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Distributions of temperature and thermoelastic stresses in a thin disk active element with an arbitrary optical density

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Abstract. **Stationary distributions of temperature and thermoelastic stresses (thermal tensions) are obtained in an active disk element with an arbitrary optical density upon a double-pass pumping. It is shown that the temperature distribution is determined by the sum of three terms: two exponential and a linear one, the exponential terms being preserved with changing the boundary conditions while the linear term producing no thermal tensions changes. It is found that thermal tensions decrease with increasing the absorption coefficient both for the constant thickness of the disk and for the constant optical density. The assessed values of the temperature are calculated during the local heating of a thin disk when the diameter of the pumped region is comparable with the disk thickness.**

Keywords: *active disk elements, thermoelastic stresses.*

Disc solid-state lasers have been developed for a long time [1]. To absorb pumping more completely, a multipass scheme is used in these lasers, which significantly complicates the laser scheme. Based on the analysis of distributions of temperature and thermoelastic stresses (thermal tensions) and on the experimental results, it was proposed in [2] to use optically dense media for manufacturing active disk elements, in which pump radiation is almost completely absorbed per one – two passes.

In this paper we calculated the stationary distributions of temperature and thermoelastic stresses in active disk elements with an arbitrary optical density.

Consider a thin disk pumped from one side ($x = 0$) and cooled from both sides (for $x = 0$ and $x = d$, where d is the disk thickness). We will denote the temperatures of media by t_1 and t_2, which cool the disk surfaces with the coordinates $x = 0$ and $x = d$, respectively. We will consider the double-pass scheme, when exciting radiation propagating through the crystal is incident on a mirror with the reflectivity R, and after reflection from it, propagates through the crystal in the backward direction.

The stationary heat conduction equation for this particular case has the form

$$\frac{\partial^2 t}{\partial x^2} = -\frac{\eta_t P_p k}{\lambda S}\left(e^{-kx} + Re^{kx-2kd}\right), \tag{1}$$

where t is the temperature; P_p is the pump power propagated through the front face of the disk after partial reflection from it; k and λ are absorption and heat conduction coefficients, respectively; S is the disk area; η_t is the fraction of the absorbed pump power transformed to heat.

The boundary conditions (of the third kind) can be written in the form:

$$\lambda \frac{\partial t}{\partial x}\bigg|_{x=0} = a[t(0) - t_1], \tag{2}$$

$$\lambda \frac{\partial t}{\partial x}\bigg|_{x=d} = -b[t(d) - t_2], \tag{3}$$

where a and b are the coefficients of the heat exchange with cooling media having the temperatures t_1 and t_2; $t(0)$ and $t(d)$ are the temperatures of the disk surfaces with coordinates $x = 0$ and $x = d$. Below, we will assume for definiteness in specific calculations that the pumped disk surface ($x = 0$) is cooled by air or water, while the other side of the disk ($x = d$) is cooled by water.

The heat power released in the crystal is

$$P_t = \eta_t P_p\left(1 - e^{-kd}\right)\left(1 + Re^{-kd}\right). \tag{4}$$

In the stationary regime, heat released in the disk is equal to heat released for the same time to the cooling media:

$$\eta_t P_p\left(1 - e^{-kd}\right)\left(1 + Re^{-kd}\right) = S\{a[t(0) - t_1] + b[t(d) - t_2]\}. \tag{5}$$

The amount of heat released within 1 s through the face $x = 0$ is

$$Q_1 = P_t \frac{a[t(0) - t_1]}{a[t(0) - t_1] + b[t(d) - t_2]} = \lambda S \frac{\partial t}{\partial x}\bigg|_{x=0}, \tag{6}$$

and through the face $x = d$ is

$$Q_2 = P_t \frac{b[t(d) - t_2]}{a[t(0) - t_1] + b[t(d) - t_2]} = -\lambda S \frac{\partial t}{\partial x}\bigg|_{x=d}. \tag{7}$$

The solution of expression (1) has the from

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$$t(x)=\frac{\eta_t P_p}{\lambda S}\left\{\frac{1}{k}\left(e^{-kd}-e^{-kx}\right)+(d-x)\left[1-Re^{-2kd}\right.\right.$$
$$\left.-\frac{a[t(0)-t_1](1-e^{-kd})}{a[t(0)-t_1]+b[t(d)-t_2]}\left(1+Re^{-kd}\right)\right]$$
$$\left.+\frac{Re^{-kd}}{k}\left[1-e^{k(x-d)}\right]\right\}+t(d). \quad (8)$$

Solution (8) consists of three terms: two exponential terms,

$$-\frac{\eta_t P_p}{k\lambda S}\left[e^{-kx}+\left(Re^{-2kd}\right)e^{kx}\right], \quad (9)$$

and the linear term. Because the boundary conditions affect only the linear term, which, is as known [3, 4], does not cause thermoelastic stresses, we can conclude that in the stationary case the boundary conditions (naturally, reasonable ones, which do not lead, for example, to the crystal melting) do not influence thermal tensions appearing in a free thin disk.

Let us show that for small optical densities $D=kd$, solution (8) is transformed into a known parabolic solution typical of the uniform distribution of heat release sources. The heat conduction equation for this case has the form

$$\frac{\partial^2 t}{\partial x^2}\simeq-\frac{\eta_t P_p k(1+R)}{\lambda S}, \quad (10)$$

and the boundary conditions are

$$\lambda\left.\frac{\partial t}{\partial x}\right|_{x=0}=a[t(0)-t_1], \quad (11)$$

$$-\lambda\left.\frac{\partial t}{\partial x}\right|_{x=d}=b[t(d)-t_2]. \quad (12)$$

The solution of expression (10) has the form

$$t(x)=\frac{\eta_t P_p k(1+R)}{\lambda S}\left\{\frac{d^2}{2}-\frac{x^2}{2}\right.$$
$$\left.+(x-d)\frac{a[t(0)-t_1]d}{a[t(0)-t_1]+b[t(d)-t_2]}\right\}+t(d). \quad (13)$$

Let us show that for $D\ll 1$ solution (8) obtained above transforms into (13). For this, we will rearrange the terms in braces in (8):

$$\frac{1}{k}\left(e^{-kd}-e^{-kx}\right)+(d-x)+Re^{-2kd}(x-d)+\frac{Re^{-kd}}{k}$$
$$\times\left[1-e^{k(x-d)}\right]+\frac{a[t(0)-t_1]}{a[t(0)-t_1]+b[t(d)-t_2]}$$
$$\times\left(1-e^{-kd}\right)\left(1+Re^{-kd}\right)(x-d). \quad (8')$$

Expression (8′) consists of five terms. The sum of the first two terms for small D is

$$\frac{1}{k}\left(e^{-kd}-e^{-kx}\right)+(d-x)\approx\frac{1}{k}\left(1-kd+\frac{k^2d^2}{2}-1+\right.$$
$$\left.+kx-\frac{k^2x^2}{2}\right)+(d-x)=k\left(\frac{d^2}{2}-\frac{x^2}{2}\right), \quad (14)$$

and the sum of the second two terms is

$$R\left\{e^{-2kd}(x-d)+\frac{e^{-kd}}{k}\left[1-e^{k(x-d)}\right]\right\}$$
$$\approx R\left\{(1-2kd)(x-d)+\frac{1-kd}{k}\left[kd-kx\right.\right.$$
$$\left.\left.-\frac{k^2(x-d)^2}{2}\right]\right\}=kR\left(\frac{d^2}{2}-\frac{x^2}{2}\right). \quad (15)$$

Thus, the sum of the first four terms in (8′) is

$$k\left(\frac{d^2}{2}-\frac{x^2}{2}\right)(1+R). \quad (16)$$

The last term in expression (8′) is

$$\frac{a[t(0)-t_1]}{a[t(0)-t_1]+b[t(d)-t_2]}\left(1-e^{-kd}\right)\left(1+Re^{-kd}\right)(x-d)$$
$$\approx\frac{a[t(0)-t_1]}{a[t(0)-t_1]+b[t(d)-t_2]}kd(1+R)(x-d), \quad (17)$$

and expression (8) for $D\ll 1$ transforms into the expression

$$t(x)=\frac{\eta_t P_p}{\lambda S}\left\{k(R+1)\left(\frac{d^2}{2}-\frac{x^2}{2}\right)\right.$$
$$\left.+\frac{a[t(0)-t_1]}{a[t(0)-t_1]+b[t(d)-t_2]}d(x-d)k(1+R)\right\}+t(d)$$
$$=\frac{\eta_t P_p k(1+R)}{\lambda S}\left\{\frac{d^2}{2}-\frac{x^2}{2}\right.$$
$$\left.+\frac{a[t(0)-t_1]d(x-d)}{a[t(0)-t_1]+b[t(d)-t_2]}\right\}+t(d), \quad (18)$$

which precisely coincides with the above solution of (13) for the uniform distribution of heat release sources.

We will show how to use solution (8). Let the temperatures t_1 and t_2 of cooling media and the pump power P_p be specified. To calculate $t(x)$ it is necessary to determine the temperatures $t(0)$ and $t(d)$ of disk surfaces, which enter (8). We have two equations:

$$t(0)=\frac{\eta_t P_p}{\lambda S}\left\{\frac{1}{k}\left(e^{-kd}-1\right)+d\left[1-Re^{-2kd}\right.\right.$$
$$\left.-\frac{a[t(0)-t_1]}{a[t(0)-t_1]+b[t(d)-t_2]}\left(1-e^{-kd}\right)\left(1+Re^{-kd}\right)\right]$$
$$\left.+\frac{Re^{-kd}}{k}\left(1-e^{-kd}\right)\right\}+t(d) \quad (19)$$

and expression (5). By solving (5) and (19) with respect to $t(0)$ and $t(d)$, we have

$$t(0)=\frac{1}{1+a/b+ad/\lambda}\left\{\frac{\eta_t P_p}{\lambda S}\left[\frac{e^{-kd}}{k}-\frac{1}{k}+d-dRe^{-2kd}+\right.\right.$$

$$+\frac{adSt_1}{\eta_t P_p}+\frac{R\mathrm{e}^{-kd}}{k}\left(1-\mathrm{e}^{-kd}\right)\Bigg]+\frac{\eta_t P_p\left(1-\mathrm{e}^{-}kd\right)\left(1+R\mathrm{e}^{-kd}\right)}{bS}$$

$$+\frac{at_1}{b}+t_2\Bigg\}, \tag{20}$$

$$t(d)=\frac{\eta_t P_p\left(1-\mathrm{e}^{-kd}\right)\left(1+R\mathrm{e}^{-kd}\right)-aS[t(0)-t_1]}{bS}+t_2. \tag{21}$$

By substituting the obtained values of $t(0)$ and $t(d)$ into (8), we will find the temperature distribution by the disk thickness $t(x)$.

Let us present another notation of the solution of expression (1) with boundary conditions (2) and (3):

$$t(x)=-\frac{\eta_t P_p}{\lambda Sk}\left(\mathrm{e}^{-kx}+R\mathrm{e}^{kx-2kd}\right)+C_1x+C_2,$$

where

$$C_1=ab\Bigg\{t_2-t_1+\frac{\eta_t P_p}{S}\left[\frac{\mathrm{e}^{-kd}(R-1)}{b}+\frac{\mathrm{e}^{-kd}(R+1)}{\lambda k}-\frac{1}{a}\right.$$

$$\left.+\frac{R\mathrm{e}^{-2kd}}{a}-\frac{1}{\lambda k}-\frac{R\mathrm{e}^{-2kd}}{\lambda k}\right]\Bigg\}(abd+a\lambda+b\lambda)^{-1}.$$

$$C_2=\frac{\eta_t P_p}{S}\left(\frac{1}{a}-\frac{R\mathrm{e}^{-2kd}}{a}+\frac{1}{\lambda k}+\frac{R\mathrm{e}^{-2kd}}{\lambda k}\right)+t_1+\frac{\lambda}{a}C_1$$

The coordinate $x_{\max}$ at which the temperature has a maximum, is easy to determine by equating the derivative $\mathrm{d}t/\mathrm{d}x$ to zero:

$$\frac{\mathrm{d}t}{\mathrm{d}x}=\frac{\eta_t P_p}{\lambda S}\Bigg\{\mathrm{e}^{-kx}-\left[1-R\mathrm{e}^{-2kd}-\frac{a[t(0)-t_1]}{a[t(0)-t_1]+b[t(d)-t_2]}\right.$$

$$\left.\times\left(1-\mathrm{e}^{-kd}\right)\left(1+R\mathrm{e}^{-kd}\right)\right]-R\mathrm{e}^{-2kd}\mathrm{e}^{kx}\Bigg\}=0. \tag{22}$$

By denoting the expression in square brackets by C, we obtain

$$R\mathrm{e}^{-2kd}\mathrm{e}^{2kx_{\max}}+C\mathrm{e}^{kx_{\max}}-1=0, \tag{23}$$

which yields

$$\mathrm{e}^{kx_{\max}}=\frac{-C+\left(C^2+4R\mathrm{e}^{-2kd}\right)^{1/2}}{2R\mathrm{e}^{-2kd}}, \tag{24}$$

$$x_{\max}=\frac{1}{k}\ln\left[\frac{\left(C^2+4R\mathrm{e}^{-2kd}\right)^{1/2}-C}{2R\mathrm{e}^{-2kd}}\right]. \tag{25}$$

We calculated the thermoelastic stresses σ by using the expression [3]

$$\sigma=\frac{\alpha E}{1-v}\left[-t(x)+\frac{1}{d}\int_0^d t(x)\mathrm{d}x\right.$$

$$\left.+\frac{3(x-d/2)}{2(d/2)^3}\int_0^d t(x)(x-d/2)\mathrm{d}x\right]. \tag{26}$$

Here, α is the coefficient of thermal expansion; E is the elasticity modulus; v is Poisson's coefficient (the medium is assumed isotropic). By substituting the distribution $t(x)$ (8) into (26), we obtain

$$\sigma(x)=\frac{\alpha E}{1-v}\frac{\eta_t P_p}{k\lambda S}\left[\mathrm{e}^{-kx}+R\mathrm{e}^{-2kd+kx}+\frac{\mathrm{e}^{-kd}}{kd}-\frac{1}{kd}+\frac{R\mathrm{e}^{-2kd}}{kd}\right.$$

$$-\frac{R\mathrm{e}^{-kd}}{kd}+\frac{12(x-d/2)}{d^3k}\left(\frac{d\mathrm{e}^{-kd}}{2}-\frac{1}{k}+\frac{\mathrm{e}^{-kd}}{k}+\frac{d}{2}\right.$$

$$\left.\left.-\frac{Rd\mathrm{e}^{-kd}}{2}+\frac{R\mathrm{e}^{-kd}}{k}-\frac{R\mathrm{e}^{-2kd}}{k}-\frac{Rd\mathrm{e}^{-2kd}}{2}\right)\right]. \tag{27}$$

Below we present the results of calculations of the temperature and stresses obtained from above expressions. The calculations were performed for the heat power P_t', the same for all the absorption coefficients, which was calculated per unit of the disk surface area: $P_t'=P_t/S=50$ W cm^{-2}. In this case, the ratio $\eta_t P_p/S$ was replaced by

$$\frac{P_t'}{\left(1-\mathrm{e}^{-kd}\right)\left(1+R\mathrm{e}^{-kd}\right)}. \tag{28}$$

We used in calculations the heat conduction coefficient $\lambda=0.1$ W cm^{-1} K^{-1}, the reflectivity $R=1$, $v=0.25$, the temperatures of the cooling media $t_1=t_2=20\,^{\circ}$C, the heat exchange coefficients $b=0.75$ W cm^{-2} K^{-1} (cooling by water) and $a=0.015$ (cooling by air) or 0.75 W cm^{-2} K^{-1} (cooling by water). The parameters $\alpha=7\times10^{-6}$ K^{-1} and $E=2\times10^6$ kgf cm^{-2} were taken, for example, corresponding to the YAG crystal. Note that the used value of the specific heat power corresponded to a $\sim$100-kW hypothetical laser for the disk area of 1000 cm^2 [2].

Figure 1 presents the temperature distribution $t(x)$ along the axis of a 0.1-cm-thick disk cooled from one side by air

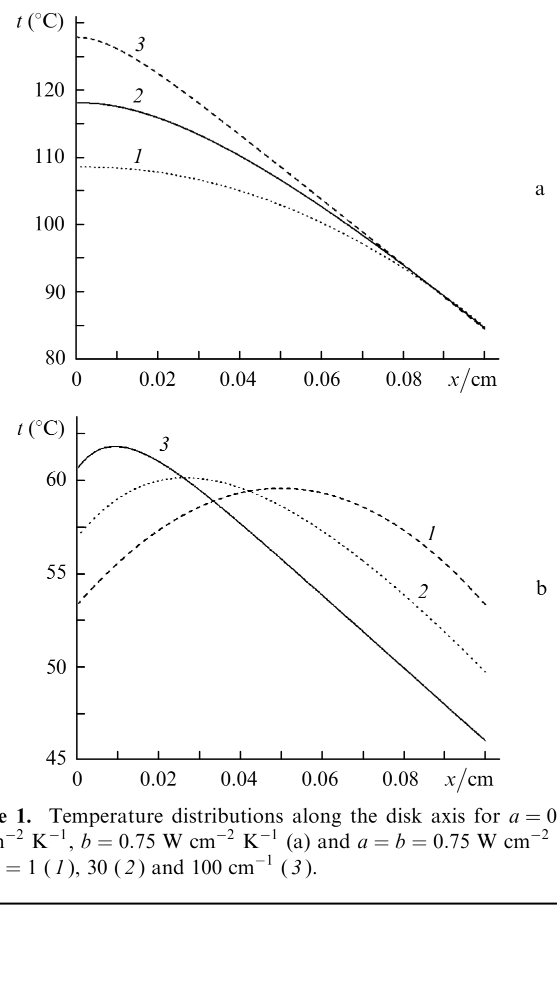

Figure 1. Temperature distributions along the disk axis for $a=0.015$ W cm^{-2} K^{-1}, $b=0.75$ W cm^{-2} K^{-1} (a) and $a=b=0.75$ W cm^{-2} K^{-1} (b), $k=1$ (*1*), 30 (*2*) and 100 cm^{-1} (*3*).

and from the other side by water ($a = 0.015$ W cm^{-2} K^{-1}, $b = 0.75$ W cm^{-2} K^{-1}) and by water from both sides ($a = b = 0.75$ W cm^{-2} K^{-1}) for the absorption coefficients $k = 1$, 30 and 100 cm^{-1}. One can see that the temperature on the pumped disk surface ($x = 0$) is higher than that on the opposite surface ($x = d$). The maximum temperature is achieved for $x = x_{max}$ [see expression (25)], which is so much the closer to zero the lower the ratio a/b. For small values of the optical density (for example, $D = 0.1$ for $k = 1$ cm^{-1}), we obtain the known parabolic distribution, in this case, for $a = b$, the position of the parabola vertex (where the temperature is maximal) almost coincides with the disk centre ($x_{max} \simeq d/2$). When k is increased, the temperature $t(0)$ increases and $t(d)$ decreases, and the distribution $t(x)$ tends to be quasi-linear one. In this case, the thermal tensions in the sample should decrease because the linear dependence $t(x)$ does not cause stresses [3, 4]. This is confirmed by calculations of thermal tensions $\sigma(x)$.

Note once again that the quantity $\sigma(x)$ in our specific case is independent of the temperature of the cooling media and heat exchange coefficients and is determined only by the heat power P_t scattered in the crystal, the disk area S and thickness d as well as by the coefficients of absorption (k), heat conduction (λ) and reflection (R) of exciting radiation from the rear mirror, which directly enter expression (27).

Figure 2 shows the dependences $\sigma(x)$ calculated for $d = 0.1$ cm, $k = 1$, 30 and 100 cm^{-1}. The results of calculations are valid for both types of cooling (air – water, water – water). For small optical densities ($D = 0.1$), we have a parabolic distribution. Note that in this case the pump absorption in the active element proceeds during a rather large number of passes. The maximal value σ_{max} is achieved for $x = 0$ (i.e. on the pumped disk surface). The stresses σ_{max} first slightly increase (approximately by 10 %) and then decrease with increasing k and for $k > 60$ cm^{-1} ($D > 6$) they become lower than at small optical densities, which is explained by the quasi-linearisation of the temperature dependence mentioned above. For the specified value of D, the stresses σ_{max} decrease with increasing k. For the specified k, the stresses σ_{max} increase with increasing the disk thickness.

The presented results confirm the conclusion [2] that thin disks based on optically dense active media, in which pump radiation is absorbed per one – two passes, have lower thermal tensions than disks with a small optical density, which require multipass pumping.

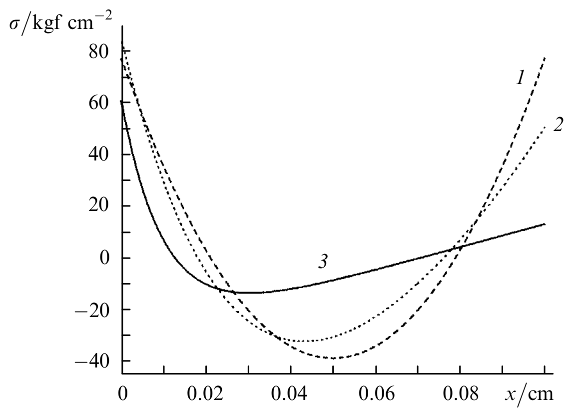

Figure 2. Distribution of thermoelastic stresses along the disk axis for $k = 1$ (*1*), 30 (*2*) and 100 cm^{-1} (*3*).

Note that the performed calculations did not take into account the increase in the heat conduction coefficient λ with increasing temperature [5, 6]. The calculations showed that, for example, for $k = 100$ cm^{-1} a decrease in λ by 30 % leads to an increase in temperature of the 'external' disk face (at which pump radiation is incident) approximately by 18 K. The maximal thermal tension appearing on the same face and changing, as seen from expression (27), inversely proportional to λ, increases in this case by 40 %.

Note that expression (8) for the temperature distribution was obtained for thin disks, which are cooled through plane faces and whose heat release through the side surface is small. However, in a real laser experiment, the cooling conditions can be different. Thus, in [2] pump radiation focused into a spot of radius $R_1 = 0.01$ cm was incident on a 0.035-cm-thick disk of radius $R_0 = 0.2$ cm. In this case the ratio of the side surface area of the pumped region to the area of its faces was ~ 3.5 and the heat could effectively 'spread' over the entire crystal. First, this decreased the temperature, and second, increased the area from which the heat escaped to the cooler – flowing water, which significantly increased the heat removal and also decreased the crystal temperature. If we neglect these processes and assume that the heat is extracted only from the faces of the pumped region, the temperature under conditions of the experiment described in [2], in which the value of P_t (the heat power released in the crystal) was ~ 0.5 W, is estimated to be ~ 1500 °C! However, if the area, from which the heat is extracted, is increased up to ~ 1 mm^2, the temperature decreases by an order of magnitude. This increase in the area in a real experiment occurs due to the 'spreading' of the heat along the crystal.

Because the analytic solution of the problem on the heat 'spreading' in the crystal under conditions of the experiment described in [2] is problematic, we will consider a simpler model to estimate the temperature. Because the disk thickness is small, we will assume that the heat being released is distributed uniformly over the pumped volume and the temperature fall between the disk faces is small, i.e. we consider the temperature to be the function of only the distance r from the centre of the pump spot along the disk surface. The 'external' disk face is cooled by the flowing water having temperature $t_2 = 20$ °C. Then, the heat conduction equation in cylindrical coordinates will have the form:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial t}{\partial r}\right) = -\frac{P_t}{\lambda S_1 d}, \qquad (29)$$

where

$$S_1 = \pi R_1^2.$$

Solution (29) has the form

$$t = \frac{P_t}{4\lambda S_1 d}\left[-r^2 + R_1^2\left(1 + 2\ln\frac{R_0}{R_1}\right)\right] + t(R_0)$$

$$(0 \leqslant r \leqslant R_1), \qquad (30)$$

$$t = \frac{P_t R_1^2}{2\lambda S d}\ln\frac{R_0}{r} + t(R_0) \quad (R_1 \leqslant r \leqslant R_0), \qquad (31)$$

where $t(R_0)$ is the temperature at the disk edge (for $r = R_0$).

As a boundary condition of the first kind, we specify the temperature $t(R_0)$. It is obvious that $t(R_0)$ cannot be smaller than the temperature $t_2 = 20\,^{\circ}\mathrm{C}$ of the cooling liquid. On the other hand, if we assume that the heat power P_{t} is distributed uniformly in the entire disk, the face temperature $t(d)$ cooled by water will be determined from the relation

$$P_{\mathrm{t}} = b[t(d) - t_2]\pi R_0^2, \quad (32)$$

which yields $t(d) = 25\,^{\circ}\mathrm{C}$. Because the temperature of the disk edges in the case when the heat is distributed over the entire volume, will be higher than in the case when the heat is released in a small pumped region, $25\,^{\circ}\mathrm{C}$ is the upper boundary of the temperature $t(R_0)$. Thus, by assuming that $20\,^{\circ}\mathrm{C} \leqslant t(R_0) \leqslant 25\,^{\circ}\mathrm{C}$, we obtain the temperature at the disk centre $100\,^{\circ}\mathrm{C} \leqslant t(r = 0) \leqslant 105\,^{\circ}\mathrm{C}$.

Let us calculate the heat removal P_{t}'' from the cooled disk face according to the boundary condition of the third kind:

$$P_{\mathrm{t}}'' = b\int_0^{R_0} 2\pi r[t(r) - t_2]\mathrm{d}r. \quad (33)$$

By substituting $t(r)$ determined from (30), (31), we obtain $P_{\mathrm{t}}'' \simeq 1.35$ W, while the input heat power is 0.5 W.

This diskrepancy is caused obviously by the fact that expressions (30), (31) were obtained by neglecting the integral heat extraction through the face to water but by taking into account the cooling by means of introducing temperature $t(R_0)$ at the disk edge. At the same time, because the heat removal is proportional to the difference in temperatures of the cooled surface and water and occurs from the entire disk face being cooled, i.e. from area πR_0^2, the real temperature is lower than the calculated one. This circumstance in the first approximation can be taken into account by introducing expressions (30), (31) correction coefficient $\beta < 1$ as

$$t'(r) = \beta[t(r) - t_2] + t_2. \quad (34)$$

where $t(r)$ is the temperature calculated by expressions (30), (31) and $t'(r)$ is the corrected value of the temperature. According to (34) the temperature decrease is higher, the larger the difference $t(r) - t_2$, i.e. the higher the heat 'sink' to water, the character of the dependence $t'(r)$ being the same as $t(r)$. It is obvious that $\beta = P_{\mathrm{t}}/P_{\mathrm{t}}'' \approx 0.37$ and the temperature at the disk centre is

$$t'(r = 0) = \beta[t(r = 0) - t_2] + t_2 \approx 50\,^{\circ}\mathrm{C}. \quad (35)$$

The value of the temperature, that is lower than that under the boundary conditions of the first kind, is explained by a more complete, than in the case of specifying the temperature at the disk edge, account for cooling under the boundary condition of the third kind, according to which the heat is extracted from the entire disk edge being cooled, which takes place in the experiment [5].

Therefore, the model under study taking into account the heat 'spreading' over the laser crystal yields a realistic estimate of its temperature.

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