrm{c}}$ is the radius vector of the point nearest to the centre of the receiving aperture.

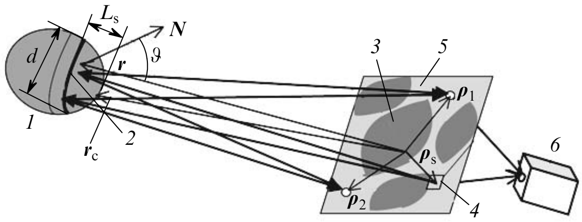

Figure 1. Measurement of approximate transverse coherence functions $B_{\mathrm{t}} = \langle E_1E_2^*\rangle_{\mathrm{t}}$ and $B_{\mathrm{t}}' = \langle E_1'E_2'^*\rangle_{\mathrm{t}}$ of the received fields $E(\boldsymbol{\rho}, t)$ and $E'(\boldsymbol{\rho}, t)$ with a Young – Michelson interferometer [$E_{\mathrm{i}} = E(\boldsymbol{\rho}_{\mathrm{i}}, t)$ и $E_{\mathrm{i}}' = E_{\mathrm{i}}'(\boldsymbol{\rho}_{\mathrm{i}}, t)$; $\boldsymbol{\rho}$ is the radius vector in the receiving aperture plane; rays propagating from the source to object, scattered by the object and transmitted through two apertures in the receiving aperture are indicated by thick arrows]: (*1*) rough object; (*2*) boundary of the visible region of the object; (*3*) speckle pattern of the received field; (*4*) source aperture; (*5*) receiving aperture; (*6*) unit for formation of functions B_{t} and B_{t}'.

A self-luminous object will be represented in the form of an extended source with a nonplanar and rough surface on which point sources are densely located. A similar model of an extended source was proposed in [9]. According to this model, the field $E'(\boldsymbol{\rho}, t)$ formed by this object can be written in the form

$$E(\boldsymbol{\rho}, t) \approx \frac{\mathrm{i}\omega_0}{r_{\mathrm{c}}c}\int\!\!\int E_{\mathrm{s}}(\boldsymbol{r}_\Sigma)\exp\left[\mathrm{i}\omega_0\left(t - \frac{|\boldsymbol{r}_\Sigma - \boldsymbol{\rho}|}{c}\right)\right] \times u\left(\boldsymbol{r}_\Sigma, t - \frac{|\boldsymbol{r}_\Sigma - \boldsymbol{\rho}|}{c}\right)\mathrm{d}\boldsymbol{r}_\Sigma, \quad (1')$$

where $E_{\mathrm{s}}'(\boldsymbol{r}_\Sigma) = Ak(\boldsymbol{r}_\Sigma)$ is the distribution of the field emitted by the self-luminous object along its mean surface; $u(\boldsymbol{r}_\Sigma, t)$ is the local function of radiation modulation on the object surface; and $k(\boldsymbol{r}_\Sigma) \leqslant 1$ is the relative distribution of radiation amplitudes on the object surface.

The radiation modulation function $u(t)$ can be written in the form $u(t) = |u(t)|\exp[\mathrm{i}\psi(t)]$, where $\psi(t)$ – $\arctan[\mathrm{Im}\,u(t)/\mathrm{Re}\,u(t)]$. According to its definition, $|u(t)| \leqslant 1$. The dependence of this function on $\boldsymbol{r}_\Sigma$ is omitted here for simplicity. Usually, the function $|u(t)|$ has few extrema during the observation time T, while the function $\psi(t)$ has many extrema. Therefore, it is appropriate to represent the function $|u(t)|$ in the form a determinate process and the function $\psi(t)$ – in the form of nondeterminate (random) process. In this case, the averaging $\langle\rangle_u$ over different realisations of the function $u(t)$ can be replaced by the averaging $\langle\rangle_\psi$ over different realisations of the function $\psi(t)$, which gives the relation $B(\tau) = \langle u(t)u^*(t+\tau)\rangle_\psi$.

By assuming that ψ has the Gaussian distribution $w_\mathrm{i}(\psi) = (2\pi)^{-1/2}\sigma_\psi^{-1}\exp[-\psi^2/(2\sigma_\psi^2)]$ with the correlation function $\langle\psi(t)\psi(t+\tau)\rangle_\psi = \sigma_\psi^2(t)\exp[-\tau^2/\tau_\psi^2(t)]$, where σ_ψ and τ_ψ are the root-mean-square deviation and correlation time of the function ψ, respectively, we have under condition $\sigma_\psi \gg 1$,

$$\left\langle B_u\left(\frac{\tau}{\tau_\mathrm{c}}\right)\right\rangle_\psi = \langle\langle u(t)u^*(t+\tau)\rangle\rangle_{\mathrm{t}\psi}$$

$$= \frac{1}{T}\int_{t_0}^{t_0+T}|u(t)u(t+\tau)|\langle\exp\{\mathrm{i}[\psi(t)-\psi(t+\tau)]\}\rangle_\psi \mathrm{d}t$$

$$= \frac{1}{T}\int_{t_0}^{t_0+T}|u(t)u(t+\tau)|\exp\left[-\frac{\langle[\psi(t)-\psi(t+\tau)]^2\rangle_\psi}{2}\right]$$

$$= B_{u\mathrm{d}}\left(\frac{\tau}{\tau_\mathrm{c}}\right)\exp\left\{-\sigma_\psi^2\left[-\exp\left(-\frac{\tau^2}{\tau_\psi^2}\right)\right]\right\}$$

$$\approx B_{u\mathrm{d}}(\tau)\exp\left(-\frac{\tau^2}{\tau_\mathrm{i}^2}\right),$$

where $\tau_\mathrm{i} = \tau_\psi/\sigma_\psi$; $B_{u\mathrm{d}}(\tau/\tau_\mathrm{d}) = \langle|u(t)u(t+\tau)|\rangle_\mathrm{t}$; and τ_d is the half-width of the function $|u(t)|$. For example, in the case of probe radiation formed by a pulsed laser with the Gaussian modulus of the modulation function $|u(t)| = \exp(-t^2/\tau_\mathrm{d}^2)$ (τ_d is the pulse duration), we have $B_{u\mathrm{d}}(\tau/\tau_\mathrm{d}) \approx \exp(-\tau^2/\tau_\mathrm{d}^2)$ and $\langle B_u(\tau/\tau_\mathrm{c})\rangle_\psi \approx \exp(-\tau^2/\tau_\mathrm{c}^2)$, where $\tau_\mathrm{c} = \tau_\mathrm{d}\tau_\mathrm{i}(\tau_\mathrm{d}^2+\tau_\mathrm{i}^2)^{-1/2}$ is the coherence time of detected radiation. In particular, for a Q-switched pulsed laser, we have $\tau_\mathrm{i} \gg \tau_\mathrm{d}$ and $\tau_\mathrm{c} \approx \tau_\mathrm{d}$, i.e. the coherence time τ_c coincides with the pulse duration. Then, with the high accuracy, $u(t) \approx |u(t)| = \exp(-t^2/\tau_\mathrm{d}^2)$, i.e. $u(t)$ is a determinate function. In the opposite case, when $\tau_\mathrm{d} \gg \tau_\mathrm{i}$, we have $\langle B_u(\tau/\tau_\mathrm{c})\rangle_\psi \approx \exp(-\tau^2/\tau_\mathrm{i}^2)$ and $\tau_\mathrm{c} \approx \tau_\mathrm{i}$. which takes place, for example, for a cw laser. The accuracy of the approximation of the function $B(\tau)$ by the function B_u was estimated in Appendix 1 by using the results presented above under the condition that $T \gg \tau_\mathrm{c}$:

$$B_u\left(\frac{\tau}{\tau_\mathrm{c}}\right) = \langle u(t)u^*(t+\tau)\rangle_\mathrm{t} \approx B(\tau)$$

$$= \langle u(t)u^*(t+\tau)\rangle_\psi \approx \exp\left(-\frac{\tau^2}{\tau_\mathrm{i}^2}\right). \qquad (2)$$

Note here that in the case of a self-luminous object, the coherence time τ_c and, therefore, the coherence length $L_\mathrm{c} = c\tau_\mathrm{c}$ are different for different sites of its surface, so that $\tau_\mathrm{c} = \tau_\mathrm{c}(\boldsymbol{r}_\Sigma)$ and $L_\mathrm{c} = L_\mathrm{c}(\boldsymbol{r}_\Sigma)$.

Let us now analyse the fields formed by rough passively scattering and self-luminous nonplanar objects. For this purpose, we first introduce the roughness height $\xi(\boldsymbol{r})$ as the deviation of the real surface of the object under study on its mean surface along the normal to the mean surface at the point with the radius vector $\boldsymbol{r} = \boldsymbol{r}(u,v)$, where u and v are the surface orthogonal coordinates on the mean surface. Then, $\boldsymbol{r}_\Sigma \approx \boldsymbol{r} + \boldsymbol{N}(\boldsymbol{r})\xi(\boldsymbol{r})$, where $\boldsymbol{N}(\boldsymbol{r})$ is the normal to the mean surface. If $\xi(\boldsymbol{r}) \ll r_\mathrm{c}$, we have

$$E(\boldsymbol{\rho},t,\xi) \sim \int V_\mathrm{s}(\boldsymbol{r})F(\boldsymbol{r},\boldsymbol{\rho}_\mathrm{s},\boldsymbol{\rho},t)\mathrm{d}\boldsymbol{r},$$
$$E'(\boldsymbol{\rho},t,\xi) \sim \int V'_\mathrm{s}(\boldsymbol{r})F'(\boldsymbol{r},\boldsymbol{\rho}_\mathrm{s},\boldsymbol{\rho},t)\mathrm{d}\boldsymbol{r}, \qquad (3)$$

where

$$F(\boldsymbol{r},\boldsymbol{\rho}_\mathrm{s},\boldsymbol{\rho},t) = \exp\left(-\mathrm{i}\omega_0\frac{|\boldsymbol{r}-\boldsymbol{\rho}_\mathrm{s}|+|\boldsymbol{r}-\boldsymbol{\rho}|}{c}\right)$$
$$\times u\left[t - \frac{|\boldsymbol{r}-\boldsymbol{\rho}_\mathrm{s}|+|\boldsymbol{r}-\boldsymbol{\rho}|+2\cos\vartheta(\boldsymbol{r})\xi(\boldsymbol{r})}{c}\right],$$
$$F'(\boldsymbol{r},\boldsymbol{\rho}_\mathrm{s},\boldsymbol{\rho},t) = \exp\left(-\mathrm{i}\omega_0\frac{|\boldsymbol{r}-\boldsymbol{\rho}|}{c}\right)$$
$$\times u\left[t - \frac{|\boldsymbol{r}-\boldsymbol{\rho}|+\cos\vartheta(\boldsymbol{r})\xi(\boldsymbol{r})}{c}\right] \qquad (4)$$

are spherical waves propagating from the object surface;

$$V_\mathrm{s}(\boldsymbol{r}) = E_\mathrm{s}(\boldsymbol{r})\exp[2\mathrm{i}\Psi_\mathrm{r}(\boldsymbol{r})], \quad V'_\mathrm{s}(\boldsymbol{r}) = E'_\mathrm{s}(\boldsymbol{r})\exp[\mathrm{i}\Psi_\mathrm{r}(\boldsymbol{r})] \qquad (5)$$

is the field distribution on the object surface; $\Psi_\mathrm{r}(\boldsymbol{r}) = 2\pi\mathrm{i}\omega_0\cos\vartheta(\boldsymbol{r})\xi(\boldsymbol{r})/\lambda_0$ is the phase incursion of radiation propagating within the surface roughness; and $\vartheta(\boldsymbol{r})$ is the angle between the normal $\boldsymbol{N}(\boldsymbol{r})$ and direction to the object. In the case of a rough object, the function $\xi(\boldsymbol{r})$ has a random distribution with the dispersion $\sigma^2(\boldsymbol{r}) = \int\xi^2 w_1(\xi)\mathrm{d}\xi$, the correlation function $B_{12}(\boldsymbol{r}_1,\boldsymbol{r}_2) = \int\int\xi_1\xi_2 w_{12}(\xi_1,\xi_2)\mathrm{d}\xi_1\mathrm{d}\xi_2$, and the correlation radius $l(\boldsymbol{r}) = \{\int[B_{12}(\boldsymbol{r},\boldsymbol{r}_1)\times[\sigma^2(\boldsymbol{r}_1)]^{-1}]\mathrm{d}\boldsymbol{r}_1\}^{-1/2}$, where $w_1(\xi)$ and $w_{12}(\xi_1,\xi_2)$ are the one- and two-dimensional probability densities of the roughness height distribution [5]. Knowing the function $w_{12}(\xi_1,\xi_2)$, the averaging $\langle\rangle_\mathrm{f}$ over the ensemble of analysed fields in the calculation of the coherence functions B_f and B'_f can be replaced by averaging $\langle\rangle_\mathrm{r}$ over different realisations of roughness heights:

$$B_\mathrm{f}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,t) = \langle E(\boldsymbol{\rho}_1,t_1)E^*(\boldsymbol{\rho}_2,t_2)\rangle_\mathrm{r}$$
$$= \int\int E(\boldsymbol{\rho}_1,t_1,\xi_1)E^*(\boldsymbol{\rho}_2,t_2,\xi_2)w_{12}(\xi_1,\xi_2)\mathrm{d}\xi_1\mathrm{d}\xi_2$$
$$\sim \int\int\langle V_\mathrm{s}(\boldsymbol{r}_1)V_\mathrm{s}^*(\boldsymbol{r}_2)F(\boldsymbol{r}_1,\boldsymbol{\rho}_\mathrm{s},\boldsymbol{\rho}_1,t_1)$$
$$\times F^*(\boldsymbol{r}_2,\boldsymbol{\rho}_\mathrm{s},\boldsymbol{\rho}_2,t_2)\rangle_\mathrm{r}\mathrm{d}\boldsymbol{r}_1\mathrm{d}\boldsymbol{r}_2. \qquad (6)$$

Similar relations can be obtained for B'_f.

3. Approximate transverse functions of field and intensity coherence formed by passively scattering and self-luminous objects

In practice, the functions $B_{\rm f}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,t)$ and $B'_{\rm f}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,t)$ are often replaced by approximate transverse coherence functions of the fields E and E' formed by remote passively scattering and self-luminous objects $B_{\rm t}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)=\langle E(\boldsymbol{\rho}_1,t)\times E^*(\boldsymbol{\rho}_2,t)\rangle_{\rm t}$ and $B'_{\rm t}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)=\langle E'(\boldsymbol{\rho}_1,t)E'^*(\boldsymbol{\rho}_2,t)\rangle_{\rm t}$. Let us calculate the mean value $\langle B_{\rm t}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)\rangle_{\rm r}$ of the function $B_{\rm t}$. For $L_{\rm c}\gg\sigma\gg\lambda_0$, assuming that the probability density w_1 is Gaussian, $w_1(\xi)=(2\pi)^{-1/2}\sigma^{-1}\exp(-\xi^2/2\sigma^2)$ [5], and taking into account relations (2)–(6), we obtain

$$\langle B_{\rm t}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)\rangle_{\rm r}\sim\int\!\!\int\langle V_{\rm s}(\boldsymbol{r}_1)V_{\rm s}^*(\boldsymbol{r}_2)\rangle_{\rm r}\times\exp\left[{\rm i}\omega_0\,\frac{\Psi(\boldsymbol{r}_2,\boldsymbol{\rho}_2)-\Psi(\boldsymbol{r}_1,\boldsymbol{\rho}_1)}{c}\right]\times B_u\left[\frac{\Psi(\boldsymbol{r}_2,\boldsymbol{\rho}_2)-\Psi(\boldsymbol{r}_1,\boldsymbol{\rho}_1)}{L_{\rm c}}\right]{\rm d}\boldsymbol{r}_1{\rm d}\boldsymbol{r}_2,$$

where

$$\langle V_{\rm s}(\boldsymbol{r}_1)V_{\rm s}^*(\boldsymbol{r}_2)\rangle_r=E_{\rm s}(\boldsymbol{r}_1)E_{\rm s}^*(\boldsymbol{r}_2)\langle\Phi(\xi_1-\xi_2)\rangle_{\rm r}\approx|A|^2|k(\boldsymbol{r}_1)|^2\exp\left[-\frac{(\boldsymbol{r}_1-\boldsymbol{r}_2)^2}{\rho_\varphi^2(\boldsymbol{r}_1)}\right];\qquad(7)$$

$\xi_1=\xi(\boldsymbol{r}_1)$ and $\xi_2=\xi(\boldsymbol{r}_2)$; $\Phi(\xi_1-\xi_2)=\exp[4\pi{\rm i}(\xi_1-\xi_2)\times\cos\vartheta(\boldsymbol{r})/\lambda_0]$; $\Psi(\boldsymbol{r},\boldsymbol{\rho})=|\boldsymbol{r}-\boldsymbol{\rho}_{\rm s}|+|\boldsymbol{r}-\boldsymbol{\rho}\boldsymbol{\alpha}\boldsymbol{\xi}\boldsymbol{\delta}|$; $\rho_\varphi(\boldsymbol{r})\approx\lambda_0 l(\boldsymbol{r})\times[2\cos\vartheta(\boldsymbol{r})\sigma(\boldsymbol{r})]^{-1}$ is the correlation radius of the function $V_{\rm s}(\boldsymbol{r})$. Then, we can show that for $\rho_\varphi(\boldsymbol{r})\ll\Lambda(\boldsymbol{r})$, where $\Lambda(\boldsymbol{r})=\{|k(u,v)/[\partial^2k(u,v)/\partial^2u+k(u,v)]/[\partial^2k(u,v)/\partial^2v]\}^{-1/2}$,

$$\langle B_{\rm t}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)\rangle_{\rm r}\sim\int I_{\rm is}(\boldsymbol{r})\exp\left[2\pi{\rm i}\,\frac{\boldsymbol{r}(\boldsymbol{\rho}_1-\boldsymbol{\rho}_2)}{\lambda_0 r_{\rm c}}\right]\times B_u\left[\frac{\boldsymbol{r}(\boldsymbol{\rho}_1-\boldsymbol{\rho}_2)}{L_{\rm c}r_{\rm c}}\right]{\rm d}\boldsymbol{r}.\qquad(8)$$

Here, $I_{\rm is}(\boldsymbol{r})=|AD(\boldsymbol{r}_{\rm c})|^2k_{\rm i}(\boldsymbol{r})$ is the intensity distribution of the image of a passively scattering object at the scale $\mu\approx r_{\rm c}/f$; f is the focal distance of an optical system forming this image; and $k_{\rm i}(\boldsymbol{r})\approx|\rho_\varphi(\boldsymbol{r})k(\boldsymbol{r})/\lambda_0|^2\exp\{-[\tan\vartheta(\boldsymbol{r})l]^2\times\sigma^{-2}]\}$. The quantity $\Lambda(\boldsymbol{r})$ can be treated as the size of a local detail of the function $k(\boldsymbol{r})$ [5].

Similarly, in the case of a self-luminous object, taking into account the relation

$$\langle V'_{\rm s}(\boldsymbol{r}_1)V'^*_{\rm s}(\boldsymbol{r}_2)\rangle_{\rm r}\approx|A'|^2|k(\boldsymbol{r}_1)|^2\exp\left[-\frac{(\boldsymbol{r}_1-\boldsymbol{r}_2)^2}{\rho'^2_\varphi(\boldsymbol{r}_1)}\right],\qquad(7')$$

where $\rho'_\varphi(\boldsymbol{r})=2\rho_\varphi(\boldsymbol{r})$ is the correlation radius of the function $V'(\boldsymbol{r})$, we obtain

$$\langle B'_{\rm t}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)\rangle_{\rm r}\sim\int I'_{\rm is}(\boldsymbol{r})\exp\left[2\pi{\rm i}\,\frac{\boldsymbol{r}(\boldsymbol{\rho}_1-\boldsymbol{\rho}_2)}{\lambda_0 r_{\rm c}}\right]\times B_u\left[\frac{\boldsymbol{r}(\boldsymbol{\rho}_1-\boldsymbol{\rho}_2)}{L_{\rm c}(\boldsymbol{r})r_{\rm c}}\right]{\rm d}\boldsymbol{r}.\qquad(8')$$

Here, $I'_{\rm is}(\boldsymbol{r})=|A'|^2k'_{\rm i}(\boldsymbol{r})$ is the intensity distribution in the image of a self-luminous object at the scale $\mu\approx r_{\rm c}/f$; $k'_{\rm i}(\boldsymbol{r})\approx|\rho'_\varphi(\boldsymbol{r})k(\boldsymbol{r})/\lambda_0|^2\exp\{-[2\tan\vartheta(\boldsymbol{r})l]^2/\sigma^2]\}$; and $L_{\rm c}(\boldsymbol{r})=c\tau_{\rm c}(\boldsymbol{r})$ is the local coherence length of radiation from the surface of the self-luminous object. For example, in the case of the Lorentzian correlation function [3], when $B_u(\tau,\boldsymbol{r})=\exp[-|\tau|/\tau_{\rm c}(\boldsymbol{r})]$, we have

$$\langle B'_{\rm t}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)\rangle_{\rm r}\sim\int I'_{\rm is}(\boldsymbol{r})\exp\left[2\pi{\rm i}\,\frac{\boldsymbol{r}(\boldsymbol{\rho}_1-\boldsymbol{\rho}_2)}{\lambda_0 r_{\rm c}}\right]\times\exp\left[-\left|\frac{\boldsymbol{r}(\boldsymbol{\rho}_1-\boldsymbol{\rho}_2)}{L_{\rm c}(\boldsymbol{r})r_{\rm c}}\right|\right]{\rm d}\boldsymbol{r}.$$

Note that the relation $\rho'_\varphi(\boldsymbol{r})=2\rho_\varphi(\boldsymbol{r})$ takes place because in the case of a passively scattering object, radiation passes through surface irregularities twice, while in the case a self-luminous object – once. Therefore, the phase Ψ' of the function $V'_{\rm s}$ along the object surface changes two times slower than the phase Ψ of the function $V_{\rm s}$ and, hence, the correlation radius of the function $V'_{\rm s}$ noticeably exceeds that of the function $V_{\rm s}$.

For $L_{\rm c}\gg L_{\rm s}$, radiation forming the fields $E(\boldsymbol{\rho},t)$ and $E'(\boldsymbol{\rho},t)$ is monochromatic [7]. In this case, $u(t)\approx 1$ and $\langle B_{\rm t}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)\rangle_{\rm r}=B_{\rm fm}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)$, $\langle B'_{\rm t}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)\rangle_{\rm r}=B'_{\rm fm}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)$, where $B_{\rm fm}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)\sim\int I_{\rm is}(\boldsymbol{r})\exp[2\pi{\rm i}\boldsymbol{r}(\boldsymbol{\rho}_1-\boldsymbol{\rho}_2)/(\lambda_0 r_{\rm c}]{\rm d}\boldsymbol{r}$ and $B'_{\rm fm}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)\sim\int I'_{\rm is}(\boldsymbol{r})\exp[2\pi{\rm i}\boldsymbol{r}(\boldsymbol{\rho}_1-\boldsymbol{\rho}_2)/(\lambda_0 r_{\rm c})]{\rm d}\boldsymbol{r}$ are the Fourier transforms of the intensity distribution in the image of a nonplanar object. The two last relations are the generalisation of the Van Cittert–Zernicke theorem to the case of a nonplanar object and monochromatic radiation propagating from the object. Taking into account relations (8) and (8′) and the fact that $L_{\rm c}\geqslant 20\lambda_0$ in practice, the functions $B_{\rm t}$ and $B'_{\rm t}$ are recorded in the region $|\boldsymbol{\rho}_1-\boldsymbol{\rho}_2|\sim\lambda_0 r_{\rm c}/d$, where d is the size of the visible region of the object, and it is possible to show that we have $\langle B_{\rm t}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)\rangle_{\rm r}=B_{\rm f}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)$ and $\langle B'_{\rm t}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)\rangle_{\rm r}=B'_{\rm f}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)$ for $L_{\rm s}\gg L_{\rm c}$. Under the condition $L_{\rm s}\gg L_{\rm c}\geqslant 20\lambda_0$ (see Appendix 2), we have $\eta=\langle|B_{\rm t}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)-B_{\rm f}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)|^2\rangle_{\rm r}/|B_{\rm f}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)|^2\ll 1$ and $\eta'=\langle|B'_{\rm t}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)-B'_{\rm f}(\boldsymbol{\rho}_1,\ \boldsymbol{\rho}_2)|^2\rangle_{\rm r}/|B'_{\rm f}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)|^2\ll 1$. It follows from this that

$$B_{\rm t}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)\approx B_{\rm f}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)\sim\int I_{\rm is}(\boldsymbol{r})\exp\left[2\pi{\rm i}\,\frac{\boldsymbol{r}(\boldsymbol{\rho}_1-\boldsymbol{\rho}_2)}{\lambda_0 r_{\rm c}}\right]{\rm d}\boldsymbol{r},\qquad(9)$$

$$B'_{\rm t}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)\approx B'_{\rm f}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)\sim\int I'_{\rm is}(\boldsymbol{r})\exp\left[2\pi{\rm i}\,\frac{\boldsymbol{r}(\boldsymbol{\rho}_1-\boldsymbol{\rho}_2)}{\lambda_0 r_{\rm c}}\right]{\rm d}\boldsymbol{r}.\qquad(9')$$

This means that for $L_{\rm s}\gg L_{\rm c}\geqslant 20\lambda_0$, not only mean values, but also individual realisations of the functions $B_{\rm t}$ and $B'_{\rm t}$ are proportional to the Fourier transform of the intensity distribution in the image of a nonplanar object. This fact is the generalisation of the Van Cittert–Zernicke theorem to the case of a nonpolar object and quasi-monochromatic or polychromatic radiation propagating from the object, for which $L_{\rm s}\gg L_{\rm c}$ [7]. In particular, for $L_{\rm s}\gg L_{\rm c}\geqslant 20\lambda_0$, we have $B_{\rm t}(\boldsymbol{\rho},\boldsymbol{\rho})=\bar I(\boldsymbol{\rho})\approx B_{\rm fm}(\boldsymbol{\rho},\boldsymbol{\rho})\sim\int I_{\rm is}(\boldsymbol{r}){\rm d}\boldsymbol{r}$ and $B'_{\rm t}(\boldsymbol{\rho},\boldsymbol{\rho})=\bar I'(\boldsymbol{\rho})\approx B'_{\rm fm}(\boldsymbol{\rho},\boldsymbol{\rho})\sim\int I'_{\rm is}(\boldsymbol{r}){\rm d}\boldsymbol{r}$. In the opposite case, when $L_{\rm c}\gg L_{\rm s}$, approximate equalities (9) and (9′) are not fulfilled due to strong fluctuations of the fields E and E' and, hence, functions $B_{\rm t}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)$ and $\bar I(\boldsymbol{\rho})$.

In the case of an average planar object, when $\sin\vartheta(\boldsymbol{r})=0$, for $L_{\rm s}\gg L_{\rm c}\geqslant 20\lambda_0$, we have

$$B_{\rm t}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)\sim\int\rho_\varphi^2(\boldsymbol{r})I_{\rm s}(\boldsymbol{r})\exp\left[2\pi{\rm i}\,\frac{\boldsymbol{r}(\boldsymbol{\rho}_1-\boldsymbol{\rho}_2)}{\lambda_0 r_{\rm c}}\right]{\rm d}\boldsymbol{r},\qquad(10)$$

$$B_t'(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) \sim \int \rho_\varphi'^2(\boldsymbol{r}) I_s'(\boldsymbol{r}) \exp\left[2\pi i \frac{\boldsymbol{r}(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2)}{\lambda_0 r_c}\right] d\boldsymbol{r}, \quad (10')$$

where $I_s(\boldsymbol{r}) = |E_s(\boldsymbol{r})|^2 = |V_s(\boldsymbol{r})|^2$ and $I_s'(\boldsymbol{r}) = |E_s'(\boldsymbol{r})|^2 = |V_s'(\boldsymbol{r})|^2$ are the intensity distributions of the fields $V_s(\boldsymbol{r})$ and $V_s'(\boldsymbol{r})$ on the object surface. In the case of sharp irregularities, when $l(\boldsymbol{r}) \approx \sigma(\boldsymbol{r})$, we have $B_t(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) \sim \int I_s(\boldsymbol{r}) \exp[2\pi i \boldsymbol{r}(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2)/(\lambda_0 r_c)] d\boldsymbol{r}$ and $B_t'(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) \sim \int I_s'(\boldsymbol{r}) \times \exp[2\pi i \boldsymbol{r}(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2)/(\lambda_0 r_c)] d\boldsymbol{r}$. The last relation virtually coincides with expression (5.5.8) from [1] for the coherence function of the field formed by a self-luminous planar object. Thus, the function B_t' is proportional to the Fourier transform of the intensity distribution $I_s'(\boldsymbol{r})$ on the planar object surface only in the case of sharp irregularities of the object surface, which is the known Van Cittert–Zernicke theorem. And only in this case, as in paper [1], the correlation radius ρ_φ' of the function V_s' is constant ($\rho_\varphi' \approx \lambda_0$) over the entire object surface.

The moduli of the approximate transverse coherence functions B_t and B_t' can be determined with the help of a Young–Michelson interferometer [1–4] by measuring the visibility $V_s(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2)$ of the interference pattern near the receiving aperture axis, which was formed by beams propagated through two small holes in the aperture (Fig. 1). They are calculated from the expressions $B_t(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) = V_s(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2)[\bar{I}(\boldsymbol{\rho}_1)\bar{I}(\boldsymbol{\rho}_2)]^{1/2}$ and $B_t'(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) = V_s'(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2)[\bar{I}'(\boldsymbol{\rho}_1) \times \bar{I}'(\boldsymbol{\rho}_2)]^{1/2}$. By using these expressions and relations (9) and (9′), we can determine the angular size α of the visible region of a remote object [1–4] and also in principle the intensity distributions $I_{is}(\boldsymbol{r})$ and $I_{is}'(\boldsymbol{r})$ in the image. It can be shown that the relative accuracy of measuring α is $\eta_\alpha' \approx L_c/L_s$ for a self-luminous object and $\eta_\alpha \approx L_c/2L_s$ for a passively scattering object. The angular size α can be measured quite accurately when the coherence length satisfies the inequality $L_c \leqslant 0.1 L_s$.

Consider now the properties of approximate transverse intensity coherence functions $B_{ti}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, t) = \langle I_1 I_2 \rangle_t - \bar{I}_1 \bar{I}_2$ and $B_{ti}'(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, t) = \langle I_1' I_2' \rangle_t - \bar{I}_1' \bar{I}_2'$, where $I_j = |E(\boldsymbol{\rho}_j, t)|^2$, $I_j' = |E'(\boldsymbol{\rho}_j, t)|^2$, $\bar{I}_j(\boldsymbol{\rho}) = B_t(\boldsymbol{\rho}_j, \boldsymbol{\rho}_j)$, and $\bar{I}_j'(\boldsymbol{\rho}) = B_t'(\boldsymbol{\rho}_j, \boldsymbol{\rho}_j)$. To determine the parameters of remote nonplanar objects, these functions can be recorded with a Brown–Twiss interferometer in the variant presented in Fig. 2. Although this interferometer is most often used in practice to obtain information on a remote self-luminous object, in particular, in the stellar interferometry, Fig. 2 presents as an example the field $E(\boldsymbol{\rho}, t)$ formed by a passively scattering object. It was assumed that if, for example, for $T > 10\tau_c$ the function $u(t)$ is random, then $B_u(\tau) \approx B(\tau) = \langle u(t) u^*(t + \tau) \rangle_u$. Taking into account that the fields $E(\boldsymbol{\rho}, t)$ and $E'(\boldsymbol{\rho}, t)$ under real conditions have random Gaussian distributions and by using the results obtained in Appendices 1 and 2, we can show that in the Fresnel approximation under the condition $L_s \gg L_c \geqslant 20\lambda_0$, which is usually fulfilled in practice, the relations

$$B_{ti}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) = \left| \int I_{is}(\boldsymbol{r}) \exp\left[2\pi i \frac{\boldsymbol{r}(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2)}{\lambda_0 r_c}\right] d\boldsymbol{r} \right|^2 + F_{12}, \quad (11)$$

$$B_{ti}'(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) = \left| \int I_{is}'(\boldsymbol{r}) \exp\left[2\pi i \frac{\boldsymbol{r}(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2)}{\lambda_0 r_c}\right] d\boldsymbol{r} \right|^2 + F_{12}' \quad (11')$$

take place with an accuracy to insignificant factors, where

$$F_{12}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) = \int\int I_{is}(\boldsymbol{r}_1) I_{is}(\boldsymbol{r}_2) \left| B_u\left(\frac{2(r_2 - r_1)}{c}\right) \right|^2 \times \exp\left[2\pi i \frac{(\boldsymbol{r}_1 - \boldsymbol{r}_2)(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2)}{\lambda_0 r_c}\right] d\boldsymbol{r};$$

$$F_{12}'(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) = \int\int I_{is}'(\boldsymbol{r}_1) I_{is}'(\boldsymbol{r}_2) \left| B_u\left(\frac{r_2 - r_1}{c}\right) \right|^2 \times \exp\left[2\pi i \frac{(\boldsymbol{r}_1 - \boldsymbol{r}_2)(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2)}{\lambda_0 r_c}\right] d\boldsymbol{r}.$$

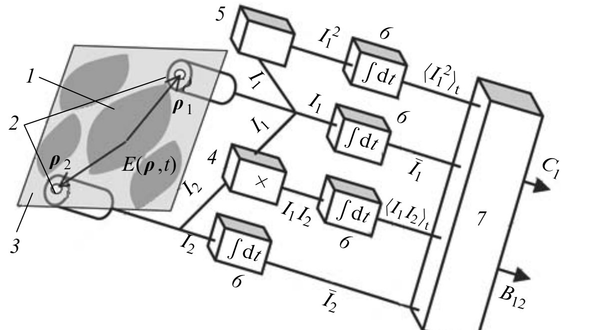

Figure 2. Functional scheme of a Brown–Twiss interferometer [$I_i = I(\boldsymbol{\rho}_i, t) = |E(\boldsymbol{\rho}_i, t)|^2$, $C_1 = \langle I_1^2 \rangle_t / \bar{I}_1^2 - 2$, $B_{12} = B_{ti}/(\bar{I}_1 \bar{I}_2) = [\langle I_1 I_2 \rangle_t / (\bar{I}_1 \bar{I}_2)] - 1$ is the normalised approximate coherence function of the intensity distribution $I(\boldsymbol{\rho}, t)$]: (*1*) speckle pattern of the processed field $E(\boldsymbol{\rho}, t)$; (*2*) photomultipliers; (*3*) receiving aperture; (*4*) multiplicators; (*5*) quadrature unit; (*6*) integrators; (*7*) unit for formation of the function B_{12} and parameter C_1.

Taking also into account that $F_{12}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) \leqslant F_{12}(\boldsymbol{\rho}, \boldsymbol{\rho}) = \int\int I_{is}(\boldsymbol{r}_1) I_{is}(\boldsymbol{r}_2) |B_u(2(r_2 - r_1)/c)|^2 d\boldsymbol{r}$ and $F_{12}'(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) \leqslant F_{12}'(\boldsymbol{\rho}, \boldsymbol{\rho}) \int\int I_{is}'(\boldsymbol{r}_1) I_{is}'(\boldsymbol{r}_2) |B_u((r_2 - r_1)/c)|^2 d\boldsymbol{r}$ and by using expression (A2.4) under the condition $L_s \gg L_c$, at which the fields $E(\boldsymbol{\rho}, t)$ and $E'(\boldsymbol{\rho}, t)$ are formed either in quasi-monochromatic or polychromatic radiation [7], we can show that

$$F_{12}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) \leqslant F_{12}(\boldsymbol{\rho}, \boldsymbol{\rho}) \approx C_s L_c \int I_{is}^2(\boldsymbol{r}) d\boldsymbol{r},$$
$$F_{12}'(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) \leqslant F_{12}'(\boldsymbol{\rho}, \boldsymbol{\rho}) \approx C_s' L_c \int I_{is}'^2(\boldsymbol{r}) d\boldsymbol{r}, \quad (12)$$

where $C_s \sim 1$ and $C_s' \sim 1$ are constants and $F_{12}/B_{ti} \ll 1$ and $F_{12}'/B_{ti}' \ll 1$. Under this condition, the equalities $B_{ti}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) = |B_t(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2)|^2$ and $B_{ti}'(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) = |B_t'(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2)|^2$ are fulfilled, i.e. the functions $B_{ti}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) = |B_t(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2)|^2$ and $B_{ti}'(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) = |B_t'(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2)|^2$ are proportional to the squares of the modulus of Fourier transforms of the intensity distributions $I_s(\boldsymbol{r})$ and $I_s'(\boldsymbol{r})$ in the images of nonplanar passively scattering and self-luminous objects, respectively. This statement is the generalisation of the known fact to the case of a few objects [1–4] that the intensity coherence function on the receiving aperture is proportional to the square of the modulus of Fourier transforms of intensity distributions $I_s(\boldsymbol{r})$ and $I_s'(\boldsymbol{r})$ on the object surface. It should be emphasised here that this is valid in fact only for a planar object with sharp surface irregularities when $l(\boldsymbol{r}) \approx \sigma(\boldsymbol{r})$. Only in this case, we have

$$B_{\rm ti}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)\approx\left|\int I_{\rm s}(\boldsymbol{r})\exp\left[2\pi{\rm i}\,\frac{\boldsymbol{r}(\boldsymbol{\rho}_1-\boldsymbol{\rho}_2)}{\lambda_0 r_{\rm c}}\right]{\rm d}\boldsymbol{r}\right|^2,$$

$$B'_{\rm ti}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)\approx\left|\int I'_{\rm s}(\boldsymbol{r})\exp\left[2\pi{\rm i}\,\frac{\boldsymbol{r}(\boldsymbol{\rho}_1-\boldsymbol{\rho}_2)}{\lambda_0 r_{\rm c}}\right]{\rm d}\boldsymbol{r}\right|^2.$$

For $L_{\rm s}\gg L_{\rm c}$, relations (11), (11′), and (12) can be used for determining the angular size α of the visible region of a remote object, for example, from the half-widths of functions $B_{\rm ti}$ and $B'_{\rm ti}$ [1–4]. These functions can be also used for determining the radius of curvature ρ_0 of objects. For example, in the case of a passively scattering object, it follows from relations (10)–(12) and (A2.4) that $\rho_0\approx(C_{\rm s}\alpha^2 r_{\rm c}^2)/(C_{\rm t}L_{\rm c})$, where $C_{\rm t}(\boldsymbol{\rho})=\langle I^2(\boldsymbol{\rho})\rangle_{\rm t}/\bar I^2(\boldsymbol{\rho})-2$. It can be shown that the values of α and ρ_0 can be estimated quite accurately by probing objects by radiation with $L_{\rm c}\leqslant 0.1L_{\rm s}$.

4. Conclusions

(i) The non-spectral approach based on the use of the time correlation function $B_u(\tau)=\langle u(t)u^*(t+\tau)\rangle_{\rm t}$ has been used to substantiate the possibility of employing Young–Michelson and Brown–Twiss interferometers for measuring angular dimensions and surface shape parameters of passively scattering and self-luminous nonplanar rough objects by optical radiation propagating from them. This possibility is realised with the help of approximate coherence of the field coherence $B_{\rm t}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)=\langle E(\boldsymbol{\rho}_1,t)\times E^*(\boldsymbol{\rho}_2,t)\rangle_{\rm t}$ and $B'_{\rm t}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)=\langle E'(\boldsymbol{\rho}_1,t)E'^*(\boldsymbol{\rho}_2,t)\rangle_{\rm t}$ and intensity $B_{\rm ti}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)=\langle I_1I_2\rangle_{\rm t}-\bar I_1\bar I_2$ and $B'_{\rm ti}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)=\langle I'_1I'_2\rangle_{\rm t}-\bar I'_1\bar I'_2$, where $\bar I_j=T^{-1}\int_{t_0}^{t_0+T}I_j(\boldsymbol{\rho},t){\rm d}t$ and $\bar I'_j=T^{-1}\times\int_{t_0}^{t_0+T}I'_j(\boldsymbol{\rho},t){\rm d}t$, $I_j=|E(\boldsymbol{\rho}_j,t)|^2$, $I'_j=|E'(\boldsymbol{\rho}_j,t)|^2$.

(ii) If radiation is quasi-monochromatic or polychromatic, which is the case for $L_{\rm s}\gg L_{\rm c}$, then under the addition condition $L_{\rm c}\geqslant 20\lambda_0$, the functions $B_{\rm t}$ and $B'_{\rm t}$ are proportional to the Fourier transform of the intensity distribution in the object image. This fact is the generalisation of the known Van Cittert–Zernicke theorem to the case of nonplanar objects: the coherence function of the detected fields E and E' is proportional to the Fourier transform of the intensity distribution on the object surface. In fact this takes place only in the case of fields formed by an object planar on average if its surface has sharp irregularities. By recording the moduli of functions $B_{\rm t}$ and $B'_{\rm t}$ with a Michelson interferometer, we can determine the angular dimensions of visible regions of remote nonplanar objects.

(iii) For $L_{\rm s}\gg L_{\rm c}\geqslant 20\lambda_0$, we have $B_{\rm ti}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)\approx|B_{\rm t}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)|^2$ and $B'_{\rm ti}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)\approx|B'_{\rm t}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)|^2$. This means that functions $B_{\rm ti}$ and $B'_{\rm ti}$ are proportional to the squares of the modulus of the Fourier transform of the intensity distribution in object images. These functions can be recorded with a Brown–Twiss interferometer for determining the angular dimensions of the visible region of a remote nonplanar object and the object surface parameters, including the radius of curvature of the object surface.

Appendix 1

Calculation of the accuracy of approximation of the correlation function $B(\tau)$ by the time correlation function under the condition $\tau_{\rm c}\ll T$

The accuracy of the approximation of the function $B(\tau)=\langle u(t)u^*(t+\tau)\rangle_\psi$ by the time correlation function $B_u(\tau)=\langle u(t)u^*(t+\tau)\rangle_{\rm t}$ will be estimated by the parameter $\eta_{\rm t}=[\langle[B(\tau)-B_u(\tau)]^2\rangle_\psi]^{1/2}\times B^{-1}(\tau)$ under the condition that the phase $\psi(t)=\arctan[{\rm Im}\,u(t)/{\rm Re}\,u(t)]$ of the modulation function $u(t)$ is random, which is the case, for example, for a cw probe laser or thermal radiation from a self-luminous object. In this case, $B(\tau)=\langle u(t)u^*(t+\tau)\rangle_u=\langle u(t)u^*(t+\tau)\rangle_\psi$ and assuming that ψ has the Gaussian distribution with the correlation function $\langle\psi(t)\psi(t+\tau)\rangle_\psi=\sigma_\psi^2(t)\exp[-\tau^2/\tau_\psi^2(t)]$, we have

$$\langle B_u(\tau)^2\rangle_\psi=\frac{1}{T^2}\int_{t_0}^{t_0+T}\int_{t_0}^{t_0+T}\exp\langle\{-[\psi(t_1)-\psi(t_1+\tau)+\psi(t_2)$$

$$-\psi(t_2+\tau)]^2/2\}\rangle_\psi|u(t_1)||u(t_1+\tau)||u(t_2)||u(t_2+\tau)|{\rm d}t_1{\rm d}t_2$$

$$=\frac{1}{T^2}\int_{t_0}^{t_0+T}\int_{t_0}^{t_0+T}F(t_1,t_2,\tau)|u(t_1)u(t_1+\tau)|$$

$$\times|u(t_2)u(t_2+\tau)|{\rm d}t_1{\rm d}t_2,\qquad\text{(A1.1)}$$

where

$$F(t_1,t_2,\tau)=\exp\left\{-\sigma_\psi^2\left[2-2\exp\left(-\frac{\tau^2}{\tau_\psi^2}\right)\right.\right.$$

$$+2\exp\left[-\frac{(t_2-t_1)^2}{\tau_\psi^2}\right]-\exp\left[-\frac{(t_2+\tau-t_1)^2}{\tau_\psi^2}\right]$$

$$\left.\left.-\exp\left[-\frac{(t_1+\tau-t_2)^2}{\tau_\psi^2}\right]\right]\right\}$$

is the function having two maxima equal to unity: one at the intersection of planes $t_2+\tau-t_1=0$ and $t_1+\tau-t_2=0$ and the other in the plane $\tau=0$. All these planes intersect along the line $t_1=t_2$. Let us assume that $\sigma_\psi\gg 1$. Then, $F(t_1,t_2,\tau)\approx\exp(-2\tau^2/\tau_{\rm i}^2)$ in the vicinity of the first maximum and $F(t_1,t_2,\tau)\approx\exp\{-2[\tau^2/\tau_{\rm i}^2+(t_1-t_2)\tau_{\rm i}^{-2}]\}$ in the vicinity of the second maximum, where $\tau_{\rm i}=\tau_\psi/\sigma_\psi$. The width of each maximum is of the order of $\tau_{\rm i}$. Because $\tau_{\rm i}\ll T$, except the straight line $t_1=t_2$, these maxima and their nearest vicinities are strongly separated and the approximation $F(t_1,t_2,\tau)\approx\exp(-2\tau^2/\tau_{\rm i}^2)+\exp\{-2[\tau^2/\tau_{\rm i}^2+(t_1-t_2)^2/\tau_{\rm i}^2]\}$ can be used. By substituting this relation into (A1.1), we obtain

$$\langle B_u(\tau)^2\rangle_\psi\approx\langle B_u(\tau)\rangle_\psi^2+\exp\left(-2\,\frac{\tau^2}{\tau_{\rm i}^2}\right)$$

$$\times\frac{1}{T^2}\int_{t_0}^{t_0+T}\int_{t_0}^{t_0+T}\exp\left[-\frac{(t_1-t_2)^2}{\tau_{\rm i}^2}\right]|u(t_1)||u(t_1+\tau)|$$

$$\times|u(t_2)||u(t_2+\tau)|{\rm d}t_1{\rm d}t_2,$$

where $\langle B_u(\tau)\rangle_\psi\approx B_{ud}(\tau)\exp(-\tau^2/\tau_i^2)$. We will assume that the function $|u(t)|$ has the half-width $\tau_{\rm d}\gg\tau_{\rm i}$. Then, $\langle B_u(\tau)^2\rangle_\psi\approx\langle B_u(\tau)\rangle_\psi^2+(\tau_{\rm i}/T)\exp(-2\tau^2/\tau_{\rm i}^2)\langle|u(t)|u(t+\tau)|^2\rangle_{\rm t}$. By substituting this relation into expression $\eta_{\rm t}=[\langle[B(\tau)-B_u(\tau)]^2\rangle]_\psi/B(\tau)$ and taking into account that $\langle|u(t)||u(t+\tau)|^2\rangle_{\rm t}\leqslant 1$ and for $\tau_{\rm d}\gg\tau_{\rm i}$ the relation $\tau_{\rm c}\approx\tau_{\rm i}$ is valid, we obtain finally for $\tau_{\rm c}\ll T$ that $\eta_{\rm t}\approx(\tau_{\rm c}/T)^{1/2}\ll 1$, i.e. $B(\tau)\approx B_u(\tau)$ with the accuracy $\eta_{\rm t}\approx(\tau_{\rm c}/T)^{1/2}$.

Appendix 2

Calculation of the accuracy η of approximation of the function B_t by the function B_{fm}

Without loss of generality, we calculate the parameter η for a passively scattering object illuminated by quasi-monochromatic probe radiation. The coherence length L_c of this radiation is considerably smaller than the depth L_s of the visible region of the object (see Fig. 1) and noticeably exceeds the root-mean-square deviation σ of the height of its surface irregularities [7]. In this case, $\eta = \langle |B_t(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) - B_{fm}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2)|^2 \rangle_r / |B_f(\boldsymbol{\rho}, \boldsymbol{\rho})|^2$. Taking into account the Gaussian distribution of the field $E(\boldsymbol{\rho}, t)$, we obtain

$$\langle |B_t(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2)|^2 \rangle_r = \frac{1}{T^2} \int_{t_0}^{t_0+T} \int_{t_0}^{t_0+T} \langle E(\boldsymbol{\rho}_1, t_1) E^*(\boldsymbol{\rho}_2, t_1) E^*(\boldsymbol{\rho}_1, t_2)$$

$$\times E(\boldsymbol{\rho}_2, t_2) \rangle_r \mathrm{d}t_1 \mathrm{d}t_2 = \frac{1}{T^2} \left| \int_{t_0}^{t_0+T} B_f(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, t) \mathrm{d}t \right|^2$$

$$+ \frac{1}{T^2} \int_{t_0}^{t_0+T} \int_{t_0}^{t_0+T} \langle E(\boldsymbol{\rho}_1, t_1) E^*(\boldsymbol{\rho}_1, t_2) \rangle$$

$$\times \langle E^*(\boldsymbol{\rho}_2, t_1) E(\boldsymbol{\rho}_2, t_2) \rangle \mathrm{d}t_1 \mathrm{d}t_2. \qquad (A2.1)$$

Then, taking into account relations (6) and (A2.1) and that $\langle u(t) u^*(t+\tau) \rangle_t = B_u(\tau/\tau_c)$, we can obtain in the Fresnel approximation

$$\langle |B_t(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) - B_{fm}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2)|^2 \rangle_r \sim \frac{1}{T^2} \int k_i(\boldsymbol{r}_1) k_i(\boldsymbol{r}_2)$$

$$\times \int_{t_0}^{t_0+T} \int_{t_0}^{t_0+T} u\left(t_1 - 2r_1 - \frac{\boldsymbol{r}_1 \boldsymbol{\rho}_1}{c r_c}\right) u^*\left(t_2 - \frac{2r_1}{c} - \frac{\boldsymbol{r}_1 \boldsymbol{\rho}_1}{c r_c}\right)$$

$$\times u^*\left(t_1 - 2r_2 - \frac{\boldsymbol{r}_2 \boldsymbol{\rho}_2}{c r_c}\right) u\left(t_2 - 2r_2 - \frac{\boldsymbol{r}_2 \boldsymbol{\rho}_2}{c r_c}\right) \mathrm{d}t_1 \mathrm{d}t_2 \mathrm{d}\boldsymbol{r}_1 \mathrm{d}\boldsymbol{r}_2$$

$$= \iint k_i(\boldsymbol{r}_1) k_i(\boldsymbol{r}_2) \left| B_u\left(\frac{2(r_2 - r_1)}{L_c} - \frac{\boldsymbol{r}_1 \boldsymbol{\rho}_1 - \boldsymbol{r}_2 \boldsymbol{\rho}_2}{L_c r_c}\right) \right|^2 \mathrm{d}\boldsymbol{r}_1 \mathrm{d}\boldsymbol{r}_2.$$

We will assume that the probe radiation is narrowband. This means that $L_c > 4\lambda_0 M^{1/2}$, where $M = (dd_\rho)^2/(\lambda_0 r_c)^2$ is the number of spots of the processed field on the receiving aperture, d is the size of the visible region of the object, and d_ρ is the receiving aperture size [7]. Then,

$$\langle |B_t(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) - B_{fm}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2)|^2 \rangle_r \sim \iint k_i(\boldsymbol{r}_1) k_i(\boldsymbol{r}_2)$$

$$\times \left| B_u\left(\frac{2(r_2 - r_1)}{c}\right) \right|^2 \mathrm{d}\boldsymbol{r}_1 \mathrm{d}\boldsymbol{r}_2. \qquad (A2.2)$$

This integral was analysed in paper [7] by approximating the surface under study by a paraboloid of revolution $r \approx r_c + R^2/2\rho_0$, where R is the distance from the surface axis to a point with the radius vector $\boldsymbol{r}_c$ and ρ_0 is the radius of curvature of the surface. In the case of such an approximation, we have $k_i(\boldsymbol{r}) \sim (l/\sigma)^2 |k(R)|^2 \times \exp\{-Rl]^2/\sigma^2\}$. It follows from this approximation that

$$d \approx \frac{\rho_0 \sigma}{l}, \quad L_s \approx \frac{\rho_0 \sigma^2}{8 l^2}. \qquad (A2.3)$$

For $L_c \ll L_s$, we have

$$\iint k_i(\boldsymbol{r}_1) k_i(\boldsymbol{r}_2) \left| B_u\left(\frac{2(r_2 - r_1)}{c}\right) \right|^2 \mathrm{d}\boldsymbol{r}_1 \mathrm{d}\boldsymbol{r}_2$$

$$\approx L_c \rho_0 \int k_i^2(\boldsymbol{r}) \mathrm{d}\boldsymbol{r}. \qquad (A2.4)$$

Taking into account (A2.2)–(A2.4) and that $|B_{fm}(\boldsymbol{\rho}, \boldsymbol{\rho})|^2 \sim (\int k_i(\boldsymbol{r}) \mathrm{d}\boldsymbol{r})^2$, for the condition $L_c \ll L_s$, we obtain finally $\eta \approx L_c/2L_s \ll 1$, and hence, $B_t(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) \approx B_{fm}(\boldsymbol{\rho}, \boldsymbol{\rho})$. In the opposite case, when $L_s \ll L_c$, we have $|B_u([2(r_2 - r_1)/c)]|^2 \approx 1$ and $\eta \approx 1$. This means that the functions B_t and B_{fm} are substantially different. Similar calculations show that for $L_c \ll L_s$, we have $B'_t(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) \approx B'_{fm}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2)$. It can be shown that for $L_c < \sigma$, which corresponds to the propagation of polychromatic radiation from the object, we have $\eta \sim L_c/L_s \ll 1$. In this case, we can assume that the equalities $B_t(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) = B_{fm}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2)$ and $B'_t(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) = B'_{fm}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2)$ are fulfilled.

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