

LIGHT PRESSURE

PACS numbers: 42.50.Wk; 42.25.Hz
DOI: 10.1070/QE2008v038n12ABEH013821

Theoretical study of the light pressure force acting on a spherical dielectric particle of an arbitrary size in the interference field of two plane monochromatic electromagnetic waves

D.V. Guzatov, L.S. Gaida, A.A. Afanas'ev

Abstract**. The light pressure force acting on a spherical dielectric particle in the interference field of two plane monochromatic electromagnetic waves is studied in detail for different particle radii and angles of incidence of waves.**

Keywords: *light pressure force, interference of electromagnetic waves, light scattering.*

1. Introduction

In 1986 a device allowing holding and manipulating small dielectric particles by laser radiation, was demonstrated for the first time [1]. A year later, the scientists from the same research group managed to displace a living cell by an IR laser beam without damaging it [2]. The device making it possible to move small dielectric particles and biological objects without their damage by using laser radiation was termed 'laser pincers'. At present laser pincers finds more and more applications in biology and medicine to study viruses and bacteria [3], DNA molecules [4], processes proceeding inside a living cell [5], etc. The manipulation of small dielectric particles in the laser radiation field becomes possible due to the action of the light pressure force, which is conventionally divided into two parts: gradient force and scattering force [6]. In the case of small dielectric particles, the scattering force can be, as a rule, neglected compared to the gradient force directed along the intensity gradient of the electromagnetic field [7, 8]. Under the action of the gradient force, depending on the polarisability sign, a small dielectric particle tends to move to the region of the maximum (minimum) field intensity, where the force action is minimal.

Apart from biological applications, the light pressure force can be used to produce artificial heterogeneous media representing liquid suspensions of suspended dielectric particles with controlled optical properties. These media can have large Kerr optical coefficients and be applied as broadband nonlinear media irradiated by low intensity long laser pulses [9 – 11]. The simplest way of producing regular intensity modulation of light fields is the interference of two light beams resulting in harmonic modulation of laser radiation intensity. The control of the intensity modulation period by changing the convergence angle of interfering beams allows the action of the light pressure force on the dielectric particle to be varied. Figure 1 shows the experimental results [12] on ordering 5.8-μm polystyrene balls suspended in a physiological solution in the interference field of two beams from a He – Ne laser. One can see that under the action of the light pressure force, the balls initially distributed disorderly (Fig. 1a) concentrate in the regions of the interference field maxima (Fig. 1b). After switching off

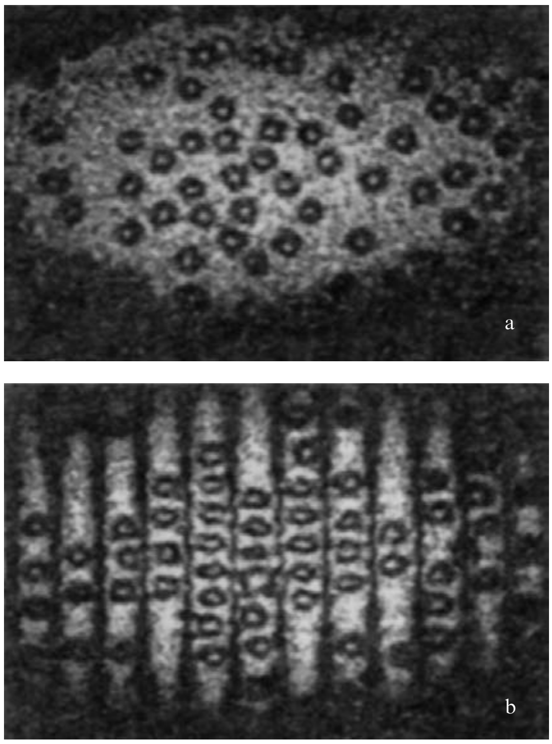

Figure 1. Spatial distributions of polystyrene microspheres suspended in physiological solution in the interference field [12] before the experiment (a) and during the experiment (b). The diameter of spherical particles is 5.8 μm and the modulation period of the light field is 8.2 μm.

D.V. Guzatov, L.S. Gaida Research Center of Resource Savings, National Academy of Sciences of Belarus, pl. Tizengauza 7, 230023 Grodno, Belarus; e-mail: dm_guzatov@mail.ru, gls@grsu.by;
A.A. Afanas'ev B.I. Stepanov Institute of Physics, National Academy of Sciences of Belarus, prosp. Nezavisimosti 68, 220072 Minsk, Belarus; e-mail: lvp@dragon.bas-net.by

Received 5 February 2008; revision received 4 April 2008
Kvantovaya Elektronika **38** (12) 1155 – 1162 (2008)
Translated by I.A. Ulitkin

the setup, the ordered pattern is preserved within several seconds and then, due to diffusion processes the distribution of the balls becomes disordered. Note that nonspherical dielectric particles behave similarly. In this case, they tend not only to enter into the regions of the interference field maxima but also spread along the interference fringe [12].

The light pressure force acting on a spherical particle in the field of a plane monochromatic electromagnetic wave was first calculated and studied by Debye in 1909 [13]. At present, many papers are devoted to the calculation of the light pressure force produced by electromagnetic radiation on dielectric particles. In these papers, the affect of laser (Gaussian) beams on spherical particles is mainly studied [14–19]. The aim of this paper is to solve analytically the problem of the light pressure force acting on a spherical particle of an arbitrary size (transparent dielectric or metal) and arbitrary radius in a simplest interference field of two plane monochromatic electromagnetic waves. In this case, main attention is paid to the case of a dielectric particle. The problem of interference field scattering of two plane electromagnetic waves from a dielectric spherical particle was considered earlier, however, the light pressure force acting on the particle by the field has not been calculated so far [20–22]. In this paper, we will try to fill in this gap. We calculate the light pressure field acting on the dielectric (metal) particle by using the formalism of Maxwell's stress tensor [23], which allows deriving the analytic expression for the force in the case of a spherical particle of an arbitrary radius.

2. Scattering of the interference field of two plane monochromatic electromagnetic waves on a dielectric spherical particle

The solution of the problem on scattering of a plane electromagnetic field from a dielectric (metal) spherical particle (the Mie solution) has been known already for a hundred years [24]. It is used to solve many problems related to scattering of electromagnetic radiation from spherical particles [25]. The wide application of the Mie solution is caused by the convenient analytic computational method and good agreement of the theoretical results with the experimental data. In this section we will generalise the Mie solution for the incident electromagnetic field of the type

$$\begin{aligned}
\boldsymbol{E}^{\mathrm{i}} &= \boldsymbol{e}_y \exp(-\mathrm{i}\omega t)(E_1 + E_2) \\
&= \boldsymbol{e}_y \exp(-\mathrm{i}\omega t)\{E_{10}\exp[\mathrm{i}k_{\mathrm{m}}(x\sin\alpha_1 + z\cos\alpha_1)] \\
&\quad + E_{20}\exp[\mathrm{i}k_{\mathrm{m}}(x\sin\alpha_2 + z\cos\alpha_2)]\},
\end{aligned}$$

$$\begin{aligned}
H^{\mathrm{i}} &= -\frac{\mathrm{i}}{k_{\mathrm{m}}}\sqrt{\frac{\varepsilon_{\mathrm{m}}}{\mu_m}}\,\mathrm{rot}\,\boldsymbol{E}^{\mathrm{i}} = -\sqrt{\frac{\varepsilon_{\mathrm{m}}}{\mu_{\mathrm{m}}}}E_{10} \\
&\times \exp[\mathrm{i}k_{\mathrm{m}}(x\sin\alpha_1 + z\cos\alpha_1) - \mathrm{i}\omega t](\boldsymbol{e}_x\cos\alpha_1 - \boldsymbol{e}_z\sin\alpha_1) \qquad (1) \\
&- \sqrt{\frac{\varepsilon_{\mathrm{m}}}{\mu_{\mathrm{m}}}}E_{20}\exp[\mathrm{i}k_{\mathrm{m}}(x\sin\alpha_2 + z\cos\alpha_2) - \mathrm{i}\omega t] \\
&\times (\boldsymbol{e}_x\cos\alpha_2 - \boldsymbol{e}_z\sin\alpha_2),
\end{aligned}$$

where $\boldsymbol{e}_x$, $\boldsymbol{e}_y$, $\boldsymbol{e}_z$ are the unit vectors of the Cartesian coordinate system; ω is the field oscillation frequency; E_{10} and E_{20} are the complex wave amplitudes; $k_{\mathrm{m}} = (\omega/c) \times \sqrt{\varepsilon_{\mathrm{m}}\mu_{\mathrm{m}}}$ is the wave number in the medium where the spherical particle is located; c is the speed of light in vacuum; ε_{m} and μ_{m} are the dielectric and magnetic permeabilities of the medium under study, respectively; α_1 and α_2 are the angles between the wave directions and the positive direction of the z axis ($0 \leqslant \alpha_1, \alpha_2 \leqslant \pi$) (Fig. 2). We represent expression (1) for the incident field in the form of expansion in the fundamental system of spherical vector functions, which are the solutions of the vector Helmholtz equation in the spherical coordinates r, θ, φ ($0 \leqslant r < \infty$, $0 \leqslant \theta \leqslant \pi$, $0 \leqslant \varphi < 2\pi$) [23]. In the general case, the expression for spherical vector functions (vector functions below) can be written in the from

$$\left\{\begin{array}{c}\boldsymbol{m}_{mne}\\ \boldsymbol{m}_{mno}\end{array}\right\} = \frac{m}{kr\sin\theta}Z_n(kr)P_n^m(\cos\theta)\left\{\begin{array}{c}-\sin(m\varphi)\\ \cos(m\varphi)\end{array}\right\}\boldsymbol{e}_\theta - \frac{1}{kr}Z_n(kr)\left[\frac{\partial}{\partial\theta}P_n^m(\cos\theta)\right]\left\{\begin{array}{c}\cos(m\varphi)\\ \sin(m\varphi)\end{array}\right\}\boldsymbol{e}_\varphi, \qquad (2)$$

$$\begin{aligned}
\left\{\begin{array}{c}\boldsymbol{n}_{mne}\\ \boldsymbol{n}_{mno}\end{array}\right\} &= \frac{n(n+1)}{(kr)^2}Z_n(kr)P_n^m(\cos\theta)\left\{\begin{array}{c}\cos(m\varphi)\\ \sin(m\varphi)\end{array}\right\}\boldsymbol{e}_r \\
&+ \frac{1}{kr}Z'_n(kr)\left[\frac{\partial}{\partial\theta}P_n^m(\cos\theta)\right]\left\{\begin{array}{c}\cos(m\varphi)\\ \sin(m\varphi)\end{array}\right\}\boldsymbol{e}_\theta \\
&+ \frac{m}{kr\sin\theta}Z'_n(kr)P_n^m(\cos\theta)\left\{\begin{array}{c}-\sin(m\varphi)\\ \cos(m\varphi)\end{array}\right\}\boldsymbol{e}_\varphi. \qquad (3)
\end{aligned}$$

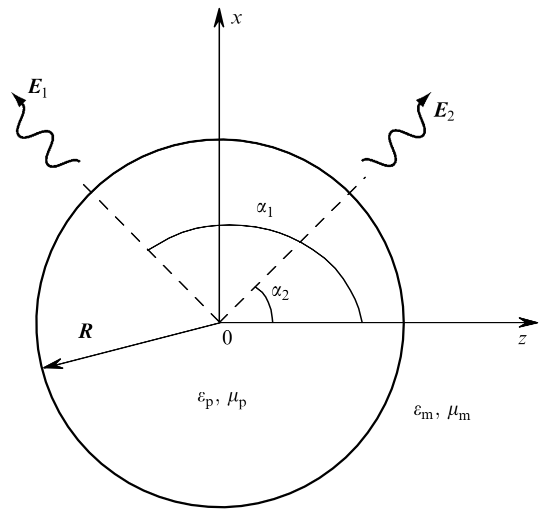

Figure 2. Geometry of the problem on the light pressure on a spherical particle.

In expressions (2) and (3), $n = 0, 1, 2$; $m = 0, 1, ..., n$; in the case of the even function (with the subscript 'e'), the upper line of the expression in braces is used and in the case of the odd function (with the subscript 'o'), the lower line is used; k is the wave number in the medium under study; $Z_n(kr)$ is one of the Riccati–Bessel functions [26] whose form depends on the region where the solution is sought for;

the prime at the function denotes hereafter the derivative of this function in its argument; P_n^m is the adjoined Legendre function [26]; $\boldsymbol{e}_r$, $\boldsymbol{e}_\theta$, $\boldsymbol{e}_\varphi$ are the unit vectors of the spherical coordinate system. By using expressions (2) and (3), expressions for the incident electric and magnetic fields can be represented in the form:

$$\boldsymbol{E}^{\mathrm{i}} = \exp(-\mathrm{i}\omega t)\sum_{n=0}^{\infty}\sum_{m=0}^{n}(G_{mn\mathrm{e}}\boldsymbol{m}_{mn\mathrm{e}}^{\mathrm{i}} + Q_{mn\mathrm{e}}\boldsymbol{n}_{mn\mathrm{e}}^{\mathrm{i}}) + \exp(-\mathrm{i}\omega t)\sum_{n=0}^{\infty}\sum_{m=0}^{n}(G_{mn\mathrm{o}}\boldsymbol{m}_{mn\mathrm{o}}^{\mathrm{i}} + Q_{mn\mathrm{o}}\boldsymbol{n}_{mn\mathrm{o}}^{\mathrm{i}}),$$

$$\boldsymbol{H}^{\mathrm{i}} = -\mathrm{i}\sqrt{\frac{\varepsilon_{\mathrm{m}}}{\mu_{\mathrm{m}}}}\exp(-\mathrm{i}\omega \boldsymbol{t})\sum_{n=0}^{\infty}\sum_{m=0}^{n}(Q_{mn\mathrm{e}}\boldsymbol{m}_{mn\mathrm{e}}^{\mathrm{i}} + G_{mn\mathrm{e}}\boldsymbol{n}_{mn\mathrm{e}}^{\mathrm{i}}) - \mathrm{i}\sqrt{\frac{\varepsilon_{\mathrm{m}}}{\mu_{\mathrm{m}}}}\exp(-\mathrm{i}\omega t)\sum_{n=0}^{\infty}\sum_{m=0}^{n}(Q_{mn\mathrm{o}}\boldsymbol{m}_{mn\mathrm{o}}^{\mathrm{i}} + G_{mn\mathrm{o}}\boldsymbol{n}_{mn\mathrm{o}}^{\mathrm{i}}). \tag{4}$$

Expressions for the vector functions $\boldsymbol{m}_{mn\mathrm{e}}^{\mathrm{i}}$, $\boldsymbol{m}_{mn\mathrm{o}}^{\mathrm{i}}$ and $\boldsymbol{n}_{mn\mathrm{e}}^{\mathrm{i}}$, $\boldsymbol{n}_{mn\mathrm{o}}^{\mathrm{i}}$, entering (4) are derived from (2) and (3) by substituting $k \to k_{\mathrm{m}}$ and $Z_n(k_{\mathrm{m}}r) \to \psi_n(k_{\mathrm{m}}r) = \sqrt{\pi k_{\mathrm{m}} r/2} \times J_{n+1/2}(k_{\mathrm{m}}r)$, where $J_{n+1/2}(k_{\mathrm{m}}r)$ is the Bessel function [26]. We will find the unknown coefficients $G_{mn\mathrm{e}}$, $G_{mn\mathrm{o}}$ and $Q_{mn\mathrm{e}}$, $Q_{mn\mathrm{o}}$ in expansions (4) by using the orthogonality properties of vector functions [23]. By integrating the scalar products $\boldsymbol{E}^{\mathrm{i}}$ (4) over the angles of the spherical coordinate system in complex conjugated functions $\boldsymbol{m}_{mn\mathrm{e}}^{\mathrm{i}*}$ and $\boldsymbol{m}_{mn\mathrm{o}}^{\mathrm{i}*}$, we obtain expressions for $G_{mn\mathrm{e}}$ and $G_{mn\mathrm{o}}$, respectively. Similarly, by integrating the scalar products $\boldsymbol{H}^{\mathrm{i}}$ (4) over the angles of the spherical coordinate system in complex conjugated vector function $\boldsymbol{m}_{mn\mathrm{e}}^{\mathrm{i}*}$ and $\boldsymbol{m}_{mn\mathrm{o}}^{\mathrm{i}*}$, we obtain expressions for $Q_{mn\mathrm{e}}$ and $Q_{mn\mathrm{o}}$. As a result, for the sought coefficients, we obtain relations

$$G_{mn\mathrm{e}} = E_{10}a_{mn\mathrm{e}}^{\mathrm{i}}(\alpha_1) + E_{20}a_{mn\mathrm{e}}^{\mathrm{i}}(\alpha_2),$$
$$G_{mn\mathrm{o}} = 0,$$
$$Q_{mn\mathrm{e}} = 0, \tag{5}$$
$$Q_{mn\mathrm{o}} = E_{10}b_{mn\mathrm{o}}^{\mathrm{i}}(\alpha_1) + E_{20}b_{mn\mathrm{o}}^{\mathrm{i}}(\alpha_2),$$

where

$$a_{mn\mathrm{e}}^{\mathrm{i}}(\alpha_{1,2}) = -(2-\delta_{0m})(1-\delta_{0n})\mathrm{i}^n \times \frac{2n+1}{n(n+1)}\frac{(n-m)!}{(n+m)!}\left[\frac{\partial}{\partial\alpha}P_n^m(\cos\alpha_{1,2})\right];$$

$$b_{mn\mathrm{o}}^{\mathrm{i}}(\alpha_{1,2}) = -(1-\delta_{0n})2m\mathrm{i}^{n+1} \times \frac{2n+1}{n(n+1)}\frac{(n-m)!}{(n+m)!}\frac{P_n^m(\cos\alpha_{1,2})}{\sin\alpha_{1,2}}; \tag{6}$$

δ_{0n} is the Kronecker delta. Note that as it should be, in the limiting case, $E_{20} = 0$, $\alpha_1 = 0$, π and expressions (5) are transformed into well-known expressions for the plane wave polarised along the y axis and propagating along the z axis [23].

Thus, taking (5) into account, expressions for the incident electromagnetic wave can be represented in the form

$$\boldsymbol{E}^{\mathrm{i}} = \exp(-\mathrm{i}\omega t)\sum_{n=1}^{\infty}\sum_{m=0}^{n}(G_{mn\mathrm{e}}\boldsymbol{m}_{mn\mathrm{e}}^{\mathrm{i}} + Q_{mn\mathrm{o}}\boldsymbol{n}_{mn\mathrm{o}}^{\mathrm{i}}),$$
$$\boldsymbol{H}^{\mathrm{i}} = -\mathrm{i}\sqrt{\frac{\varepsilon_{\mathrm{m}}}{\mu_{\mathrm{m}}}}\exp(-\mathrm{i}\omega t)\sum_{n=1}^{\infty}\sum_{m=0}^{n}(Q_{mn\mathrm{o}}\boldsymbol{m}_{mn\mathrm{o}}^{\mathrm{i}} + G_{mn\mathrm{e}}\boldsymbol{n}_{mn\mathrm{e}}^{\mathrm{i}}). \tag{7}$$

The electric $\boldsymbol{E}^{\mathrm{t}}$ and magnetic $\boldsymbol{H}^{\mathrm{t}}$ fields propagated inside ($r < R$) a spherical particle of radius R, will be sought for in the form

$$\boldsymbol{E}^{\mathrm{t}} = \exp(-\mathrm{i}\omega t)\sum_{n=1}^{\infty}\sum_{m=0}^{n}(G_{mn\mathrm{e}}^{\mathrm{t}}\boldsymbol{m}_{mn\mathrm{e}}^{\mathrm{t}} + Q_{mn\mathrm{o}}^{\mathrm{t}}\boldsymbol{n}_{mn\mathrm{o}}^{\mathrm{t}}),$$
$$\boldsymbol{H}^{\mathrm{t}} = -\mathrm{i}\sqrt{\frac{\varepsilon_{\mathrm{p}}}{\mu_{\mathrm{p}}}}\exp(-\mathrm{i}\omega t)\sum_{n=1}^{\infty}\sum_{m=0}^{n}(Q_{mn\mathrm{o}}^{\mathrm{t}}\boldsymbol{m}_{mn\mathrm{o}}^{\mathrm{t}} + G_{mn\mathrm{e}}^{\mathrm{t}}\boldsymbol{n}_{mn\mathrm{e}}^{\mathrm{t}}). \tag{8}$$

For the dielectric medium ($r > R$), in which the particle is suspended, the scattered electric $\boldsymbol{E}^{\mathrm{s}}$ and magnetic $\boldsymbol{H}^{\mathrm{s}}$ fields can be written in the form

$$\boldsymbol{E}^{\mathrm{s}} = \exp(-\mathrm{i}\omega t)\sum_{n=1}^{\infty}\sum_{m=0}^{n}(G_{mn\mathrm{e}}^{\mathrm{s}}\boldsymbol{m}_{mn\mathrm{e}}^{\mathrm{s}} + Q_{mn\mathrm{o}}^{\mathrm{s}}\boldsymbol{n}_{mn\mathrm{o}}^{\mathrm{s}}),$$
$$\boldsymbol{H}^{\mathrm{s}} = -\mathrm{i}\sqrt{\frac{\varepsilon_{\mathrm{m}}}{\mu_{\mathrm{m}}}}\exp(-\mathrm{i}\omega t)\sum_{n=1}^{\infty}\sum_{m=0}^{n}(Q_{mn\mathrm{o}}^{\mathrm{s}}\boldsymbol{m}_{mn\mathrm{o}}^{\mathrm{s}} + G_{mn\mathrm{e}}^{\mathrm{s}}\boldsymbol{n}_{mn\mathrm{e}}^{\mathrm{s}}). \tag{9}$$

In expressions (8) and (9), the coefficients $G_{mn\mathrm{e}}^{\mathrm{t}}, Q_{mn\mathrm{o}}^{\mathrm{t}}$ and $G_{mn\mathrm{e}}^{\mathrm{s}}$, $Q_{mn\mathrm{o}}^{\mathrm{s}}$ should be defined; the vector functions $\boldsymbol{m}_{mn\mathrm{e}}^{\mathrm{t}}$, $\boldsymbol{m}_{mn\mathrm{o}}^{\mathrm{t}}$ and $\boldsymbol{n}_{mn\mathrm{e}}^{\mathrm{t}}$, $\boldsymbol{n}_{mn\mathrm{o}}^{\mathrm{t}}$ are obtained from (2) and (3) by substituting $k \to k_{\mathrm{p}} = (\omega/c)\sqrt{\varepsilon_{\mathrm{p}}\mu_{\mathrm{p}}}$ and $Z_n(k_{\mathrm{p}}r) \to \psi_n(k_{\mathrm{p}}r)$, where ε_{p} and μ_{p} are the dielectric and magnetic permeabilities of the spherical particle material, respectively; the vector functions $\boldsymbol{m}_{mn\mathrm{e}}^{\mathrm{s}}$, $\boldsymbol{m}_{mn\mathrm{o}}^{\mathrm{s}}$ and $\boldsymbol{n}_{mn\mathrm{e}}^{\mathrm{s}}$, $\boldsymbol{n}_{mn\mathrm{o}}^{\mathrm{s}}$ are obtained from (2) and (3) by substituting $k \to k_{\mathrm{m}}$ and $Z_n(k_{\mathrm{m}}r) \to \zeta_n(k_{\mathrm{m}}r) = \sqrt{\pi k_{\mathrm{m}} r/2}\, H_{n+1/2}^{(1)}(k_{\mathrm{m}}r)$, where $H_{n+1/2}^{(1)}(k_{\mathrm{m}}r)$ is the Henkel function of the first kind [26]. We will find the unknown expansion coefficients (8) and (9) by using the equality conditions of the tangential components of the electromagnetic field from two sides of the spherical particle surface, from which we obtain the equations for the sought-for coefficients. By solving these equations, we find the expressions:

$$G_{mn\mathrm{e}}^{\mathrm{t}} = \frac{k_{\mathrm{p}}}{k_{\mathrm{m}}}G_{mn\mathrm{e}}\left[\frac{\psi_n(k_{\mathrm{m}}R)}{\psi_n(k_{\mathrm{p}}R)} + \sigma_n\frac{\zeta_n(k_{\mathrm{m}}R)}{\psi_n(k_{\mathrm{p}}R)}\right],$$
$$Q_{mn\mathrm{o}}^{\mathrm{t}} = \frac{k_{\mathrm{p}}}{k_{\mathrm{m}}}Q_{mn\mathrm{o}}\left[\frac{\psi'_n(k_{\mathrm{m}}R)}{\psi'_n(k_{\mathrm{p}}R)} + \tau_n\frac{\zeta'_n(k_{\mathrm{m}}R)}{\psi'_n(k_{\mathrm{p}}R)}\right],$$
$$G_{mn\mathrm{e}}^{\mathrm{s}} = G_{mn\mathrm{e}}\sigma_n, \tag{10}$$
$$Q_{mn\mathrm{o}}^{\mathrm{s}} = Q_{mn\mathrm{o}}\tau_n,$$

where expressions for the Mie coefficients for the reflected field σ_n and τ_n have the form [23]

$$\sigma_n = -\frac{\psi_n(k_{\mathrm{m}}R)}{\zeta_n(k_{\mathrm{m}}R)}\left[\sqrt{\frac{\varepsilon_{\mathrm{p}}}{\mu_{\mathrm{p}}}}\frac{\psi'_n(k_{\mathrm{p}}R)}{\psi_n(k_{\mathrm{p}}R)} - \sqrt{\frac{\varepsilon_{\mathrm{m}}}{\mu_{\mathrm{m}}}}\frac{\psi'_n(k_{\mathrm{m}}R)}{\psi_n(k_{\mathrm{m}}R)}\right]$$

$$\times\left[\sqrt{\frac{\varepsilon_{\rm p}}{\mu_{\rm p}}}\frac{\psi_n'(k_{\rm p}R)}{\psi_n(k_{\rm p}R)}-\sqrt{\frac{\varepsilon_{\rm m}}{\mu_{\rm m}}}\frac{\zeta_n'(k_{\rm m}R)}{\zeta_n(k_{\rm m}R)}\right]^{-1};$$

$$\tau_n=-\frac{\psi_n(k_{\rm m}R)}{\zeta_n(k_{\rm m}R)}\left[\sqrt{\frac{\mu_{\rm p}}{\varepsilon_{\rm p}}}\frac{\psi_n'(k_{\rm p}R)}{\psi_n(k_{\rm p}R)}-\sqrt{\frac{\mu_{\rm m}}{\varepsilon_{\rm m}}}\frac{\psi_n'(k_{\rm m}R)}{\psi_n(k_{\rm m}R)}\right] \quad (11)$$

$$\times\left[\sqrt{\frac{\mu_{\rm p}}{\varepsilon_{\rm p}}}\frac{\psi_n'(k_{\rm p}R)}{\psi_n(k_{\rm p}R)}-\sqrt{\frac{\mu_{\rm m}}{\varepsilon_{\rm m}}}\frac{\zeta_n'(k_{\rm m}R)}{\zeta_n(k_{\rm m}R)}\right]^{-1};$$

the derivatives are taken for $r=R$. Obtained coefficients (10) can be used to calculate the time-averaged scattering power [23]

$$W_{\rm sc}=\frac{c}{8\pi}{\rm Re}\int_0^{\pi}{\rm d}\theta\sin\theta\int_0^{2\pi}{\rm d}\varphi(\boldsymbol{e}_r[\boldsymbol{E}^{\rm s},\boldsymbol{H}^{\rm s*}])r^2\big|_{r\to\infty}. \quad (12)$$

By using the asymptotic of the Riccati–Bessel functions for $r\to\infty$ [26], after integration in (12), we obtain the sought-for expression

$$W_{\rm sc}=\frac{c}{4k_{\rm m}^2}\sqrt{\frac{\varepsilon_{\rm m}}{\mu_{\rm m}}}\sum_{n=1}^{\infty}\sum_{m=0}^{n}\frac{n(n+1)}{2n+1}\frac{(n+m)!}{(n-m)!}$$

$$\times[(1+\delta_{0m})|G^{\rm s}_{mne}|^2+(1-\delta_{0m})|Q^{\rm s}_{mno}|^2]. \quad (13)$$

Thus, expressions (8) and (9) for fields with coefficients (10) and (11) are the solution of the problem on scattering of the interference field of two plane monochromatic electromagnetic waves from a spherical dielectric (metal) particle.

3. Light pressure force acting on a spherical dielectric particle in the interference field of two plane monochromatic electromagnetic waves

The general expression for the light pressure force of the arbitrary electromagnetic field on a particle of an arbitrary shape and size is presented, for example, in [27]. By neglecting mechanic deformations of the particle and the dielectric medium in which it is submerged, appearing under the action of electromagnetic radiation, the expression of the time-averaged light pressure force $\boldsymbol{F}$ takes the form

$$\boldsymbol{F}=\frac{1}{2}{\rm Re}\int_S(\boldsymbol{v}\hat{T}){\rm d}S,$$

$$\hat{T}=\frac{\varepsilon_{\rm m}}{4\pi}\left(\boldsymbol{E}\otimes\boldsymbol{E}^*-\frac{1}{2}|\boldsymbol{E}|^2\hat{I}\right)+\frac{\mu_{\rm m}}{4\pi}\left(\boldsymbol{H}\otimes\boldsymbol{H}^*-\frac{1}{2}|\boldsymbol{H}|^2\hat{I}\right), \quad (14)$$

where S is an arbitrary surface surrounding the particle under study; $\boldsymbol{v}$ is the vector of the external normal to S; $\hat{T}$ is Maxwell's stress tensor [23]; $\otimes$ denotes the direct product of vectors; $\hat{I}$ is the unit tensor. To calculate force (14), it is convenient to take the surface S in the form of a sphere with the infinitely large radius. As a result, (14) is transformed into the form:

$$\boldsymbol{F}=-\frac{\varepsilon_{\rm m}}{16\pi}\int_0^{\pi}{\rm d}\theta\sin\theta\int_0^{2\pi}{\rm d}\varphi(|\boldsymbol{e}_\theta\boldsymbol{E}^{\rm i}+\boldsymbol{e}_\theta\boldsymbol{E}^{\rm s}|^2$$

$$+|\boldsymbol{e}_\varphi\boldsymbol{E}^{\rm i}+\boldsymbol{e}_\varphi\boldsymbol{E}^{\rm s}|^2)\boldsymbol{e}_r r^2\big|_{r\to\infty}$$

$$-\frac{\mu_{\rm m}}{16\pi}\int_0^{\pi}{\rm d}\theta\sin\theta\int_0^{2\pi}{\rm d}\varphi(|\boldsymbol{e}_\theta\boldsymbol{H}^{\rm i}+\boldsymbol{e}_\theta\boldsymbol{H}^{\rm s}|^2 \quad (15)$$

$$+|\boldsymbol{e}_\varphi\boldsymbol{H}^{\rm i}+\boldsymbol{e}_\varphi\boldsymbol{H}^{\rm s}|^2)\boldsymbol{e}_r r^2\big|_{r\to\infty},$$

where we took into account that for $r\to\infty$ the radial components of spherical vector functions decrease faster than $1/r$ and do not contribute to (15). By using expansion (7), we can show that

$$\boldsymbol{F}_1=-\frac{\varepsilon_{\rm m}}{16\pi}\int_0^{\pi}{\rm d}\theta\sin\theta\int_0^{2\pi}{\rm d}\varphi(|\boldsymbol{e}_\theta\boldsymbol{E}^{\rm i}|^2$$

$$+|\boldsymbol{e}_\varphi\boldsymbol{E}^{\rm i}|^2)\boldsymbol{e}_r r^2\big|_{r\to\infty}-\frac{\mu_{\rm m}}{16\pi}\int_0^{\pi}{\rm d}\theta\sin\theta \quad (16)$$

$$\times\int_0^{2\pi}{\rm d}\varphi(|\boldsymbol{e}_\theta\boldsymbol{H}^{\rm i}|^2+|\boldsymbol{e}_\varphi\boldsymbol{H}^{\rm i}|^2)\boldsymbol{e}_r r^2\big|_{r\to\infty}=0.$$

Therefore, expression (15) can be written in the form

$$\boldsymbol{F}=\boldsymbol{F}_2+\boldsymbol{F}_3, \quad (17)$$

where

$$\boldsymbol{F}_2=-\frac{\varepsilon_{\rm m}}{8\pi}{\rm Re}\int_0^{\pi}{\rm d}\theta\sin\theta\int_0^{2\pi}{\rm d}\varphi[(\boldsymbol{e}_\theta\boldsymbol{E}^{\rm i})(\boldsymbol{e}_\theta\boldsymbol{E}^{\rm s*})$$

$$+(\boldsymbol{e}_\varphi\boldsymbol{E}^{\rm i})(\boldsymbol{e}_\varphi\boldsymbol{E}^{\rm s*})]\boldsymbol{e}_r r^2\big|_{r\to\infty}-\frac{\mu_{\rm m}}{8\pi}{\rm Re}\int_0^{\pi}{\rm d}\theta\sin\theta \quad (18)$$

$$\times\int_0^{2\pi}{\rm d}\varphi[(\boldsymbol{e}_\theta\boldsymbol{H}^{\rm i})(\boldsymbol{e}_\theta\boldsymbol{H}^{\rm s*})+(\boldsymbol{e}_\varphi\boldsymbol{H}^{\rm i})(\boldsymbol{e}_\varphi\boldsymbol{H}^{\rm s*})]\boldsymbol{e}_r r^2\big|_{r\to\infty},$$

$$\boldsymbol{F}_3=-\frac{\varepsilon_{\rm m}}{16\pi}\int_0^{\pi}{\rm d}\theta\sin\theta\int_0^{2\pi}{\rm d}\varphi(|\boldsymbol{e}_\theta\boldsymbol{E}^{\rm s}|^2$$

$$+|\boldsymbol{e}_\varphi\boldsymbol{E}^{\rm s}|^2)\boldsymbol{e}_r r^2\big|_{r\to\infty}-\frac{\mu_{\rm m}}{16\pi}\int_0^{\pi}{\rm d}\theta\sin\theta \quad (19)$$

$$\times\int_0^{2\pi}{\rm d}\varphi(|\boldsymbol{e}_\theta\boldsymbol{H}^{\rm s}|^2+|\boldsymbol{e}_\varphi\boldsymbol{H}^{\rm s}|^2)\boldsymbol{e}_r r^2\big|_{r\to\infty}.$$

By using (7) and (9), after integration in (18) and (19), we obtain

$$\boldsymbol{F}_2=F_{2x}\boldsymbol{e}_x+F_{2z}\boldsymbol{e}_z,\quad \boldsymbol{F}_3=F_{3x}\boldsymbol{e}_x+F_{3z}\boldsymbol{e}_z. \quad (20)$$

The explicit form of expressions for F_{2x}, F_{2z} and F_{3x}, F_{3z}, due to their cumbersomeness is presented in Appendix. Thus, it follows from (20) that the light pressure force acting on a spherical particle in the incident field does not have a component directed along the y axis and acts in the interference plane. For definiteness, we have

$$E_{10}=E_0\exp[{\rm i}k_{\rm m}(X\sin\alpha_1+Z\cos\alpha_1)],$$

$$E_{20}=E_0\exp[{\rm i}k_{\rm m}(X\sin\alpha_2+Z\cos\alpha_2)], \quad (21)$$

where the quantities X and Z mean the displacement of the spherical particle centre with respect to the maximum of the selected interference fringe. Conditions for producing interference maxima and minima of the electric field (1) taking into account (21) have the form

$$k_{\mathrm{m}}(x+X)(\sin\alpha_1-\sin\alpha_2)+k_{\mathrm{m}}(z+Z)\times(\cos\alpha_1-\cos\alpha_2)=2q\pi,$$
$$k_{\mathrm{m}}(x+X)(\sin\alpha_1-\sin\alpha_2)+k_{\mathrm{m}}(z+Z)\times(\cos\alpha_1-\cos\alpha_2)=(2q+1)\pi, \quad (22)$$

where $q = 0, \pm 1, \pm 2, \dots$. It follows from (21) that for $X = Z = 0$, the line of the interference maxima of the incident field passes through the spherical particle centre, which coincides with the origin of the coordinate system. If X and Z are nonzero, the line of maxima of the interference pattern, generally speaking, does not pass through the particle centre, which can be interpreted as the particle displacement by the quantity X or Z with respect to the selected interference maximum (minimum).

Let us expand expression (17) for the force acting on a spherical dielectric (without losses) particle in series in powers $\omega R/c \ll 1$. By using (20), (21) and the explicit form of expressions for F_{2x}, F_{2z} and F_{3x}, F_{3z} (see Appendix), and by using the first nonvanishing term of the expansion, we obtain

$$\boldsymbol{F}\approx-\frac{1}{2}k_{\mathrm{m}}R^3|E_0|^2\varepsilon_{\mathrm{m}}\times\left[\frac{\varepsilon_{\mathrm{p}}-\varepsilon_{\mathrm{m}}}{\varepsilon_{\mathrm{p}}+2\varepsilon_{\mathrm{m}}}+\frac{\mu_{\mathrm{p}}-\mu_{\mathrm{m}}}{\mu_{\mathrm{p}}+2\mu_{\mathrm{m}}}\cos(\alpha_1-\alpha_2)\right]\times\sin\{k_{\mathrm{m}}[X(\sin\alpha_1-\sin\alpha_2)+Z(\cos\alpha_1-\cos\alpha_2)]\}\times[(\sin\alpha_1-\sin\alpha_2)\boldsymbol{e}_x+(\cos\alpha_1-\cos\alpha_2)\boldsymbol{e}_z]. \quad (23)$$

It follows from (23) that for symmetric incidence of electromagnetic waves with respect to the x axis, i.e. when the condition $\alpha_1 + \alpha_2 = \pi$ is fulfilled (see Fig. 2), force component (23) directed along the x axis is equal to zero. If for the given symmetric incidence of waves the condition $Z = 0$ is also fulfilled, both vector force components are equal to zero.

In papers [28, 29], expression ($\varepsilon_{\mathrm{m}} = 1$ and $\mu_{\mathrm{m}} = \mu_{\mathrm{p}} = 1$):

$$\boldsymbol{F}_{\mathrm{grad}}=\frac{1}{2}\Pi\,\mathrm{grad}(\mathrm{Re}\boldsymbol{E}^{\mathrm{i}})^2 \quad (24)$$

is used as gradient force acting on a spherical dielectric particle, where

$$\Pi=\frac{\varepsilon_{\mathrm{p}}-1}{\varepsilon_{\mathrm{p}}+2}R^3$$

is the polarisability of the spherical dielectric particle in a homogeneous field. In the case of interference of two plane monochromatic electromagnetic waves, we obtain after averaging in time from (24)

$$\boldsymbol{F}_{\mathrm{grad}}=-\frac{\omega}{2c}R^3|E_0|^2\frac{\varepsilon_{\mathrm{p}}-1}{\varepsilon_{\mathrm{p}}+2}\times\sin\left\{\frac{\omega}{c}[(x+X)(\sin\alpha_1-\sin\alpha_2)+(z+Z)\times(\cos\alpha_1-\cos\alpha_2)]\right\}[(\sin\alpha_1-\sin\alpha_2)\boldsymbol{e}_x+(\cos\alpha_1-\cos\alpha_2)\boldsymbol{e}_z]. \quad (25)$$

By comparing (23) and (25) ($\varepsilon_{\mathrm{m}} = 1$ and $\mu_{\mathrm{m}} = \mu_{\mathrm{p}} = 1$), one can see that these expressions are identical, if we put $x = z = 0$ in (25). Thus, as was expected, expression for gradient force (24) calculated in the centre of the spherical dielectric particle under study is the first approximation for force (14) and can be used as an asymptotic expression for the light pressure force acting on the spherical dielectric (without losses) particle with a small radius ($R \ll c/\omega$). Note that account for further terms in expansion (17) in series by powers of the small parameter $\omega R/c \ll 1$ results not only in refining of the asymptotic expression for gradient force (23) but also in taking into account of contributions of scattering force. Because the obtained expressions for F_{2x}, F_{2z} and F_{3x}, F_{3z} are cumbersome (see Appendix), it is extremely difficult to derive an explicit expression for the first nonvanishing contribution of scattering force, which does not allow us to present it in this paper.

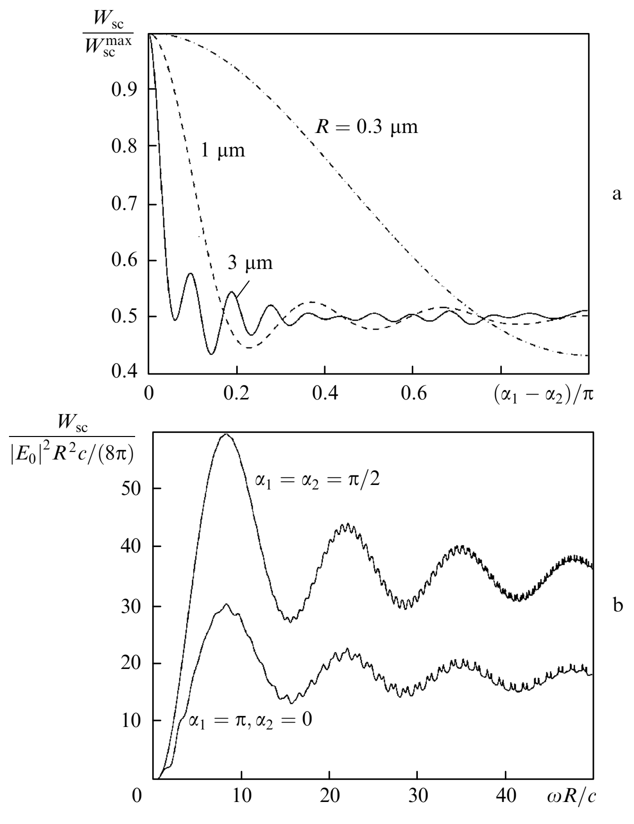

Figure 3. Dependences of normalised scattering power (13) of two plane electromagnetic waves from the quartz spherical particle ($\varepsilon_{\mathrm{p}} = 2.4$, $\mu_{\mathrm{p}} = 1$) located in water ($\varepsilon_{\mathrm{m}} = 1.7$, $\mu_{\mathrm{m}} = 1$) on the difference in the angles of incidence of plane waves (for $\alpha_1 + \alpha_2 = \pi$) for different radii of the particles (a) and on the normalised radius of the particle for the waves incident in one direction ($\alpha_1 = \alpha_2 = \pi/2$) and in the counterpropagating direction ($\alpha_1 = \pi$, $\alpha_2 = 0$) (b). The wavelength of incident radiation is 1.064 µm, the particle displacement is $X = Z = 0$.

4. Discussion of the results

To illustrate the analytic results obtained in the above sections, we will consider a quartz spherical dielectric particle ($\varepsilon_p = 2.4$, $\mu_p = 1$) located in an aqueous medium ($\varepsilon_m = 1.7$, $\mu_m = 1$). Let incident radiation have the wavelength of 1.064 μm (in vacuum). Without loss of generality, consider the symmetric incidence of plane electromagnetic waves ($\alpha_1 + \alpha_2 = \pi$) with respect to the x axis.

Figure 3a shows the scattering power (13) of two plane electromagnetic waves from a quartz particle as a function of difference in the angles of incidence for some specified values of the particle radius. One can clearly see that the highest scattering power is achieved when the waves are incident codirectionally on the spherical particle. The number of oscillations of the scattering power increases with increasing the particle radius (Fig. 3a). Figure 3b presents the scattering power as a function of the spherical particle radius for two mutual directions of incidence of plane electromagnetic waves: in one (copropagating) direction and counterpropagating direction. As was noted above, the scattering power is larger for copropagating waves (Fig. 3b). In this case, it oscillates and fast small-amplitude oscillations are observed against the background of these oscillations, which agrees with known results [25].

Figure 4a shows light pressure force (17) as a function of difference in the incidence angle of plane electromagnetic waves $\alpha_1 - \alpha_2$. For the symmetric incidence of plane waves, the force component directed along the z axis is equal to zero, which follows from the geometry of the problem (Fig. 2). The only nonzero component of the light pressure force $F_x = F_{2x} + F_{3x}$ takes the maximum value when the propagation directions of incident waves ($\alpha_1 = \alpha_2$) coincide, which also takes place for the scattering power (cf. with Fig. 3a). The light pressure force takes the minimum (zero) value when incident waves counterpropagate, as is seen from Fig. 4a. When the radius of the spherical particle is increased, the oscillation frequency of the force increases depending on the difference in the incidence angles of waves (Fig. 4a) and the particle radius (Fig. 4b). In this case, as for the scattering power, when the radius is increased, fast oscillations are observed against the background of relatively slow oscillations (Fig. 4b), which agrees with the results of paper [25].

Figure 5 presents light pressure force (17) acting on the quartz particle under study in the field of two plane electromagnetic waves propagating at angles $\alpha_1 = 3\pi/4$ and $\alpha_2 = \pi/4$ as a function of the particle displacement Z ($X = 0$). One can clearly see that the force components directed along the x (Fig. 5a) and z (Fig. 5b) axes oscillate with the period

$$\frac{\omega Z}{c} = \frac{2\pi}{\sqrt{\varepsilon_m \mu_m}|\cos\alpha_1 - \cos\alpha_2|} \approx 3.4$$

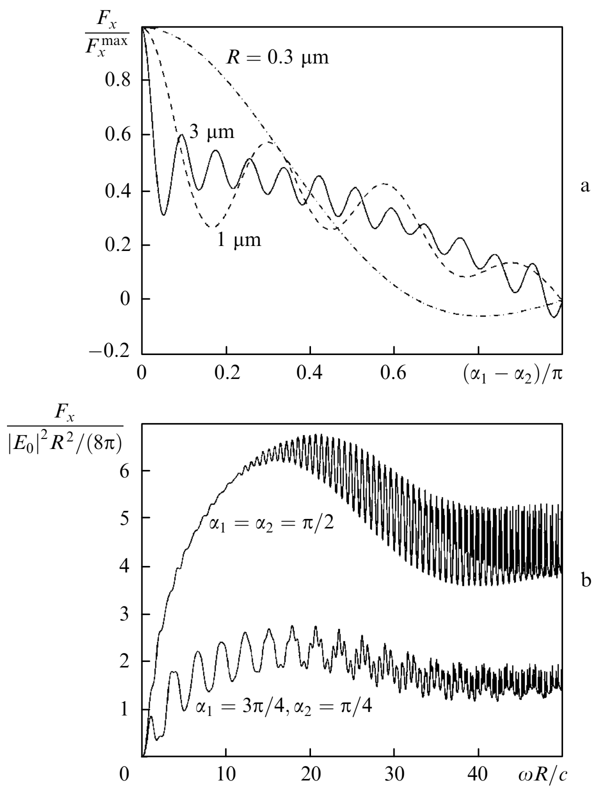

Figure 4. Dependences of the normalised component F_x of light pressure force (17) acting on the quartz spherical particle ($\varepsilon_p = 2.4$, $\mu_p = 1$) located in water ($\varepsilon_m = 1.7$, $\mu_m = 1$) in the interference field of two plane electromagnetic waves on the difference in the angles of incidence of plane waves (for $\alpha_1 + \alpha_2 = \pi$) for different radii of the particles (a) and on the normalised radius of the particle for the waves incident in one direction ($\alpha_1 = \alpha_2 = \pi/2$) and at the angle $\pi/2$ to each other ($\alpha_1 = 3\pi/4$, $\alpha_2 = \pi/4$) (b). The wavelength of incident radiation is 1.064 μm, the particle displacement is $X = Z = 0$.

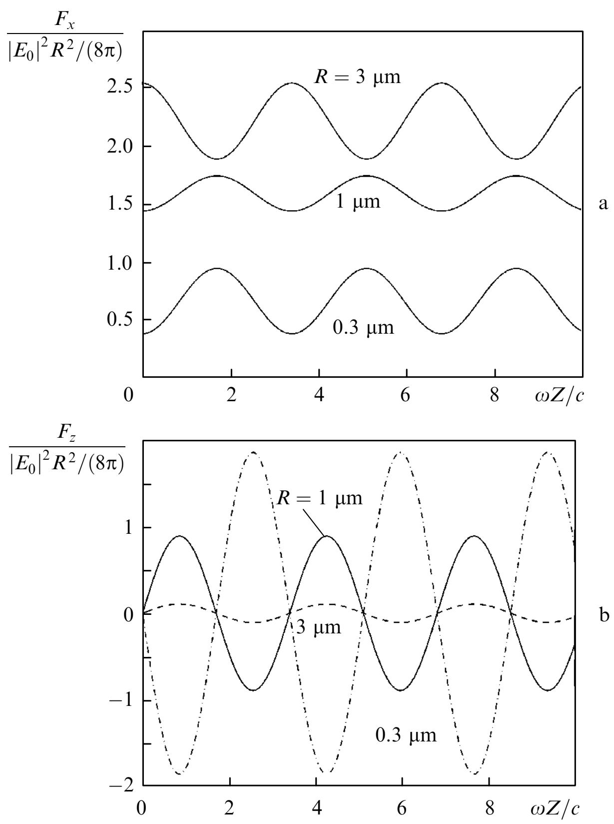

Figure 5. Dependences of the normalised components F_x (a) and F_z (b) of light pressure force (17) acting on the quartz spherical particle ($\varepsilon_p = 2.4$, $\mu_p = 1$) located in water ($\varepsilon_m = 1.7$, $\mu_m = 1$) in the interference field of two plane electromagnetic 1.064-μm waves on the normalised displacement of the particle $\omega Z/c$ ($X = 0$) for different radii of the particle. Plane waves are incident at angles $\alpha_1 = 3\pi/4$ and $\alpha_2 = \pi/4$.

with increasing the displacement. When the spherical particle radius increases, the average line, with respect to which oscillations of the x component of the light pressure force are observed, is displaced to the direction of the force increase (Fig. 5a). In the case of the z component of the gradient force, when the spherical particle radius is increased, the line, with respect to which oscillations occur, remains constant: $F_z = F_{2z} + F_{3z} = 0$ (Fig. 5b).

Figure 6 presents light pressure force (17) as a function of the radius of the particle under study for different displacements $X = Z$. Dashed curves show asymptotic solution (25). The plane electromagnetic waves are incident at angles $\alpha_1 = 3\pi/4$ and $\alpha_2 = \pi/4$. One can see from Fig. 6 that when the particle radius tends to zero, the force also tends to zero. In this case, asymptotic solution (23) for the nonzero z component of the force (Fig. 6b) agrees well with exact solution (17), if the particle radius is $R \leqslant 0.5c/\omega$. To study spherical particle with a larger particle, it is necessary to use expression (17) for calculating the light pressure force.

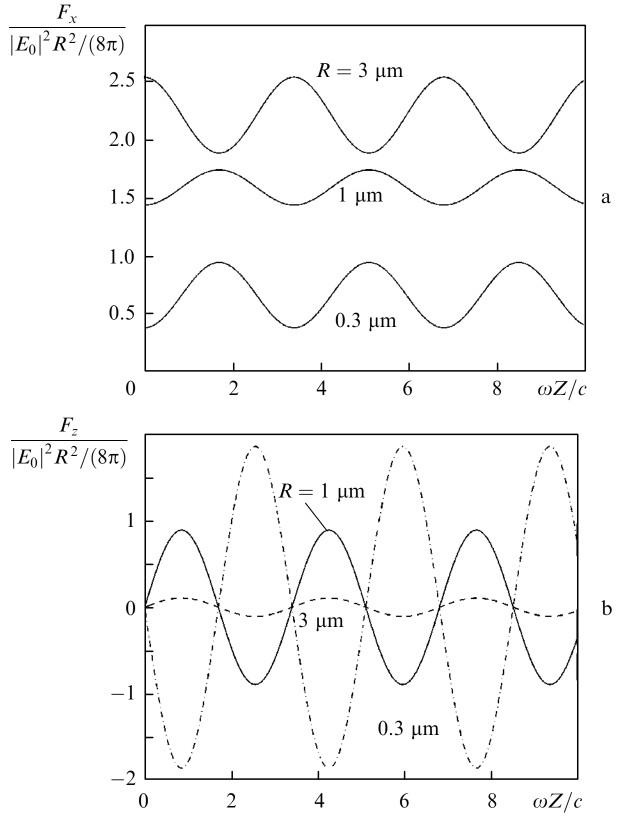

Figure 6. Dependences of the normalised components F_x (a) and F_z (b) of light pressure force (17) acting on the quartz spherical particle ($\varepsilon_p = 2.4$, $\mu_p = 1$) located in water ($\varepsilon_m = 1.7$, $\mu_m = 1$) in the interference field of two plane electromagnetic 1.064-μm waves on the normalised radius of the particle for different displacements Z ($X = 0$) of the particle. Plane waves are incident at angles $\alpha_1 = 3\pi/4$ and $\alpha_2 = \pi/4$. Dashed curves show corresponding asymptotics (23).

5. Conclusions

We have considered in detail the solution of the problem on the spherical dielectric particle in the interference field of two plane monochromatic electromagnetic waves. The dependence of the scattering power on the difference in the angles of incidence of plane waves and the radius of the spherical particle has been studied. By using Maxwell's stress tensor, the light pressure force acting on a spherical dielectric particle by the incident field has been calculated as a function of the difference in the angles of plane waves and the particle radius. The asymptotic expression has been obtained for the light pressure field for rather small radii of the dielectric (without losses) particle, which coincides with the known expression for the gradient force. Analytic expressions have been derived (see Appendix), which are suitable for calculating the light pressure force acting both on dielectric and metal spherical particles with an arbitrary radius.

The results of this paper can be used to calculate the light pressure force acting on dielectric (metal) spherical particles in the interference field of two plane monochromatic electromagnetic waves and to verify the algorithms of the numerical calculation of the light pressure force in the case of particles having a more complicated shape.

Appendix

Let us present explicit expressins for quantities F_{2x}, F_{2z} and F_{3x}, F_{3z}, entering expression (20):

$$F_{2x} = \frac{\varepsilon_\mathrm{m}}{8k_\mathrm{m}^2}\sum_{n=1}^{\infty}\sum_{m=1}^{n}(1+\delta_{1m})\frac{(n-1)(n+1)}{(2n-1)(2n+1)}\frac{(n+m)!}{(n-m)!}$$

$$\times\,\mathrm{Im}(G_{mn\mathrm{e}}G^{\mathrm{s}*}_{m-1\,n-1\,\mathrm{e}} + G^{\mathrm{s}}_{mn\mathrm{e}}G^{*}_{m-1\,n-1\,e})$$

$$+\frac{\varepsilon_\mathrm{m}}{8k_\mathrm{m}^2}\sum_{n=1}^{\infty}\sum_{m=1}^{n}(1+\delta_{1m})\frac{n(n+2)}{(2n+1)(2n+3)}\frac{(n+m)!}{(n-m)!}$$

$$\times\,\mathrm{Im}(G_{mn\mathrm{e}}G^{\mathrm{s}*}_{m-1\,n+1\,\mathrm{e}} + G^{\mathrm{s}}_{mn\mathrm{e}}G^{*}_{m-1\,n+1\,\mathrm{e}})$$

$$-\frac{\varepsilon_\mathrm{m}}{8k_\mathrm{m}^2}\sum_{n=1}^{\infty}\sum_{m=1}^{n}(1+\delta_{1m})\frac{1}{2n+1}\frac{(n+m)!}{(n-m)!}$$

$$\times\,\mathrm{Im}(Q_{mn\mathrm{o}}G^{\mathrm{s}*}_{m-1\,n\,\mathrm{e}} + Q^{\mathrm{s}}_{mn\mathrm{o}}G^{*}_{m-1\,n\,\mathrm{e}})$$

$$+\frac{\varepsilon_\mathrm{m}}{8k_\mathrm{m}^2}\sum_{n=2}^{\infty}\sum_{m=2}^{n}\frac{1}{2n+1}\frac{(n+m)!}{(n-m)!}\times$$

$$\times\,\mathrm{Im}(G_{mn\mathrm{e}}Q^{\mathrm{s}*}_{m-1\,n\,\mathrm{o}} + G^{\mathrm{s}}_{mn\mathrm{e}}Q^{*}_{m-1\,n\,\mathrm{o}})$$

$$+\frac{\varepsilon_\mathrm{m}}{8k_\mathrm{m}^2}\sum_{n=2}^{\infty}\sum_{m=2}^{n}\frac{(n-1)(n+1)}{(2n-1)(2n+1)}\frac{(n+m)!}{(n-m)!}$$

$$\times\,\mathrm{Im}(Q_{mn\mathrm{o}}Q^{\mathrm{s}*}_{m-1\,n-1\,\mathrm{o}} + Q^{\mathrm{s}}_{mn\mathrm{o}}Q^{*}_{m-1\,n-1\,\mathrm{o}})$$

$$+\frac{\varepsilon_\mathrm{m}}{8k_\mathrm{m}^2}\sum_{n=2}^{\infty}\sum_{m=2}^{n}\frac{n(n+2)}{(2n+1)(2n+3)}\frac{(n+m)!}{(n-m)!}$$

$$\times\,\mathrm{Im}(Q_{mn\mathrm{o}}Q^{\mathrm{s}*}_{m-1\,n+1\,\mathrm{o}} + Q^{\mathrm{s}}_{mn\mathrm{o}}Q^{*}_{m-1\,n+1\,\mathrm{o}}), \qquad \text{(A1)}$$

$$F_{2z} = \frac{\varepsilon_\mathrm{m}}{4k_\mathrm{m}^2}\sum_{n=1}^{\infty}\sum_{m=0}^{n}(1+\delta_{0m})\frac{n(n+2)}{(2n+1)(2n+3)}\frac{(n+m+1)!}{(n-m)!}$$

$$\times\,\mathrm{Im}(G_{mn\mathrm{e}}G^{\mathrm{s}*}_{m\,n+1\,\mathrm{e}} + G^{\mathrm{s}}_{mn\mathrm{e}}G^{*}_{m\,n+1\,\mathrm{e}}) +$$

$$+\frac{\varepsilon_{\rm m}}{4k_{\rm m}^2}\sum_{n=1}^{\infty}\sum_{m=1}^{n}\frac{m}{2n+1}\frac{(n+m)!}{(n-m)!}$$

$$\times\,\mathrm{Im}(G_{mn\rm e}Q_{mn\rm o}^{\rm s*}+G_{mn\rm e}^{\rm s}Q_{mn\rm o}^{*})$$

$$+\frac{\varepsilon_{\rm m}}{4k_{\rm m}^2}\sum_{n=1}^{\infty}\sum_{m=1}^{n}\frac{n(n+2)}{(2n+1)(2n+3)}\frac{(n+m+1)!}{(n-m)!}$$

$$\times\,\mathrm{Im}(Q_{mn\rm o}Q_{m\,n+1\,\rm o}^{\rm s*}+Q_{mn\rm o}^{\rm s}Q_{m\,n+1\,\rm o}^{*}), \qquad \text{(A2)}$$

$$F_{3x}=\frac{\varepsilon_{\rm m}}{4k_{\rm m}^2}\sum_{n=1}^{\infty}\sum_{m=1}^{n}(1+\delta_{1m})\frac{(n-1)(n+1)}{(2n-1)(2n+1)}$$

$$\times\frac{(n+m)!}{(n-m)!}\mathrm{Im}(G_{mn\rm e}^{\rm s}G_{m-1\,n-1\,\rm e}^{\rm s*})$$

$$+\frac{\varepsilon_{\rm m}}{4k_{\rm m}^2}\sum_{n=1}^{\infty}\sum_{m=1}^{n}(1+\delta_{1m})\frac{n(n+2)}{(2n+1)(2n+3)}$$

$$\times\frac{(n+m)!}{(n-m)!}\mathrm{Im}(G_{mn\rm e}^{\rm s}G_{m-1\,n+1\,\rm e}^{\rm s*})$$

$$-\frac{\varepsilon_{\rm m}}{4k_{\rm m}^2}\sum_{n=1}^{\infty}\sum_{m=1}^{n}(1+\delta_{1m})\frac{1}{2n+1}\frac{(n+m)!}{(n-m)!}\mathrm{Im}(Q_{mn\rm o}^{\rm s}G_{m-1\,n\,\rm e}^{\rm s*})$$

$$+\frac{\varepsilon_{\rm m}}{4k_{\rm m}^2}\sum_{n=2}^{\infty}\sum_{m=2}^{n}\frac{1}{2n+1}\frac{(n+m)!}{(n-m)!}\mathrm{Im}(G_{mn\rm e}^{\rm s}Q_{m-1\,n\,\rm o}^{\rm s*})$$

$$+\frac{\varepsilon_{\rm m}}{4k_{\rm m}^2}\sum_{n=2}^{\infty}\sum_{m=2}^{n}\frac{(n-1)(n+1)}{(2n-1)(2n+1)}\frac{(n+m)!}{(n-m)!}\mathrm{Im}(Q_{mn\rm o}^{\rm s}Q_{m-1\,n-1\,\rm o}^{\rm s*})$$

$$+\frac{\varepsilon_{\rm m}}{4k_{\rm m}^2}\sum_{n=2}^{\infty}\sum_{m=2}^{n}\frac{n(n+2)}{(2n+1)(2n+3)}\frac{(n+m)!}{(n-m)!}\mathrm{Im}(Q_{mn\rm o}^{\rm s}Q_{m-1\,n+1\,\rm o}^{\rm s*}), \qquad \text{(A3)}$$

$$F_{3z}=\frac{\varepsilon_{\rm m}}{2k_{\rm m}^2}\sum_{n=1}^{\infty}\sum_{m=0}^{n}(1+\delta_{0m})\frac{n(n+2)}{(2n+1)(2n+3)}\times$$

$$\times\frac{(n+m+1)!}{(n-m)!}\mathrm{Im}(G_{mn\rm e}^{\rm s}G_{m\,n+1\,\rm e}^{\rm s*})$$

$$+\frac{\varepsilon_{\rm m}}{2k_{\rm m}^2}\sum_{n=1}^{\infty}\sum_{m=1}^{n}\frac{m}{2n+1}\frac{(n+m)!}{(n-m)!}\mathrm{Im}(G_{mn\rm e}^{\rm s}Q_{mn\rm o}^{\rm s*})$$

$$+\frac{\varepsilon_{\rm m}}{2k_{\rm m}^2}\sum_{n=1}^{\infty}\sum_{m=1}^{n}\frac{n(n+2)}{(2n+1)(2n+3)}$$

$$\times\frac{(n+m+1)!}{(n-m)!}\mathrm{Im}(Q_{mn\rm o}^{\rm s}Q_{m\,n+1\,\rm o}^{\rm s*}). \qquad \text{(A4)}$$

References

1. Ashkin A., Dziedzic J.M., Bjorkholm J.E., Chu S. *Opt. Lett.,* **11**, 288 (1986).
2. Ashkin A., Dziedzic J.M., Yamane T. *Nature*, **330**, 769 (1987).
3. Ashkin A., Dziedzic J.M. *Science*, **235**, 1517 (1987).
4. Austin R.H., Brody J.P., Cox E.C., Duke T., Volkmuth W. *Phys. Today*, **50**, 32 (1997).
5. *Live Cell Imaging*. Ed. by R.D. Goldman, D.L. Spector (Cold Spring Harbor, NY: CSHL Press, 2005).
6. Rohrbach A., Stelzer E.H.K. *J. Opt. Soc. Am. A*, **18**, 839 (2001).
7. Askar'yan G.A. *Zh. Eksp. Teor. Fiz*., **42**, 1567 (1962).
8. Ashkin A. *Phys. Rev. Lett*., **24**, 156 (1970).
9. Smith P.W., Ashkin A., Tomlinson W.J. *Opt. Lett*., **6**, 284 (1981).
10. Palmer A.J. *Opt. Lett*., **5**, 54 (1980).
11. Afanas'ev A.A., Rubinov A.N., Mikhnevich S.Yu., Ermolaev I.E. *Zh. Eksp. Teor. Fiz*., **128**, 451 (2005).
12. Afanas'ev A.A., Katarkevich V.M., Rubinov A.N., Efendiev T.Sh. *Zh. Prikl. Spektrosk*., **69**, 675 (2002).
13. Debye P. *Ann. Physik*, **30**, 57 (1909).
14. Davis L.W. *Phys. Rev. A,* **19**, 1177 (1979).
15. Gouesbet G., Maheu B., Grehan G. *J. Opt. Soc. Am. A*, **5**, 1427 (1988).
16. Barton J.P., Alexander D.R. *J. Appl. Phys*., **66**, 2800 (1989).
17. Gouesbet G. *Appl. Opt*., **35**, 1543 (1996).
18. Neves A.A.R., Fontes A., Pozzo L.D.Y., de Thomaz A.A., Chillce E., Rodriguez E., Barbosa L.C., Cesar C.L. *Opt. Express*, **14**, 13101 (2006).
19. Niemen T.A., Loke V.L.Y., Stigoe A.B., Knoner G., Branczyk A.M., Heckenberg N.R., Rubinsztein-Dunlop H. *J. Opt. A: Pure Appl. Opt*., **9**, S196 (2007).
20. Jones A.R. *J. Phys. D: Appl. Phys*., **7**, 1369 (1974).
21. Chu W.P., Robinson D.M. *Appl. Opt*., **16**, 619 (1977).
22. Li J., Chylek P. *J. Opt. Soc. Am. A*, **10**, 687 (1993).
23. Stratton J.A. *Electromagnetic Theory* (New York: McGraw-Hill, 1941; Moscow: GITTL, 1948).
24. Mie G. *Ann. Physik*, **25**, 377 (1908).
25. Kerker M. *The Scattering of Light* (New York: Acad. Press, 1969).
26. Abramowitz M., Stigun I. (Eds) *Handbook of Mathematical Functions* (New York: Dover, 1964; Moscow: Nauka, 1979).
27. Landau L.D., Livshits E.M. *Electrodynamics of Continuous Media* (Oxford: Pergamon Press, 1984; Moscow: Nauka, 1986).
28. Harada Y., Asakura T. *Opt. Commun*., **124**, 529 (1996).
29. Rubinov A.N., Afanas'ev A.A., Kurochkin Yu.A., Mikhnevich S.Yu. *Nonlin. Phen. Compl. Syst*., **4**, 123 (2001).