NONLINEAR OPTICAL PHENOMENA

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Phase conjugation upon SBS of a focused laser speckle beam

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Abstract. The phase conjugation (PC) of a focused Gaussian laser beam with a partial spatial coherence of the wave front is studied numerically and theoretically upon SBS within the framework of a three-dimensional nonstationary SBS model, which takes into account transient processes and SBS saturation. The dependences of the PC coefficient h on the laser radiation power are obtained for different excesses of the angular divergence over the diffraction limit ξ . It is found that for the given reflectance of laser radiation from the SBS mirror, the PC coefficient monotonically decreases with increasing the divergence. For example, under the near threshold SBS conditions, h decreases from 70% to 30%, when ξ increases from 1 to 10. For the given divergence, the PC coefficient increases with increasing the reflectance and approaches the ideal one (h > 90%) upon deep SBS saturation, when the reflectance exceeds 90% - 95%.

Keywords: stimulated Brillouin scattering, phase conjugation.

1. Introduction

Since the discovery of the phase conjugation (PC) effect upon stimulated Brillouin scattering (SBS) of laser radiation [1], many papers devoted to its study has been published (see monographs [2–4]) and the interest in this effect has not weakened so far. One of the problems of this paper, having an important practical significance, consists in the investigation of the dependence of the PC coefficient characterising the PC quality on the angular divergence of the laser pump beam (to be more exact, on excesses of the angular divergence over the diffraction limit ξ). This dependence can differ in different SBS excitation schemes: focusing of the pump beam into a waveguide or volume SBS medium (see, for example, [5–9]).

The beam focusing into the bulk SBS medium is simpler. In addition, this focusing is reasonable in the case of highpower laser radiation due to the radiation resistance problem. The PC quality is affected by 'snake' distortions increasing with decreasing ξ and by Stokes modes uncorre-

Received 11 December 2007 *Kvantovaya Elektronika* **38** (9) 849–854 (2008) Translated by I.A. Ulitkin lated with the pumping, whose contribution, on the contrary, increases with increasing ξ [2]. These factors acting in opposite directions in combination with the spatial inhomogeneity of the pump beam envelope make it difficult to determine the optimal ξ from simple theoretical considerations. According to the existing views [2], the PC quality is worse for a focused weakly distorted ($\xi \ge 1$) laser beam than in the limiting cases of the ideal Gaussian beam ($\xi = 1$) and a beam with noticeable distortions ($\xi \ge 1$, but $\xi \le 10^3$ for the discrimination of non-conjugated noises). The experimental results contradicted each other and conclusions of the theory, which was pointed out in [10]. This is not surprising if we take into account that ξ is not the only determining parameter in a real experiment. Indeed, the PC quality depends on the character of the pump beam distortions, the SBS saturation level (the reflectance of the SBS mirror), the effect of the process nonstationarity, the degree of pump radiation depolarisation, the appearance of parasitic nonlinear processes (self-action of the pump beam and optical breakdown), etc. The unsolved problem of the exact measurement of the PC coefficient, especially urgent for $\xi > 1$, can make its contribution into these contradictions.

The PC theory upon SBS [2] was developed in the stationary linear SBS approximation, i.e. for near-threshold conditions, while in experiments the SBS saturation is always present to a certain degree. Moreover, it is desirable to achieve the deep saturation regime of the SBS mirror to ensure the significant reflection of radiation. The PC process upon SBS under conditions close to real can be studied by the numerical simulation method. Numerical models and computer codes have been developed recently to calculate the SBS in the case of the narrowband [11-13] and broadband [14] pumping. They allow one to simulate complex SBS mirrors, used in the experiment [11, 15, 16] and to optimise laser systems with the phase conjugation upon SBS [12, 17, 18]. In the existing numerical models it is possible to take into account the medium three dimensionality, the hypersonic noise in the medium volume, diffraction, transient processes related to the finite time of the hypersound relaxation, the SBS saturation, optical inhomogeneities and radiation self-action in the SBS medium, i.e. main known effects accompanying the SBS. New calculation models, apart from the solution of practical problems, help not only test theory and hypotheses formulated at early stages of studies on the PS upon SBS [11, 13, 19, 20] but also find new PC features (for example, interesting phenomena upon SBS of focused vortex laser beams [21, 22]).

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The aim of this work is to study numerically and theoretically the PC coefficient of a focused laser pump beam with a partially coherent wave front upon SBS as a function of the excess of its angular divergence over the diffraction limit under near-threshold conditions and upon the influence of the SBS saturation.

2. Formulation of the numerical experiment

Calculations were performed with the help of the model describing the process of the nonstationary SBS in a threedimensional medium [11, 14] by neglecting the self-action effects. The SBS is described by a system of equations

$$\frac{n_0}{c}\frac{\partial A_{\rm L}}{\partial t} + \frac{\partial A_{\rm L}}{\partial z} - \frac{1}{2ik}\nabla_{\perp}^2 A_{\rm L} = -\frac{{\rm i}}{2}pA_{\rm S},\tag{1}$$

$$\frac{n_0}{c}\frac{\partial A_{\rm S}}{\partial t} - \frac{\partial A_{\rm S}}{\partial z} - \frac{1}{2ik}\nabla_{\perp}^2 A_{\rm S} = -\frac{{\rm i}}{2}p^*A_{\rm L},\tag{2}$$

$$\frac{1}{2\mathrm{i}\Omega}\frac{\partial^2 p}{\partial t^2} + \frac{\partial p}{\partial t} + \frac{p}{\tau} = -\frac{\mathrm{i}g}{\tau}A_{\mathrm{L}}A_{\mathrm{S}}^* + S,\tag{3}$$

where A_{L} , A_{S} and p are the slowly varying complex amplitudes of the laser, Stokes and hypersonic fields, respectively; g is the SBS gain; τ and Ω are the decay time and the hypersound frequency, respectively; ω_0 is the laser radiation frequency; n_0 is the average refractive index; $k = n_0 \omega_0 / c$; S is the Langevin fluctuation strength (the source of the hypersonic noise in the medium volume); ∇^2_{\perp} is the Laplacian over the transverse coordinate r; z is the longitudinal coordinate. In the system of equations (1)-(3), the radiation field amplitudes are normalised so that the flux densities of laser and Stokes radiations are equal to $|A_{\rm L}|^2$ and $|A_{\rm S}|^2$, respectively. In the case when the pump coherence time is significantly longer than the hypersound period, the second derivative in (3) can be neglected. The source of the hypersonic noise in (3) is delta correlated in time:

$$\langle S(\mathbf{r}_1, z_1, t_1) S^*(\mathbf{r}_2, z_2, t_2) \rangle = B_p(\mathbf{r}_1 - \mathbf{r}_2, z_1 - z_2) \delta(t_1 - t_2),$$
(4)

where B_p is the spatial correlation function of the noise source.

To solve the system of equations (1)-(3), it is necessary to know the average and correlation characteristics of the hypersonic noise, which should be in agreement with the values observed in the experiments. The relation of the initial hypersound amplitude p_0 and the noise source power should be such as the average hypersonic noise background constant in time was maintained in the absence of radiation in the SBS medium, which is provided when the relations

$$\langle p_0(\mathbf{r}_1, z_1) p_0^*(\mathbf{r}_2, z_2) \rangle = 4\varkappa F_p(\mathbf{r}_1 - \mathbf{r}_2)\delta(z_1 - z_2),$$
(5)

$$\langle S(\mathbf{r}_1, z_1, t_1) S^*(\mathbf{r}_2, z_2, t_2) \rangle$$

= $(2/\tau) \langle p_0(\mathbf{r}_1, z_1) p_0^*(\mathbf{r}_2, z_2) \rangle \delta(t_1 - t_2)$ (6)

are fulfilled, where \varkappa the coefficient of spontaneous laser radiation scattering (in cm⁻¹ sr⁻¹); $F_p(\mathbf{r}_1 - \mathbf{r}_2)$ is the dimensionless correlation function, which gives the unit axial brightness of the Stokes radiation obtained due spontaneous scattering. The coefficient \varkappa is borrowed from experimental data or estimated theoretically [for a gaseous medium $\kappa = 4\pi^2 (n_0 - 1)^2 (\lambda_0^4 N_0)^{-1}$, where N_0 is the concentration of gas molecules and $\lambda_0 = 2\pi c/\omega_0$].

The adaptation of conditions (5) and (6) to the concrete numerical scheme yields

$$\langle p_0(\mathbf{r}_1, z_1) \, p_0^*(\mathbf{r}_2, z_2) \rangle = 4\varkappa F_p(\mathbf{r}_1 - \mathbf{r}_2) \delta_{z_1 z_2} / \Delta z,$$
 (7)

$$\langle S(\mathbf{r}_1, z_1, t_1) S^*(\mathbf{r}_2, z_2, t_2) \rangle = \langle p_0(\mathbf{r}_1, z_1) p_0^*(\mathbf{r}_2, z_2) \rangle \beta \delta_{t_1 t_2}, \quad (8)$$

where $\beta = [2/(\tau \Delta t)](1 - \alpha + \alpha^2/2 - \alpha^3/8)/(1 - \alpha/2)^2$; $\alpha = \Delta t/\tau$; Δz and Δt are longitudinal and time steps; $\delta_{z_1 z_2}$ and $\delta_{t_1 t_2}$, are the Kronecker delta.

The procedure of finding the random distribution (realisations) of the initial hypersound amplitude $p_0(\mathbf{r}, z)$ by its correlation function $\langle p_0(\mathbf{r}_1, z) p_0^*(\mathbf{r}_2, z) \rangle$ is reduced to determining the random function f_p by the correlation function $F_p(\mathbf{r}_1 - \mathbf{r}_2)$ [23].

In the calculations the SBS mirror was considered in which the laser radiation is focused by a lens into a bulk SBS cell without reflection from its side surface. The laser field distribution on the lens is a random realisation of a field with the Gaussian correlation function of the type:

$$\langle A_{\mathrm{L}}(\mathbf{r}_{1},z)A_{\mathrm{L}}^{*}(\mathbf{r}_{2},z)\rangle = J_{0}\exp\left(-2\frac{\boldsymbol{\rho}^{2}}{a^{2}}-\frac{\boldsymbol{r}^{\prime 2}}{l_{\mathrm{c}}^{2}}\right),\tag{9}$$

where $\rho = (\mathbf{r}_1 + \mathbf{r}_2)/2$; $\mathbf{r}' = \mathbf{r}_1 - \mathbf{r}_2$; J_0 is the axial density of the radiation flux in the plane of the focusing lens; l_c is the correlation radius of radiation in the plane of the lens (the lens is placed at the cell input); a is the laser beam radius. The ratio of the angular radiation divergence to the diffraction limit is characterised by the number $\xi = \sqrt{2}a/l_c$, which is the average number of the inhomogeneities along the line lying in the beam cross section and intersecting the optical axis. This field distribution is produced after the propagation of an ideal Gaussian laser beam through the random phase plate with a Gaussian statistics of surface inhomogeneities [1-4].

Figure 1 presents random realisations of the intensity distribution of the laser pump beam at the wavelength of $\lambda = 0.53 \,\mu\text{m}$, which were obtained based on correlation function (9) for the beam radius a = 0.15 cm and parameter $\xi = 1, 2, 5, 10$. When the diffraction limit divergence is slightly exceeded (Fig. 1b), the intensity distribution significantly changes from realisation to realisation and when ξ qualitatively exceeds the unity, the beam does not change noticeable from realisation to realisation and has a distinctly pronounced speckle structure with a 100 % intensity modulation (Fig. 1d).

In calculations we used the following parameters of the SBS medium: the cell length L = 80 cm, g = 23 cm GW⁻¹, $\tau = 5$ ns, which corresponds to the compressed Xe at a pressure of about 30 atm [11]. The laser radiation is focused into the cell by a lens with the focal length F = 50 cm. The beam waist length is substantially smaller than the length of the SBS cell for the above values of ξ .

3. Results of calculations

The SBS and PC were calculated by changing the output power of the laser beam from the near-threshold SBS excitation power to the power corresponding to the establishment of the regime of its deep saturation when



Figure 1. Transverse intensity distributions of laser radiation at the cell input for the case of an ideal Gaussian beam ($\xi = 1$) (a) and their single realisations for $\xi = 2$ (b), 5 (c) and 10 (d).

an overhelming part of the pump energy is transferred to the Stokes component. Because the main attention in this paper is drawn to the influence of the spatial inhomogeneity of the laser radiation intensity on the PC quality, stationary pumping (9) was considered to exclude the effect of the nonstationarity on the results of calculations.

The influence of the gain saturation on the PC quality will be considered by the example of calculations performed for $\xi = 5$. Figure 2a shows typical time dependences of the laser radiation power at the input (z = 0) and output (z = L) of the SBS cell and Stokes radiation power at its intput. Figure 2b demonstrates the dynamics of the PC coefficient (normalised overlap integral) at the cell input [2]

$$h(z,t) = \frac{\left| \iint A_{\rm L}(\mathbf{r},z,t) A_{\rm S}(\mathbf{r},z,t) d\mathbf{r} \right|^2}{\iint |A_{\rm L}(\mathbf{r},z,t)|^2 d\mathbf{r} \iint |A_{\rm S}(\mathbf{r},z,t)|^2 d\mathbf{r}}.$$
 (10)

The laser beam powers at the cell output and Stokes beam at its input as well as the PC coefficient for the time $(10-20)\tau$ achieves stationary values (we will ignore rare and deep fluctuations of the power and the PC coefficient caused by the random nature of the hypersound source and accompanied by the rearrangement of the hypersonic grating in the medium and formation of the reflected signal in the form of spikes [11]). Below we present the established values of the coefficient *h* (10). The reflectance of laser radiation is determined as $R = P_{\rm S}^{\rm in}/P_{\rm L}^{\rm in}$, where $P_{\rm L}^{\rm in}$ and $P_{\rm S}^{\rm in}$ are the laser radiation power and the established



Figure 2. Dynamics of the laser radiation power *P* at the input (1) and output (2) of the cell and of the Stokes radiation power at its input (3) (a) and dynamics of the PC coefficient *h* for $\xi = 5$ and R = 70 % (b).



Stokes radiation power at the cell input. In calculations the fulfillment of the law of the conservation of energy was controlled: the input laser radiation energy is equal to the sum of the radiation energy propagated through the cell and the Stokes component energy (with the accuracy to the energy of a hypersonic wave, which is negligibly small).

Figure 3 presents the transverse intensity distributions of the Stokes radiation at the medium input for different pump levels (the pump intensity distribution is shown in Fig. 1c) at those instants when the SBS became stationary. One can see from Fig. 3a that in the near-threshold SBS regime, when $R \approx 3\%$, the PC quality is low (h = 45%). In addition, the effect of 'snake' distortions [2] typical of the linear SBS is strongly pronounced in Fig. 3a, i.e. the 'pulling' of the Stokes radiation into the brightest speckles of the pump radiation. This leads to the fact that only these bright spots of the pump beam are reproduced in the reflected speckle pattern. As the fraction of the reflected radiation is increased, the saturation process begins affecting the PC, which leads to smoothing the inhomogeneities of the gain envelope and thus to the increase in the PC coefficient. One can see from Fig. 3b that even at a high enough SBS saturation (R = 63 %), the PC quality remains rather low (h = 68 %). Only when R > 90 %, the coefficient h approaches 90% and the beams of the laser and Stokes radiation become similar to each other visually (cf. Fig. 3c and Fig. 1c).

Figure 4 shows the dependences of the reflection and PC coefficients on the pump power at the cell input for different ξ including $\xi = 1$ [11]. The calculation results of one realisation of the pump field are presented for each $\xi > 1$. One can see that the higher the pump radiation divergence, the larger the SBS threshold power (Fig. 4a) and the worse the PC quality (Fig. 4b). A lower PC quality in the case of a laser beam containing a speckle pattern compared to the case $\xi = 1$ under near-threshold conditions can be explained by the fact that the Stokes radiation in different pump speckles increases independently, i.e. mutual phasing of the Stokes beam speckles being produced is absent. A noticeable diffraction mixing of angular components of the Stokes radiation occurs only outside the waist upon approaching the input window of the cell but here the amplification of the Stokes radiation is small compared to the amplification in the waist. In other words, in the region, where the main amplification of the Stokes radiation occurs, mutual phasing of its angular components is absent, while in the region, where they are phased, a noticeable amplification is absent. Only the SBS saturation leads to an increase in the gain in the vicinity of the output window of the cell and to the efficiency of this phasing.



Figure 4. Dependences of the reflectance *R* (a) and the PC coefficient *h* (b) of laser radiation on the pump power $P_{\rm L}^{\rm in}$ for $\xi = 1$ (\odot), 2 (\triangle), 5 (\Box) and 10 (\bigtriangledown).

Figure 5 shows the dependences of the conjugation coefficient on the laser beam reflectance h(R) obtained for different realisations of the pump beam for $\xi = 2$ and 10. One can see that a scatter in the values of hdecreases with increasing ξ , which is caused by an increase in the degree of the statistical homogeneity of the pump beam. Figure 6 presents the dependences $\langle h(R) \rangle$ for different ξ averaged over five realisations of the pump beam. As in the case of an ideal Gaussian beam [11], for a speckle beam $\langle h \rangle$ increases with increasing R. However, for one and the same laser beam reflectance, the PC quality decreases with increasing ξ . This decrease is especially significant under near-threshold conditions and has a monotonic character, without any peculiarities at small values of ξ . The obtained results agree with the data of the experiments [10], in which the monotonic decrease in the PC quality was recorded when pump beam aberrations increased (and R decreased, respectively) in the case of a deep SBS saturation. If for $\xi = 1$ the high PC quality (h > 90 %) is achieved for R > 80 %, for $\xi = 10 R$ should be larger than 90% - 95% to provide a high PC quality.



Figure 5. Dependences of the PC coefficient *h* on the reflectance *R* for $\xi = 2$ (a) and 10 (b) for different realisations of the pump beam containing speckle structures.



Figure 6. Dependences of the average PC coefficient $\langle h \rangle$ on the reflectance *R* for $\xi = 1$ (1), 2 (2), 5 (3) and 10 (4).

Calculations in the case of a further increase in ξ require significant computational resources, therefore they were presented within the framework of a linear stationary SBS model in two-dimensional and three-dimensional approximations. The gain increment of the Stokes radiation in the calculations was set equal to ~ 25 , which is typical of the linear SBS [2]. Figure 7 shows the dependences $\langle h(\xi) \rangle$ obtained for the SBS medium length exceeding the beam waist length by several times and representing the averaging over the statistical ensembles of realisations of the pump beam and noise Stokes radiation. One can see that the values of $\langle h \rangle$ in the two-dimensional case serve as an upper estimate for values obtained in three-dimensional calculations. For $\xi = 1 - 10$ the results well agree with the data determined by using the complete SBS model under the near-threshold conditions (see Fig. 6). When ξ is increased to 100 - 200 the quantity $\langle h \rangle$ monotonically decreases down to 15% but if the SBS saturation increases, it will increase.



Figure 7. Dependences of the average PC coefficient $\langle h \rangle$ on ξ for the twodimensional (\Box) and three-dimensional (**\blacksquare**) models of the medium upon linear stationary SBS.

Therefore, an increase in the pump radiation divergence leads both to an increase in the SBS threshold and to the deterioration of the PC quality of a focused beam. It is reasonable only in connection with the danger of the development of competing nonlinear optical processes in the SBS medium (striction self-focusing, thermal self-defocusing, optical breakdown), which can appear at large reflectances and lead to a disruption in the SBS. To obtain a high-quality PC of high-power laser radiation not only at large but also at small and moderate reflectances, it is necessary to use SBS mirrors of a more complex configuration [11, 15–17].

4. Conclusions

The dependence of the PC quality of a focused laser pump beam containing speckle structures upon SBS on the excesses of the angular divergence over the diffraction limit ξ has been studied numerically and theoretically in this paper. The dependences of the reflectance R and the PC coefficient h of laser radiation on the pump power for different ξ have been obtained by using the three-dimensional nonstationary SBS model [11]. It is found that for the given reflectance of laser radiation the PC quality monotonically decreases with increasing the pump beam divergence. The PC quality for the fixed divergence increases with increasing the reflectance. It is shown that the PC quality close to the ideal one (h > 90%) of a focused beam containg speckle structures is achieved at deep saturation (R > 90% - 95%) when there is a risk of the development of competing parasitic nonlinear processes in the SBS medium. This risk can be reduced by embedding a phase plate in the SBS mirror scheme and by increasing the pump beam divergence but due to a decrease in the PC coefficient.

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