Quantum Electronics **38** (5) 482 – 485 (2008)

CONTROL OF LASER RADIATION PARAMETERS

PACS numbers: 42.55.Rz; 42.60.Da; 42.60.Fc
DOI: 10.1070/QE2008v038n05ABEH013775

Phase dynamics in the self-modulation oscillation regime in a solid-state ring laser

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***Abstract*. The dynamics of the phase difference of counterpropagating waves is studied theoretically and experimentally in the self-modulation oscillation regime in a solid-state ring laser. It is found that in the case of a small enough frequency nonreciprocity of the ring resonator, the phase difference of counterpropagating waves changes within a limited range, performing periodic oscillations with the intensity self-modulation frequency. The instant frequency difference of counterpropagating waves also changes periodically in time; however, its mean value is zero (the frequency locking for counterpropagating waves takes place). The width of the frequency locking region is measured. It is shown that the phase difference of the coupling coefficients considerably affects the phase dynamics. This opens up new possibilities for determining the phase difference of coupling coefficients of counterpropagating waves.**

Keywords: *solid-state ring laser, phase dynamics, self-modulation lasing regime of the first kind, frequency nonreciprocity, phase locking*.

1. Introduction

The studies of the nonlinear dynamics of radiation of solid-state ring lasers (SRLs) have been usually devoted to the time and spectral characteristics of radiation of counterpropagating waves, whereas the dynamics of their optical phases (phase dynamics of radiation) has received little attention (see, for example, review [1]). This is explained by the fact that the direct measurement of optical phases of counterpropagating waves and their difference is a challenging technical problem. The phase information can be obtained by analysing a photomixing (heterodyne) radiation signal from two lasers [2, 3] and also signals obtained by photomixing of optical fields of counterpropagating waves [4]. The phase dynamics of SRLs in dynamic chaos regimes was studied theoretically and experimentally in papers [4 – 6].

It is known that numerous different nonstationary oscillation regimes can be observed in SRLs, which can exist both in autonomous ring lasers [1] and non-autonomous lasers with periodically modulated parameters [7]. One of the most often encountered nonstationary regimes in autonomous SRLs is the self-modulation regime of the first kind [1].

Some features of the phase dynamics in this lasing regime were theoretically studied in papers [1, 8, 9] for a particular case of the coupling coefficients of counterpropagating waves close to complex conjugate waves. It was shown in [8] that in the case of a small enough frequency nonreciprocity of a ring resonator, the mutual frequency locking of counterpropagating waves occurs. In this case, the phase difference of counterpropagating waves changes within a limited region, by performing periodic oscillations at the intensity self-modulation frequency $\omega_{\rm m}$. The phase-locking regime exits in a finite region $|\Omega| < \Omega_{\rm cr}$ of the frequency nonreciprocity of the resonator; as this region is increased ($|\Omega| > \Omega_{\rm cr}$), the phase-locking regime passes to the beat regime. In this regime, the phase difference Φ is also modulated at the self-modulation frequency and, in addition, the value of Φ increases (or decreases) linearly with time. The instant frequency difference of counterpropagating waves oscillates at the frequency $\omega_{\rm m}/2\pi$ with respect to the mean frequency difference $\langle\Phi\rangle/2\pi$. The mean value of the circular frequency difference in the beat regime is $\langle\Phi\rangle = {\rm sign}(\Omega)\omega_{\rm m}$.

By weakening the competition between counterpropagating waves with the help of a feedback circuit, it is possible to obtain the beat regime with virtually equal intensities of counterpropagating waves [10], which is similar to the well-known beat regime in gas ring lasers. The phase dynamics in this regime was studied experimentally in [11].

The aim of this paper is to study theoretically and experimentally the phase dynamics in the self-modulation lasing regime of the first kind. As far as we know, the dynamics of the phase difference of counterpropagating waves in this regime is investigated in our paper for the first time. Unlike previous studies, we analysed the phase dynamics for arbitrary phases of the coupling coefficients of counterpropagating waves.

2. Theoretical analysis

The phase dynamics was analysed by using the standard SRL model [1] described by a system of equations for the complex field amplitudes of counterpropagating waves $\tilde{E}_{1,2}(t) = E_{1,2}\exp({\rm i}\varphi_{1,2})$ and spatial harmonics of the inverse population N_0, $N_{\pm}$:

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Received 5 December 2007
Kvantovaya Elektronika **38** (5) 482 – 485 (2008)
Translated by M.N. Sapozhnikov

$$\frac{\mathrm{d}\tilde{E}_{1,2}}{\mathrm{d}t} = -\frac{\omega_c}{2Q}\tilde{E}_{1,2} \pm \mathrm{i}\frac{\Omega}{2}\tilde{E}_{1,2} + \frac{\mathrm{i}}{2}\tilde{m}_{1,2}\tilde{E}_{2,1}$$

$$+\frac{\sigma l}{2T}\left(N_0\tilde{E}_{1,2} + N_{\pm}\tilde{E}_{2,1}\right),$$

$$T_1\frac{\mathrm{d}N_0}{\mathrm{d}t} = N_{\mathrm{th}}(1+\eta) - N_0 - N_0 a\left(|E_1|^2 + |E_2|^2\right) \quad (1)$$

$$-N_+ aE_1E_2^* - N_- aE_1^*E_2,$$

$$T_1\frac{\mathrm{d}N_+}{\mathrm{d}t} = -N_+ - N_+ a\left(|E_1|^2 + |E_2|^2\right) - N_0 aE_1^*E_2.$$

Here, ω_c/Q is the bandwidth of the resonator (losses for counterpropagating waves are assumed equal); $T = L/c$ is the round-trip transit time for radiation in the resonator; T_1 is the longitudinal relaxation time; l is the active element length; $a = T_1 c\sigma/(8\hbar\omega\pi)$ is the saturation parameter; σ is the laser transition cross section; $\Omega = \omega_1 - \omega_2$ is the frequency nonreciprocity of the resonator; and ω_1 and ω_2 are the resonator eigenfrequencies for counterpropagating waves. The pumping rate is written in the form $N_{\mathrm{th}}(1+\eta)/T_1$, where N_{th} is the threshold inverse population and η is the pump power excess over the threshold. The linear coupling between counterpropagating waves is determined by phenomenologically introduced complex coupling coefficients

$$\tilde{m}_1 = m_1\exp(\mathrm{i}\vartheta_1), \quad \tilde{m}_2 = m_2\exp(-\mathrm{i}\vartheta_2), \quad (2)$$

where $m_{1,2}$ are the moduli of coupling coefficients and $\vartheta_{1,2}$ are their phases. Note that Eqns (1) are written for lasing at the gain line centre.

The self-modulation regime of the first kind, which is characterised by the out-of-phase sinusoidal modulation of the intensity of counterpropagating waves, exists in a broad range of SRL parameters [1]. The analytic solution determining the time dependence of the complex amplitudes was found in [12]. For simplicity, we present here this solution for $\Omega = 0$ (the frequency nonreciprocity is absent) in the case of the symmetrical coupling of counterpropagating waves, when coupling coefficients have the same moduli:

$$m_1 = m_2 = m. \quad (3)$$

The phase difference of the coupling coefficients is denoted by

$$\vartheta = \vartheta_1 - \vartheta_2. \quad (4)$$

The time dependences of the complex amplitudes of counterpropagating waves can be written in the form [12]

$$\tilde{E}_1 = A_1\exp\left(\mathrm{i}\frac{\omega_{\mathrm{m}}t}{2}\right) + B_1\exp\left(-\mathrm{i}\frac{\omega_{\mathrm{m}}t}{2}\right),$$
$$\tilde{E}_2 = A_2\exp\left(\mathrm{i}\frac{\omega_{\mathrm{m}}t}{2}\right) + B_2\exp\left(-\mathrm{i}\frac{\omega_{\mathrm{m}}t}{2}\right). \quad (5)$$

The intensity self-modulation frequency ω_{m} is determined by the expression

$$\omega_{\mathrm{m}} = m|\cos(\vartheta/2)|, \quad (6)$$

and constants $A_{1,2}$ and $B_{1,2}$ in the case under study are determined from the expressions

$$A_2^2 = \frac{\alpha-\beta}{2}, \quad B_2^2 = \frac{\alpha+\beta}{2},$$
$$A_1 = -\frac{A_2\omega_{\mathrm{m}}}{d}, \quad B_1 = \frac{B_2\omega_{\mathrm{m}}}{d}, \quad (7)$$

where

$$\alpha = \frac{\eta_e}{2}; \quad \beta = \frac{m^2\sin\vartheta(1+\eta)(N_{\mathrm{th}}/N_0)}{2\omega_c/Q}\frac{1}{\omega_{\mathrm{m}}};$$

$$d = -\frac{m(1+\cos\vartheta+\mathrm{i}\sin\vartheta)}{2}; \quad \frac{\sigma l}{T}N_{\mathrm{th}} = \frac{\omega_c}{Q} - m\left|\sin\frac{\vartheta}{2}\right|;$$

$$\frac{\sigma l}{T}N_0 = \frac{\omega_c}{Q}; \quad \eta_e = (1+\eta)\frac{N_{\mathrm{th}}}{N_0} - 1.$$

By using this solution, we can obtain expressions determining the time dependence of the phase difference of counterpropagating waves. The complex amplitudes of counterpropagating waves are written in the form

$$\tilde{E}_1 = E_{\mathrm{r}1} + \mathrm{i}E_{\mathrm{i}1} \quad \text{and} \quad \tilde{E}_2 = E_{\mathrm{r}2} + \mathrm{i}E_{\mathrm{i}2},$$

where $E_{\mathrm{r}j}$ and $E_{\mathrm{i}j}$ are the real and imaginary parts of the complex amplitudes of counterpropagating waves ($j = 1, 2$). The time dependence of the phase difference $\Phi = \varphi_1 - \varphi_2$ of counterpropagating waves can be found from the expressions

$$\cos\Phi = \frac{E_{\mathrm{r}1}E_{\mathrm{r}2} + E_{\mathrm{i}1}E_{\mathrm{i}2}}{E_1E_2}, \quad \sin\Phi = \frac{E_{\mathrm{i}1}E_{\mathrm{r}2} - E_{\mathrm{i}2}E_{\mathrm{r}1}}{E_1E_2}. \quad (8)$$

Consider the influence of the phase difference ϑ of coupling coefficients on the phase dynamics in the absence of the frequency nonreciprocity of the resonator ($\Omega = 0$). The time dependences $\cos\Phi$ calculated by expressions (5)–(8) are shown in Fig. 1. Some parameters used in calculations were set equal to the experimental parameters of the laser under study. The excess of the pump power over the threshold was set equal to 0.09 and the bandwidth of the resonator was determined from the relaxation frequency $\omega_{\mathrm{r}} = (\omega\eta/QT_1)^{1/2}$; for $\eta = 0.09$, we have $\omega_{\mathrm{r}}/2\pi = 65$ kHz. The self-modulation frequency was fixed ($\omega_{\mathrm{m}}/2\pi = 207$ kHz) during changing ϑ, and the values of m were calculated for the given ϑ from expression (6).

One can see from Fig. 1 that the phase difference of counterpropagating waves in the self-modulation regime of the first kind in the absence of the frequency nonreciprocity periodically changes in time in finite limits. The type of changing of $\cos\Phi$ is substantially different for coupling coefficients close to complex conjugate ones (Fig. 1a: scattering by the refractive index inhomogeneities) and for coupling coefficients that considerably differ from complex conjugate coefficients (Figs 1b–d: scattering by absorption inhomogeneities).

The phase difference of coupling coefficients affects the range of variation in the phase difference Φ of counterpropagating waves in the frequency-locking region. In this region, the mean (for the self-modulation period) frequency difference of counterpropagating waves is zero. The fre-

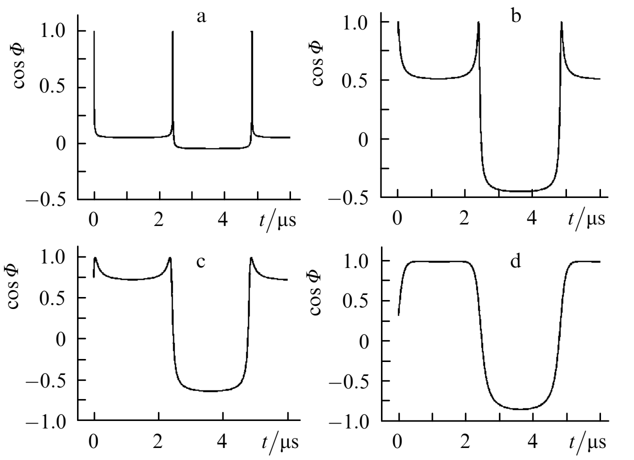

Figure 1. Theoretical time dependences of $\cos\Phi$ in the self-modulation lasing regime of the first kind in the absence of frequency nonreciprocity ($\Omega = 0$) for different phase differences and different moduli of coupling coefficients: $\vartheta = -0.1$, $m/2\pi = 207.3$ kHz (a); $\vartheta = -1$, $m/2\pi = 236$ kHz (b); $\vartheta = -1.5$, $m/2\pi = 283$ kHz (c); and $\vartheta = 1.5$, $m/2\pi = 656$ kHz (d).

quency locking is preserved in the case of small enough frequency nonreciprocities of the ring resonator (for $|\Omega| < \Omega_{cr}$).

A change in the phase difference of counterpropagating waves is caused by their linear coupling determined by the complex coupling coefficients $\tilde{m}_{1,2}$ and the nonlinear coupling due to backscattering of counterpropagating waves by inverse population gratings induced in the active medium. The self-modulation frequency for monolithic SRLs is usually considerably greater than the relaxation frequency ($\omega_m \gg \omega_r$). In this case, phase shifts due to nonlinear coupling are small, and the phase difference of counterpropagating waves is mainly modulated due to linear coupling.

The dependence of the phase-locking region width Ω_{cr} on the laser parameters can be found either analytically or by solving numerically the system of equations for the standard SRL model. It can be shown that the frequency locking exists when the inequality

$$\frac{\Omega^2}{\omega_m^2} < \frac{|\langle I_1\rangle - \langle I_2\rangle|}{\langle I_1\rangle + \langle I_2\rangle} \tag{9}$$

is fulfilled, where $\langle I_{1,2}\rangle$ are the average intensities of counterpropagating waves. By substituting into (9) the expressions for the intensity from [12], we can transform this inequality to the form $|\Omega| < \Omega_{cr}$ and obtain the expression

$$\Omega_{cr} = \frac{(1+\eta)m^2|\sin\vartheta|}{(\omega/Q)\eta} \tag{10}$$

for estimating the width Ω_{cr} of the phase-locking region.

By using expressions for frequencies ω_m and ω_r, we can rewrite (10) in the form

$$\Omega_{cr} = \frac{2(1+\eta)\omega_m^2|\tan(\vartheta/2)|}{T_1\omega_r^2}. \tag{11}$$

All the parameters entering this expression (except ϑ) can be measured experimentally and then the phase difference of coupling coefficients can be found from (11) and the modulus of the coupling coefficient can be found from (6).

3. Experiment

Experiments were performed with a diode-pumped monoblock ring Nd : YAG laser [1]. The chip laser represented a monoblock with a spherical input face and three total internal reflection faces. The geometrical perimeter of the resonator was 2.6 cm. The nonplanarity angle of the resonator was 80°. The laser was pumped by a 250-mW diode laser. The frequency nonreciprocity was varied by means of an external magnetic field produced by an electromagnet located near the chip laser. The magnetic field strength could achieve 500 Oe.

The phase dynamics was recorded by the interference method described in [4]. In this case, information on the phase dynamics is contained in the photomixing signal of counterpropagating waves, which gives the total field intensity

$$\boldsymbol{E}_{pm} = \boldsymbol{E}_1 + \boldsymbol{E}_2. \tag{12}$$

Because polarisations of counterpropagating waves are not identical in the general case, it is expedient in the interference method to separate similar (for example, linear) polarisation components in each wave. In this case, the intensity of the photomixing signal of counterpropagating waves is

$$I_{pm} = I_1 + I_2 + 2K(I_1 I_2)^{1/2}\cos\Phi, \tag{13}$$

where $I_{1,2}$ are the intensities of separated components of counterpropagating waves having the same polarisation; Φ is the phase difference of interfering waves; and the coefficient K characterises the degree of overlap of interfering beams.

The self-modulation regime of the first kind existed in the laser under study when the excess of the pump over threshold was $\eta < 0.25$. The self-modulation oscillation frequency was $\omega_m/2\pi = 207$ kHz, and the relaxation oscillation frequency was 65 kHz for the relative excess over the threshold power equal to 0.09.

The experimental intensity of counterpropagating waves and photomixing signal were computer-processed in the following way. To exclude the influence of the noise component, the real signals were approximated by the expressions $A\cos(\omega_m t + \varphi) + B$ determining these signals in the absence of noise. The value of $\cos\Phi$ was found from (13):

$$\cos\Phi = \frac{I_{pm} - I_1 - I_2}{2K(I_1 I_2)^{1/2}}. \tag{14}$$

Figure 2 presents the time dependences of $\cos\Phi$ found experimentally for different magnetic field strengths in the active medium (different frequency nonreciprocities of the ring resonator). Also, similar time dependences calculated analytically by expressions (5)–(8) are presented. The phase difference ϑ for coupling coefficients was set equal to -1.2 in calculations, while the rest of the parameters were equal to their experimental values. The experimental time dependences of $\cos\Phi$ presented in Figs 2a, b correspond to the frequency-locking regime (the phase difference changes in a

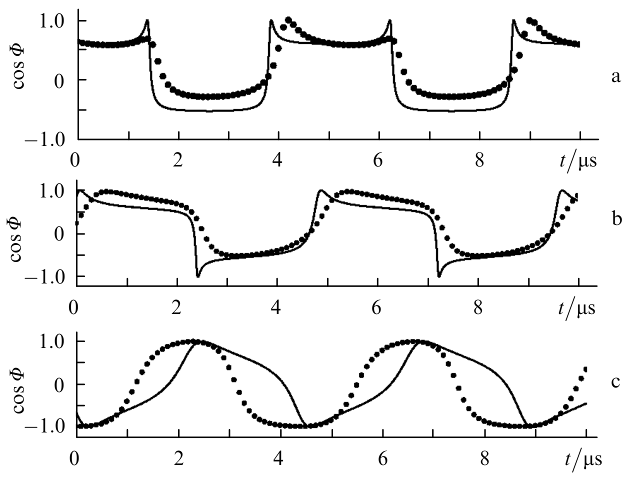

Figure 2. Time dependences of $\cos\Phi$ in the self-modulation lasing regime of the first kind in the phase-locking region for $\Omega/2\pi = 0$ (a) and 20 kHz (b) and in the beat region for $\Omega/2\pi = 100$ kHz (c). The solid curves are the theory, points are experiment data.

limited range); Fig. 2c corresponds to the beat regime. The experimental dependences well agree with calculations.

Our experimental study confirmed the theoretical prediction about the existence of the frequency locking of counterpropagating waves in the self-modulation regime of the first kind. The range of variations in the phase difference in the phase-locking region is limited, and $\cos\Phi$ changes from $+1$ to minimal values exceeding -1 (see Figs 1 and 2). In the beat regime, $\cos\Phi$ changes from -1 to $+1$ (Fig. 2c). The phase-locking region width measured in experiments is close to 20 kHz, which is in qualitative agreement with the theoretical estimate by (11).

4. Conclusions

We have studied theoretically and experimentally the dynamics of the phase difference for counterpropagating waves in the self-modulation regime of the first kind. It has been shown that in the case of small enough nonreciprocities of the ring resonator, the mutual frequency locking of counterpropagating waves takes place. The phase difference in the frequency-locking region oscillated within a limited interval with a period of self-modulation oscillations. The variation range strongly depends on the phase difference of coupling coefficients, which allows us to estimate qualitatively the phase difference for linear coupling coefficients of counterpropagating waves from the time dependence.

It was assumed earlier that coupling coefficients in monolithic ring chip lasers are determined by scattering from refractive-index inhomogeneities and therefore are close to complex conjugate coefficients ($|\vartheta| \gg 1$). The time dependence of $\cos\Phi$ measured in our experiments can be explained only if the phase difference $\vartheta - 1.2$ of coupling coefficients is large enough, i.e. the coupling coefficients in the laser under study considerably differ from complex conjugate ones.

Acknowledgements. This work was supported by the Russian Foundation for Basic Research (Grant Nos 07-023-00204 and 08-02-00217).

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