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Amplification of short light pulses with a spherical wavefront

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***Abstract*. The propagation of a conical light beam through an optical amplifier is considered. The theoretical analysis is based on an analogue of the system of Frantz – Nodvik equations taking into account the specific character of spherical waves. The conditions of applicability of reduced equations (in slowly-varying amplitude approximation) for describing spherical waves propagating in a nonlinear medium are formulated. The peculiarities of amplification for different values of the beam divergence are compared. The change in the transverse structure of a diverging beam, which appears due to the amplification saturation, is studied.**

Keywords: *optical quantum amplifier, conical light beam, two-level resonance medium, femtosecond pulses, photochemical laser.*

1. Introduction

Many recent studies have been devoted to the development of high-intensity optical femtosecond radiation sources (see review [1] and references therein). In these studies, along with the approach based on application of solid-state optical quantum amplifiers, a new direction has been outlined at present, which is related to the use of gas photochemical amplifiers in the final stages [2 – 7]. To achieve high powers in such amplifiers, multipass schemes of the type of optical traps [3 – 5] or unstable confocal resonators [6, 8] are used.

The output radiation parameters of laser systems were calculated by analysing the interaction of the light field with the amplifying medium in the saturation regime. For a number of optical schemes, this analysis can be performed using the results obtained by Frantz and Nodvik [9] for an intense short pulse with a plane wavefront. If the amplifier scheme uses an unstable resonator, the saturation effect should be considered for spherical waves. Note that this problem attracted the attention of researchers in the past in connection with the proposal to apply in the experiment solid-state amplifying elements in the form of faucets or a set of discs with successively increasing diameters [10 – 12]. Light beams with variable diameters were also considered in [13] during investigations of the stimulated Raman scattering. The specific features of amplification of diverging beams were pointed out in papers [10 – 13], where energy balance equations were used. However, the gains, which are calculated numerically, were determined for some specific variants corresponding mainly to either weak or very strong signals.

In this paper we calculate as an example the parameters of amplifiers on the XeF(C – A), Kr_2F, Xe_2Cl active gases, of interest being the amplifier parameters for which it is impossible to use the results available in the literature even for rough estimates of the output radiation energy. First of all, we will consider the applicability of balance equations for describing the propagation of spherical waves in a nonlinear medium. We will specify the conditions under which it becomes possible to describe spherical fields (strictly speaking, multicomponent fields) with the help of simplified reduced equation. In addition, we will present the reduced equations in the form, which will allow one to calculate the transverse structure deformation of a light beam caused by the saturation effect.

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2. Propagation of a conical light beam in an amplifying medium

The equation

$$\nabla \times \nabla \times \boldsymbol{E} + \frac{1}{c^2}\frac{\partial^2 \boldsymbol{D}}{\partial t^2} = 0 \qquad (1)$$

follows from Maxwell's equations in the absence of free charges and currents. Here, $\boldsymbol{E}$ is the electric field strength; $\boldsymbol{D} = \varepsilon \boldsymbol{E}$ is the electric induction vector (we assume that the permittivity ε is a scalar function of the quantity $\boldsymbol{EE}^*$). We assume that this function can be represented in the form

$$\varepsilon = \varepsilon_0 + \tilde{\varepsilon}, \qquad (2)$$

where ε_0 is a constant and the complex quantity $\tilde{\varepsilon}$ ($|\tilde{\varepsilon}| \ll \varepsilon_0$) is the contribution of a laser transition to the permittivity. Only $\tilde{\varepsilon}$ depends on the saturation, $\tilde{\varepsilon}$ changing in time and space only as a function of $\boldsymbol{EE}^*$. Because we will consider conical beams below, it is reasonable to use spherical coordinates r, θ, φ (r is the distance from the coordinate origin, θ is the polar angle and φ is the azimuth angle) and the field components E_r, E_θ, E_φ. In this case, the quantity of r will correspond to the radius of the wavefront curvature of a spherical wave, which we will consider below. In a number of problems in electrodynamics it is convenient to use not fields but vector potentials. If ε is a constant, the full description is given with the help of vector potentials for TM and TE modes, which, in turn, can be expressed in terms of two Hertz functions. Expressions for the fields in terms of the Hertz functions can be found in monographs [14, 15]. However, in our case, ε is not a constant because the saturation effect leads to the dependence of $\tilde{\varepsilon}$ on the spatial and temporal coordinates. Because of this, the components of the potentials obey more complicated equations than the equations for a homogeneous medium and the calculation procedure described in [15] is not applicable. Due to difficulties appearing in the study of the most general case, we will use a number of restrictions which correspond to the conditions typical of an optical quantum amplifier.

We will consider travelling waves with a carrier frequency ω_0, whose spatial oscillations are described by the wave number $k = \omega_0\sqrt{\varepsilon_0}/c$ and the envelope is characterised by the time scale τ_0 and the spatial scale l. Let us introduce also the transverse scale of the field change $l_\perp = r\tilde{\theta}$, where $\tilde{\theta}$ is the angular width of the initial beam (if the beam has a fine angular structure, the scale of this structure will be also involved in the estimate of $l_\perp$). We assume that the conditions $l^{-1} \ll k$, $\tau_0^{-1} \ll \omega_0$, $l_\perp^{-1} \ll k$ are fulfilled. The last inequality takes place if the value of r is not too small ($r > \lambda/\tilde{\theta}$), which is undoubtedly fulfilled in real amplifiers. This condition allows us to omit the derivatives with respect to the angular variables in equation (1) and obtain the set of equations:

$$\frac{1}{r}\frac{\partial^2}{\partial r^2}[rE_\theta(r,\theta,\varphi,t)] - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}D_\theta(r,\theta,\varphi,t) = 0, \tag{3}$$

$$\frac{1}{r}\frac{\partial^2}{\partial r^2}[rE_\varphi(r,\theta,\varphi,t)] - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}D_\varphi(r,\theta,\varphi,t) = 0, \tag{4}$$

$$-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}D_r(r,\theta,\varphi,t) = 0. \tag{5}$$

The inequality $\tau_0^{-1} \ll \omega_0$ together with the assumption with respect to the contribution of the resonance transition in ε allows us to reject the quantities of the order $(\tilde{\varepsilon}/\varepsilon_0)(\omega_0\tau)^{-1}$ in calculations of the temporal derivatives in expressions (3), (4). After this, expressions (3), (4) take the form

$$\frac{1}{r}\frac{\partial^2}{\partial r^2}[rE_j(r,\theta,\varphi,t)] - \frac{1}{c^2}\varepsilon_0\frac{\partial^2}{\partial t^2}E_j(r,\theta,\varphi,t) = \frac{1}{c^2}\tilde{\varepsilon}\frac{\partial^2}{\partial t^2}E_j(r,\theta,\varphi,t). \tag{6}$$

Here, the subscript j is the projection of the $\boldsymbol{E}$ vector in the directions θ or φ. It follows from (5) that under the above assumptions the component E_r vanishes. It is equation (6) that will be the basis of the further discussion.

Let us make several remarks concerning expression (6). Note, first of all, that it contains only derivatives with respect to r; angular variables enter into (6) only as parameters. In this case we have an analogy with the description of a quantum-mechanic wave function near a scattering centre [16]. In some problems of the theory of scattering, the structure of the wave function changes only depending on the distance to the scattering centre. In our case, however, the difference is induced by the amplification nonlinearity. Expression (6) under study involves the quantity $\tilde{\varepsilon}$ depending on $\boldsymbol{EE}^*$, i.e is nonlinear. Due to this, expression (6) contains terms describing the change of the angular structure in the saturation regime.

Expression (6) cannot be used to analyse the diffraction effects, because it neglects the derivatives with respect to the angular variables. Nevertheless, the role of diffraction is not always significant for diverging beams. Thus, if the initial divergence angle noticeably exceeds the diffraction angle, in many cases the account for diffraction leads only to a weak relative change in the total beam divergence.

Unlike diffraction effects, the saturation effects, as mentioned above, are taken into account in expression (6), which can be used to consider the changes in the angular structure during the nonlinear amplification, what is demonstrated in section 4.3.

Thus, we will take expression (6) as the initial one. Let us transform it with the help of the substitution

$$E_j(r,\theta,\varphi,t) = \frac{1}{r}U_j(r,\theta,\varphi,t). \tag{7}$$

In this case, expression (6) takes the form

$$\frac{\partial^2 U_j}{\partial r^2} - \frac{1}{c^2}\varepsilon_0\frac{\partial^2 U_j}{\partial t^2} = \frac{1}{c^2}\tilde{\varepsilon}\frac{\partial^2 U_j}{\partial t^2}. \tag{8}$$

We will use below the procedure based on the introduction of slowly varying amplitudes (see, for example, monographs [17, 18]) and use it to simplify equation (8). By representing the function U_j as a product of a slowly varying function u_j and oscillating multiplier $\exp(-\mathrm{i}\omega_0 t + \mathrm{i}kr)$, i.e. in the form

$$U_j(r,\theta,\varphi,t) = u_j(r,\theta,\varphi,t) \times \exp(-\mathrm{i}\omega_0 t + \mathrm{i}kr), \quad k = \frac{\omega_0}{v}, \tag{9}$$

where

$$v = \frac{c}{\sqrt{\varepsilon_0}} \tag{10}$$

is the phase velocity and assuming that the condition

$$\left|\frac{\partial^2 u_j}{\partial r^2}\right| \ll k\left|\frac{\partial u_j}{\partial r}\right|, \quad \left|\frac{\partial^2 u_j}{\partial t^2}\right| \ll \omega_0\left|\frac{\partial u_j}{\partial t}\right|, \tag{11}$$

is fulfilled, we find from (8)

$$2\mathrm{i}k\frac{\partial u_j}{\partial r} + 2\mathrm{i}\omega_0\frac{\varepsilon_0}{c^2}\frac{\partial u_j}{\partial t} = -\tilde{\varepsilon}\frac{\omega_0^2}{c^2}u_j(r,t). \tag{12}$$

Here, we can derive the equation for the product $u_j u_j^*$:

$$\frac{\partial}{\partial r}u_j u_j^* + \frac{\sqrt{\varepsilon_0}}{c}\frac{\partial}{\partial t}u_j u_j^* = -\frac{\mathrm{Im}\,\tilde{\varepsilon}}{\varepsilon_0}\frac{\omega_0\sqrt{\varepsilon_0}}{c}u_j u_j^*. \tag{13}$$

For an amplifying medium we have $\mathrm{Im}\,\tilde{\varepsilon} < 0$, the quantity $|\mathrm{Im}\,\tilde{\varepsilon}|$ being proportional to the gain. Let us use the notation

$$-\frac{\operatorname{Im}\tilde{\varepsilon}}{\varepsilon_0}=\beta\frac{N}{N_0}, \tag{14}$$

where N is the density of the inverse population; N_0 is the same density in the absence of saturation. Then, we can obtain from (13) an equation for the quantity $\sum_j u_j u_j^*$, which is the sum of squares of moduli of all the components u_j. By using (14) and (10), based on (13) we find

$$\left(\frac{\partial}{\partial r}+\frac{1}{v}\frac{\partial}{\partial t}\right)\sum_j u_j u_j^*=\beta\frac{\omega_0}{v}\frac{N}{N_0}\sum_j u_j u_j^*. \tag{15}$$

Consider now the electric field intensity I summed over all the components:

$$\sum_j E_j E_j^*=I. \tag{16}$$

By using (7), (9) and (15), we derive the relation

$$\sum_j u_j u_j^*=Ir^2, \tag{17}$$

and equation (15) has the from

$$\left(\frac{\partial}{\partial r}+\frac{1}{v}\frac{\partial}{\partial t}\right)(Ir^2)=\beta\frac{\omega_0}{v}\frac{N}{N_0}Ir^2. \tag{18}$$

Note that equation (18) can be derived in a simpler way if not to emphasize the difference between the electric and magnetic field amplitudes and to neglect the longitudinal components of the fields, whose contribution is only significant near the curvature centre of a divergent beam. In the course of deriving we established the conditions of applicability of expression (18): the necessity to fulfill the inequality $r > \lambda/\theta$. In addition, expression (18) allows one to study not only the dependence of the radiation intensity on the radial coordinate but also its parametric dependence on the angular variables.

The interaction between an ultrashort pulse having a diverging wavefront and a nonlinear amplifying medium will be analysed below on the basis of expression (18), which will be considered together with the equation for the medium.

3. Saturation of amplification in the case of a conical beam

Passing to the analysis of the interaction of a light field with an active medium, instead of the permittivity we will introduce the parameter, which is more convenient for discussion, i.e. amplification cross section

$$\sigma=\beta\frac{\omega_0}{vN_0}. \tag{19}$$

By using (19), expression (18) takes the from

$$\left(\frac{\partial}{\partial r}+\frac{1}{v}\frac{\partial}{\partial t}\right)(Ir^2)=\sigma NIr^2. \tag{20}$$

The derived expression formally resembles the first equation of the Frantz–Nodvik system. The difference consists in the fact that it is written here for the variable Ir^2 and not for variable I as was done in paper [9].

The expression for the inverse population density under conditions of a slow (with respect to the radiation duration) relaxations of levels, as is known, has the from

$$\frac{\partial N}{\partial t}=-\frac{\sigma v}{\hbar\omega_0}\left(1+\frac{g_2}{g_1}\right)NI, \tag{21}$$

where $g_1\,(g_2)$ is the statistical weight of the lower (upper) state of the laser transition. The case of nondegenerate levels, when $g_1=g_2=1$ was studied in paper [9]. For the transitions of excimer molecules under study, the lower state is dissociative, and we should set $g_2/g_1=0$ in (21). Then, the equation for N will have the from

$$\frac{\partial N}{\partial t}=-\frac{\sigma v}{\hbar\omega_0}NI. \tag{22}$$

Expressions (20, (22) represent a basic system of equations for problems on spherical wavefront pulses. In accordance with the Frantz–Nodvik method, we will introduce, first of all, a local time coordinate $\tau=t-r/v$, where r is a spatial coordinate. In this case, expressions (20), (22) will have the form

$$\frac{\partial}{\partial r}(Ir^2)=\sigma NIr^2, \tag{23}$$

$$\frac{\partial N}{\partial \tau}=-\frac{\sigma v}{\hbar\omega_0}NI. \tag{24}$$

Consider briefly the Frantz–Nodvik method (taking into account the peculiarities of our case). By combining expressions (23) and (24), we obtain

$$\frac{\partial}{\partial r}(Ir^2)+\frac{\hbar\omega_0}{v}r^2\frac{\partial N}{\partial\tau}=0. \tag{25}$$

In addition, by integrating (24), we derive

$$N(r,\theta,\varphi,\tau)=N(r,\theta,\varphi,-\infty)$$
$$\times\exp\left[-\frac{\sigma v}{\hbar\omega_0}\int_{-\infty}^{\tau}I(r,\theta,\varphi,\tau')\mathrm{d}\tau'\right]. \tag{26}$$

Let us introduce the notation

$$y(r,\theta,\varphi,\tau)=\frac{\sigma v}{\hbar\omega_0}\int_{-\infty}^{\tau}I(r,\theta,\varphi,\tau')\mathrm{d}\tau'. \tag{27}$$

Then, expression (26) can be written in the from:

$$N(r,\theta,\varphi,\tau)=N(r,\theta,\varphi,-\infty)\exp[-y(r,\theta,\varphi,\tau)], \tag{28}$$

after that expression (25) takes the from

$$\frac{\partial}{\partial r}\left(\frac{\partial y}{\partial\tau}r^2\right)+\sigma r^2\frac{\partial N}{\partial\tau}=0. \tag{29}$$

By changing the differentiation order in the first term in the left-hand side of Eqn (29) and integrating (29) in τ, we obtain

$$\frac{\partial}{\partial r}\{[y(r,\theta,\varphi,\tau)-y(r,\theta,\varphi,-\infty)]r^2\}$$

$$+\sigma r^2[N(r,\theta,\varphi,\tau)-N(r,\theta,\varphi,-\infty)]=0. \quad (30)$$

We will use below expression (28) and assume that $y(r,\theta,\varphi,-\infty)=0$, $N(r,\theta,\varphi,-\infty)=N_0$ (this means that for $\tau=-\infty$ radiation is absent and the medium is initially homogeneous). In this case, expression (30) is reduced to a simpler form:

$$\frac{\partial y}{\partial r}+\frac{2}{r}y=N_0\sigma[1-\exp(-y)]. \quad (31)$$

This equation should be solved together with the boundary condition specifying the shape and amplitude of a pulse at the input to the amplifying medium. Denote the initial value of the radius of the wavefront curvature by r_0 (at the input to the amplifying medium). For $r=r_0$ the parameters of the input pulse are known, i.e. the function $I(r_0,\theta,\varphi,\tau)$ is known. In this case, due to relation (27) the function $y(r_0,\theta,\varphi,\tau)$ is also known. The function

$$y(r_0,\theta,\varphi,\tau)=\frac{\sigma}{\hbar\omega_0}\int_{-\infty}^{\tau}I(r_0,\theta,\varphi,\tau')\mathrm{d}\tau'$$

is a boundary condition for expression (31), i.e. represents the function y at the input to the medium. One can see from Eqn (31) that the local time τ enters it only as a parameter. By solving the problem we can find the dependence $y(r,\theta,\varphi,\tau)$ for the arbitrary shape of the input pulse and then by differentiating it, determine the function $I(r,\theta,\varphi,\tau)$, i.e. obtain information on the changes both in the intensity and the pulse shape throughout the entire amplifier. However, a simpler problem – the quantitative estimate of the increase in the total energy contained in the pulse – is of interest for practical applications. The next section of this paper is devoted to this estimate.

4. Peculiarities of ultrashort pulse amplification in the case of a spherical wavefront

4.1 Dependence of the energy density on the coordinate

Let us analyse the changes in the electromagnetic energy density of a pulse during its propagation in the amplifier. Based on Eqn (31), we will consider the square of the modulus of the electric field strength $I(r,\theta,\varphi,\tau)$ integrated over the entire pulse duration. Let us introduce the notation

$$Y(r,\theta,\varphi)=y(r,\theta,\varphi,\infty)=\frac{\sigma}{\hbar\omega_0}\int_{-\infty}^{\infty}I(r,\theta,\varphi,\tau')\mathrm{d}\tau'. \quad (32)$$

The equation for the quantity Y

$$\frac{\partial Y}{\partial r}+\frac{2}{r}Y=N_0\sigma[1-\exp(-Y)] \quad (33)$$

has, in fact, the same structure as Eqn (31) presented above for the quantity y.

Note that significant saturation corresponds to $Y=1$ or

$$\frac{\sigma}{\hbar\omega_0}\int_{-\infty}^{\infty}I(r,\theta,\varphi,\tau')d\tau'=1,$$

The contribution of stimulated transitions to the field energy, as seen from (33), is determined by the quantity $N_0\sigma[1-\exp(-Y)]/Y$ so that the noticeable depletion of the working level population really corresponds to the quantities $Y\geqslant 1$.

In this section as well as in section 4.2 we will consider the case when the energy density of the input radiation homogeneously fills the cone with the opening angle $2\theta_0$ so that $Y(r_0,\theta,\varphi)=Y_0$ for $\theta\leqslant\theta_0$ and $Y(r_0,\theta,\varphi)=0$ for $\theta>\theta_0$.

Let us present a number of solutions of Eqn (33) in a graphic form. For calculations we chose the parameters corresponding to the photochemical amplifiers designed at present. We used the unsaturated gain $N_0\sigma=0.05$ cm^{-1}, the radius $r_0=2$ cm was chosen as the initial curvature radius in this series of calculations. The amplifier length was set equal to $r_f-r_0=100$ cm, where r_f is the curvature radius at the amplifier input. We used several values of the input energy density corresponding to the values of Y_0 equal to 0.1, 0.5 and 1.0 at the amplifier input. The dependences of Y on the dimensionless coordinate $N_0\sigma r$ (the length is divided by the amplification length) are presented in Fig. 1a. They demonstrate a nonmonotonic dependence of Y on the coordinate, which is related to the competition of two factors: the beam amplification and expansion. The most noticeable intensity attenuation due to the beam expansion occurs for the smallest values of the curvature radius r.

Note that in the limiting case of the weak saturation, i.e. for $Y\ll 1$, the solution of Eqn (33) has the from

$$Y=Y_0\frac{r_0^2}{r^2}\exp[N_0\sigma(r-r_0)], \quad (34)$$

and in the case of strong saturation, i.e. for $Y\gg 1$, we have

$$Y=Y_0\frac{r_0^2}{r^2}+\frac{N_0\sigma r}{3}-\frac{N_0\sigma r_0^3}{3r^2}. \quad (35)$$

The structure of these two expressions also indicates the nonmonotonic dependence of the energy density on the amplification length. The transition from the region of the intensity attenuation to the region of its growth is determined by the condition $N_0\sigma r>2Y/[1-\exp(-Y)]$. The limiting cases and the nonmonotonic change in the intensity are studied in papers [11, 12].

Now we include into solution of Eqn (33) the factor r^2/r_0^2, which accounts for the beam expansion The new function Yr^2/r_0^2 will monotonically depend on the coordinate. Figure 1b shows a monotonic increase in the function Yr^2/r_0^2 with increasing the variable $N_0\sigma r$.

From the formal point of view, the presented expressions as the dependences in Figs 1a and b give a complete description of the amplifier properties because the equation for the amplifier contains the only independent variable, the product $N_0\sigma r$, and the boundary condition is set by the only parameter $Y(r_0)$. However, the results of this consideration do not allow one to obtain in the explicit form the

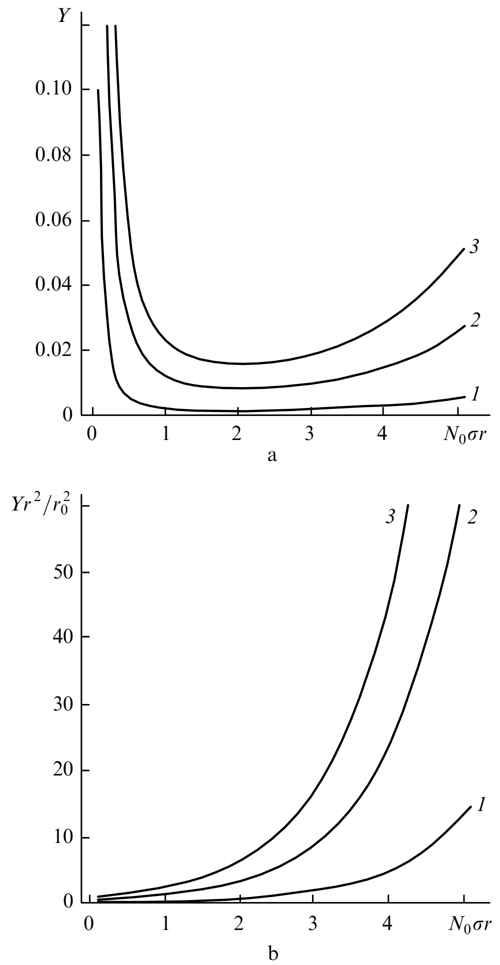

Figure 1. Dependences of the time-integrated square of the modulus of the electric field strength Y in the units of the saturation energy density (a) and the quantity Yr^2/r_0^2 (b) on the dimensionless radial coordinate $N_0\sigma r$ for $Y_0 = 0.1$ (*1*), 0.5 (*2*) and 1.0 (*3*).

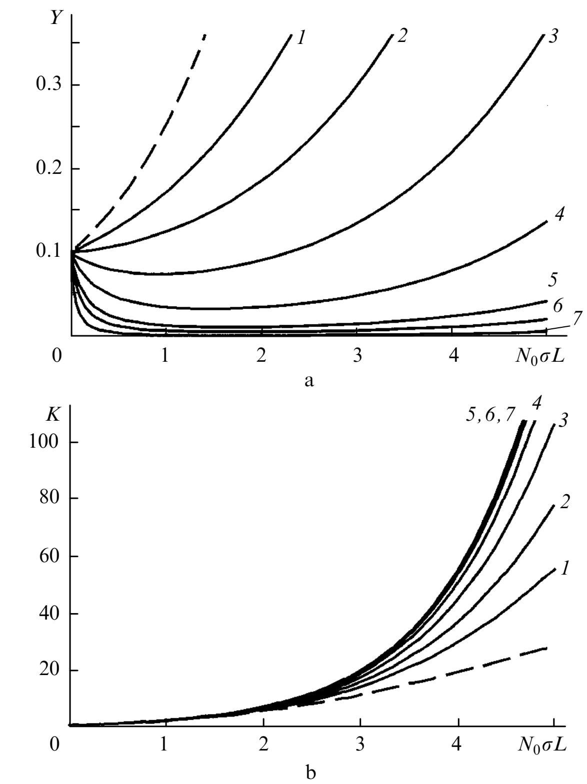

Figure 2. Dependences of the time-integrated square of the modulus of the electric field strength Y in the units of the saturation energy density (a) and the gain integrated over the time and transverse coordinate of the square of the modulus of the electric field strength K (b) on the dimensionless coordinate $N_0\sigma L \equiv N_0\sigma(r - r_0)$ for the beam divergence angles $\theta_0 = 0.625°$ (*1*), 1.25° (*2*), 2.5° (*3*), 5° (*4*), 10° (*5*), 15° (*6*) and 30° (*7*). Dashed curves correspond to the plane wavefront.

dependence of amplification on the divergence angle of the light beam. The role of this angle will be more obvious if the beam diameter at the amplifier input rather than the curvature radius is taken as its initial parameter. Let the initial beam diameter be $2a$ and the diffraction angle be $2\theta_0$. In this case the radius of the wavefront curvature at the input is $r_0 = a/\sin\theta_0$. We will construct a number of solutions of Eqn (33) by assuming that the initial diameter is $2a = 2$ cm for all the variants, the unsaturated gain is $N_0\sigma = 0.05$ cm^{-1} and the energy density in all the cases is the same and corresponds to $Y = 0.1$ (the saturation parameter is 0.1). Now we select the quantity $N_0\sigma L \equiv N_0\sigma(r - r_0)$ as an independent variable, i.e. construct the dependence on the amplifier length L by placing the point corresponding to the amplifier entrance at the coordinate origin for all variants under study. Figure 2a presents the results of the calculation. It is obvious that the nonmonotonic dependence of Y on the coordinate becomes stronger at larger divergence angles.

4.2 Dependence of the total pulse energy on the coordinate

We have dealt so far not with the total pulse energy but with the energy per unit area of the cross section. Consider now the integral characteristic, i.e. the quantity Y integrated over the transverse coordinate.

We will start by passing from the quantities under study to the conventional definitions of the energy and energy density. Denote the energy density related to the electric field components of the pulse by w. By using a standard expression for the energy density of a quasi-monochromatic field and expression (16), we can write

$$(r,\tau) = \frac{1}{16\pi}\varepsilon_0 \sum_j E_j E_j^* \equiv \frac{1}{16\pi}\varepsilon_0 I(r,\tau). \tag{36}$$

For the total energy W, we have

$$W = \int\!\!\int\!\!\int w(r,\tau) r^2 \mathrm{d}r \mathrm{d}\Omega = \frac{\Omega_0}{16\pi}\int w r^2 \mathrm{d}r, \tag{37}$$

where $\Omega_0 = 2\pi(1 - \cos\theta_0)$ is the solid angle corresponding to the conical beam. Note that the energy density w is concentrated in the spherical layer, whose approximate thickness is $\tau_p v$, where τ_p is the pulse duration. Because this quantity is small ($\tau_p v \equiv l \ll r$), the radius r can be treated constant in the localisation region of the w function. For this reason, the multiplier r^2 in (37) can be factored out of the integral. The other multiplier (w) depends on r mainly through $\tau = t - r/v$. Therefore, we can set with a good accuracy in expression (37) $\int w r^2 \mathrm{d}r = r^2 v \int w \mathrm{d}\tau$. By using this simplification and taking into account expressions (36) and (32) relating w, y, Y, we obtain from (37) the expression for the pulse energy:

$$W = \frac{\Omega_0 \varepsilon_0}{16\pi} \frac{\hbar\omega_0}{\sigma} r^2 Y. \quad (38)$$

For $r = r_0$, we find from (38) the pulse energy at the amplifier input

$$W_0 = \frac{\Omega_0 \varepsilon_0}{16\pi} \frac{\hbar\omega_0}{\sigma} r_0^2 Y_0. \quad (39)$$

According to (38), (39), the energy gain K can be written in the form

$$K = \frac{W}{W_0} = \frac{r^2}{r_0^2} \frac{Y}{Y_0}. \quad (40)$$

If to pass from the radius of the wavefront curvature to the distance $L = r - r_0$ measured from the amplifier entrance, we obtain the expression for the gain:

$$K = \frac{(r_0 + L)^2}{r_0^2} \frac{Y(r_0 + L)}{Y(r_0)}. \quad (41)$$

Figure 2b shows the dependence of the gain on the distance measured from the amplifier entrance for some angles of the beam divergence. Unlike the local energy density presented in Fig. 2a, the integral gain monotonically increases with increasing the amplifier length. Stronger amplification corresponds to larger divergence angles. One can see from Fig. 2b that for the divergence angles larger than 5°, an increase in the angle does not increase the energy gain. For comparison Fig. 2b shows also the dependence of the gain for a pulse with a plane wavefront. In this case, the gain has significantly lower values than in the case of the diverging beams.

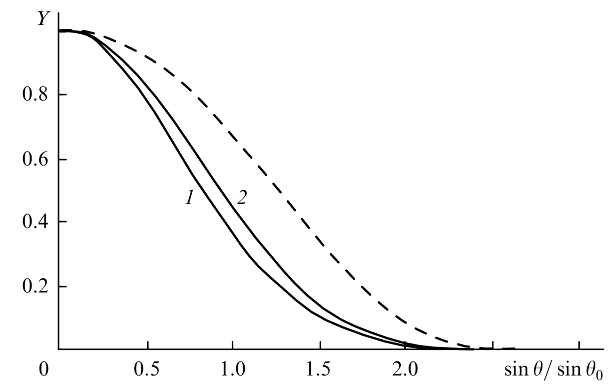

Figure 3. A transverse structure of a diverging beam at the amplifier input (*1*) and output for the divergence angle 2.5° and the gain $N_0\sigma L = 5.0$ (*2*). The dashed curve shows the transverse structure of the amplified light beam with the plane wavefront.

4.3 Change in the transverse profile of intensity radiation upon amplification

Consider an input pulse having a smooth dependence of the radiation intensity on the transverse coordinate. Assume that the transverse structure of initial emission has the from $Y(r, \theta, \varphi) = Y_0 \exp(-\sin^2\theta/\sin^2\theta_0)$, the energy density on the beam axis being 0.1 in the units of the energy density saturation. We will use the same parameters as in the previous calculations: $2a = 2$ cm, $L = 100$ cm, $\sigma_0 N = 0.05$ cm^{-1}. We will set the angle θ_0 equal to 2.5°. Figure 3 shows the initial energy density profile and the profile at the amplifier output. The shape acquired by the initial profile upon amplification of a plane–parallel beam ($\theta_0 \to 0$) is shown for comparison. One can see that the transverse structure is distorted much less in the diverging beam than that in the beam with a plane wavefront.

5. Conclusions

The present paper is related to the problem of obtaining high-power ultrashort light pulses and is aimed at establishing a theoretical approach to the optimisation of the amplifier design. Consider briefly the main results of the paper.

Conditions for the applicability of reduced equations for spherical waves propagating in a nonlinear medium have been obtained.

Based on the equation for spherical waves and equations for a two-level resonance medium whose relaxation time of the inverse population exceeds the pulse duration, the peculiarities of the change in the energy density in the amplifier have been analysed.

It has been shown that the gain integrated over the transverse coordinate proves to be larger for larger divergence angles and (for the same input beam cross sections) can exceed manifold the gain of a pulse with a plane wavefront.

The deformation of the transverse intensity profile of a light beam in the amplifier has been studied. It has been shown that distortions in the diverging beam prove to be weaker than in the plane–parallel beam. Variation in the initial divergence angle allows one to change the transverse profile of the output radiation intensity.

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