MEASUREMENT OF LASER RADIATION PARAMETERS

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Geometrical method for calculating the group velocity dispersion of a stretcher taking into account the influence of optical system parameters

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***Abstract*. It is shown that a stretcher consisting of a mirror optical system and reflecting diffraction gratings can be calculated by geometrical optics methods. Expressions are derived which describe the influence of parameters of the stretcher optical system (including its aberrations) on the stretcher group velocity dispersion.**

Keywords: *chirped pulse amplifier, compressor, stretcher, optical ray reversibility, tautochronism, caustic, point–angle eikonal, isoplanar optical system.*

1. Introduction

Modern advances in the development of optics of ultrashort and superhigh-power pulses are closely related to the method of chirped pulse amplification (CPA) [1]. To avoid the damage of an active crystal exposed to a high-power ultrashort pulse, this pulse is reversibly stretched in time ('chirped') in the CPA method by using a delay line with the positive group velocity dispersion, which is called a stretcher. Then, the energy of this pulse is amplified in the amplifier active crystal and its initial duration is restored with the help of a delay line with the negative group velocity dispersion (a compressor). The dispersion delay line is a filter with a purely phase transfer function of the type $\exp[-i\varphi(\omega)]$, and, therefore, it is completely characterised by is the group delay $\tau(\omega) \equiv \partial\varphi(\omega)/\partial\omega$ [2].

In the ideal case, to avoid the increase in the duration of the initial ultrashort laser pulse during CPA, the sum of the group delays of the stretcher [$\tau_s(\omega)$], the amplifier active body [$\tau_g(\omega)$] and the compressor [$\tau_c(\omega)$] should be frequency-independent [3]:

$$\tau_c(\omega) + \tau_g(\omega) + \tau_s(\omega) = \text{const}. \quad (1)$$

Note that the compressor and stretcher conventionally used for CPA consist only of reflecting optical elements (mirror diffraction gratings and spherical mirrors), i.e. the effect of the optical medium of these elements on the total group delay is absent. It is known that the group delay $\tau(\omega) \equiv \partial\varphi(\omega)/\partial\omega$ in the free space coincides with the phase delay $T(\omega) \equiv \varphi(\omega)/\omega$. Therefore, the group and phase delays in the compressor and stretcher are equal and, hence, expression (1) can be rewritten in the form

$$T_c(\omega) + \tau_g(\omega) + T_s(\omega) = \text{const}, \quad (2)$$

where $T_c(\omega)$ and $T_s(\omega)$ are the compressor and stretcher phase delays, respectively.

The phase delay is the time proportional to the optical path length, i.e. it is the quantity used in geometrical optics to formulate the Fermi principle and in calculations of the optical difference in the ray paths in the geometrical theory of interference and diffraction [4]. Therefore, the phase delays of the compressor and stretcher can be calculated based on geometrical optics methods. An important division of geometrical optics is, in particular, computational optics studying the methods for calculating aberrations of optical systems [4].

It was shown in papers [5, 6] that the parameters of the stretcher optical system affect the dispersion of its group velocities. The aim of this paper is to describe the effect of the optical system parameters (including its aberrations) on the group velocity dispersion of the stretcher by using geometrical optics methods.

2. Optical ray reversibility principle and its consequence

One of the fundamental principles of geometrical optics is the ray reversibility principle [4]: a ray of light will always pass through an optical system from the end point to the initial point (Fig. 1b) along the same trajectory as from the initial point to the end point (Fig. 1a). Note an important corollary of the reversibility principle: if two similar optical elements are combined so that radiation passes through one of them upon the forward and through another upon the backward passage of light rays, these rays at the input and output of the system will always be parallel (Fig. 1c). This statement will be called below the Lemma.

Although the ray reversibility principle is usually applied to systems containing only reflecting and refracting optical elements, it can be also easily extended to optical systems containing diffraction gratings. Indeed, if a beam of monochromatic radiation with the frequency ω (a monochromatic ray of light) is incident on a diffraction grating,

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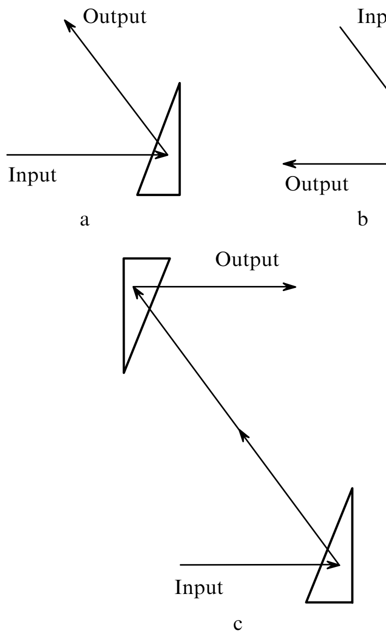

Figure 1. Optical systems for the forward (a) and backward (b) ray propagations and the geometrical interpretation of the Lemma (c).

the angle of incidence ψ and the angle of diffraction φ_ω will be related by the known expression

$$\sin\psi + \sin\varphi_\omega = l\frac{2\pi c}{d\omega}, \tag{3}$$

where d is the diffraction grating constant; c is the speed of light in vacuum; and l is the diffraction order. It is known that, by selecting properly the groove profile of a reflection diffraction grating, it is possible to concentrate the reflection of almost all the incident radiation energy to one diffraction order [7].

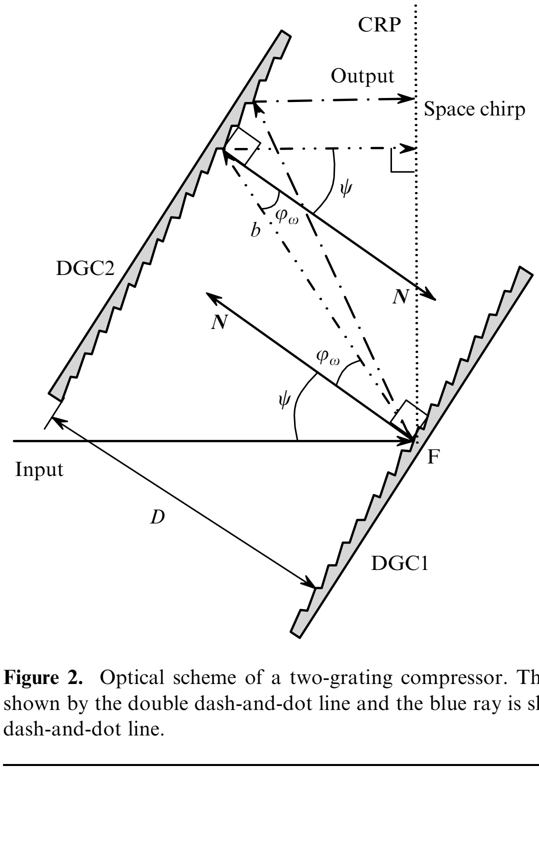

Figure 2. Optical scheme of a two-grating compressor. The red ray is shown by the double dash-and-dot line and the blue ray is shown by the dash-and-dot line.

The reversibility principle is manifested in the symmetry of diffraction grating expression (3) with respect to the interchange of angles ψ and φ_ω. Therefore, according to the Lemma, a pair of identical diffraction gratings with parallel reflecting surfaces facing each other has a unique property: a monochromatic ray of light diffracted from these gratings will exit the system in the direction parallel to the input ray (Fig. 2).

3. The group velocity dispersion of a compressor

Treacy showed [8, 9] that the above-described pair of parallel reflection diffraction gratings is the delay line with the negative group velocity dispersion. Indeed, if a narrow-wave beam with the broadband temporal spectrum (a polychromatic light ray) is incident on the first diffraction grating of the compressor at point F at an angle of ψ, the different spectral components of this beam will diffract at different angles φ_ω, by forming a one-dimensional homocentric beam of spectrally coloured rays with the centre at point F (Fig. 2). Because the first and second diffraction gratings are parallel, according to the Lemma, this homocentric beam will be transformed after its diffraction from the second diffraction grating to a beam of spectrally coloured rays (a spatial chirp), each of them being parallel to the input ray.

To calculate the phase delay $T_c(\omega)$ for different monochromatic rays from the above-mentioned parallel beam, we will introduce the unified compressor reference plane (CRP) – the intersecting plane, which is perpendicular to this beam and passes through point F (Fig. 2). The phase delay $T_c(\omega)$ of the spectral component at frequency ω with respect to this plane is equal to the path length divided by the speed of light c from point F of the ray incidence on the first diffraction grating to the intersection point of the selected monochromatic ray with the CRP [8, 9]:

$$T_c(\omega) = \frac{b[1+\cos(\psi+\varphi_\omega)]}{c}, \quad b = \frac{D}{\cos\varphi_\omega}, \tag{4}$$

where D is the distance between diffraction gratings.

4. Group velocity dispersion of a stretcher with an ideal optical system

To realise the CPA method, apart from a compressor a dispersion optical device with the positive group velocity dispersion (stretcher) is required. Martinez showed in [10, 11] that the stretcher can be formed by placing an optical system between diffraction gratings. However, the 'wave' explanation of the stretcher operation presented in these papers does not allow the description of the influence of the optical system parameters on the stretcher phase delay [5, 6].

First, we assume for simplicity that the optical system placed between the stretcher diffraction gratings is ideal, i.e. it transforms the homocentric beam of rays of light coming out from point P in the object space into a homocentric beam of rays of light converging at the optically conjugate point P′ in the image plane. The necessary and sufficient condition for the existence of an ideal optical system is the condition of the spatial tautochronism [12] according to which the propagation time for all rays of light connecting two optically conjugate points P and P′ should be the same and equal to T_0. Note that, if the ideal system has the

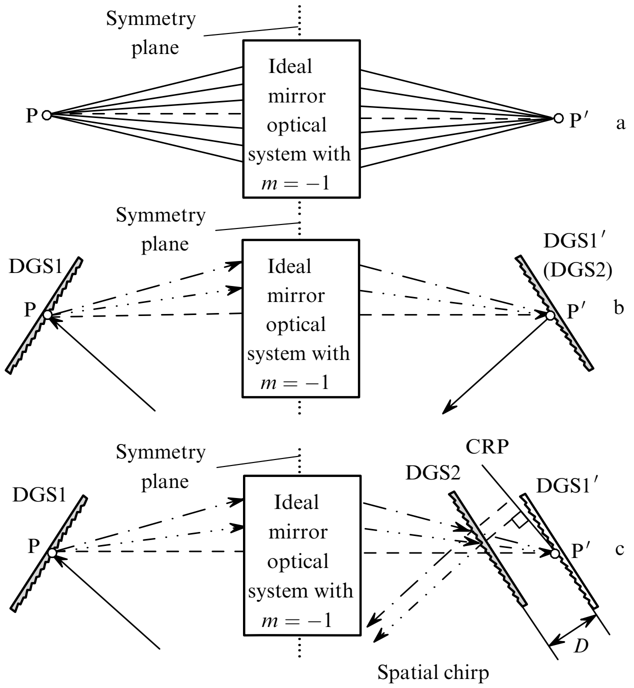

Figure 3. Optical schemes of the stretcher; ideal optical system with the mirror symmetry (a), scheme of optical conjugation of two diffraction gratings in the ideal mirror-symmetric optical system (b) and the ideal mirror-symmetric optical system and the virtual two-grating compressor DGC2–DGC1′ (c).

magnification $m = -1$, then points P and P′ are located mirror-symmetrically with respect to the symmetry plane (Fig. 3a) (although the object and the image at these points are symmetric with respect to the intersection point of this mirror symmetry plane with the optical axis of the system).

Let point P be the point of incidence of the input polychromatic light ray on the first diffraction grating of the stretcher (DGS1) with an ideal optical system. This ray is transformed after diffraction into a diverging homocentric beam of spectrally coloured rays with the centre at point P (Fig. 3b), which is then transformed by the ideal optical system into a beam of spectrally coloured rays converging at point P′. In addition, this optical system produces the image of the first diffraction grating DGS1 in the vicinity of point P′. Because the optical system under study has the magnification $m = -1$ (Fig. 3a) and optically conjugate points P and P′ lie on the optical axis of this system, the diffraction grating–object DGS1 and its image DGS1′ as well as beams of spectrally coloured rays in the object and image spaces will be mirror-symmetric with respect to the symmetry plane (Fig. 3b). According to the ray reversibility principle, if instead of the image of the first diffraction grating DGS1′, the second diffraction grating DGS2 with the same parameters as those of the image of the first grating DGS1′ is placed at point P′ (Fig. 3b), then the beam of spectrally coloured rays diffracted from the second grating will be again transformed into a polychromatic light ray emerging from point P′. In this case, the input and output chirps will be also mirror symmetric.

If the second diffraction grating is shifted along the optical axis of the system, a virtual two-grating compressor [13] (Fig. 3c) with a phase delay $T_{\mathrm{vc}}(\omega)$ will be formed in the image space of this system. Let us show that if the reference plane of this virtual compressor is treated as the reference plane of the stretcher (RPS), the stretcher phase delay $T_{\mathrm{s}}(\omega)$ calculated with respect to the RPS will be equal with an accuracy to a constant T_0 to the phase delay $T_{\mathrm{vc}}(\omega)$ of the virtual compressor with the opposite sign.

For this purpose, we construct a stretcher–compressor system in which all diffraction gratings are parallel and have the same parameters, while the distance between the gratings in the real and virtual compressors are the same and equal to D (Fig. 4), i.e. the phase delays of the real $T_{\mathrm{c}}(\omega)$ and virtual $T_{\mathrm{vc}}(\omega)$ compressors are similar:

$$T_{\mathrm{c}}(\omega) = T_{\mathrm{vc}}(\omega). \tag{5}$$

Consider this system for the backward passage of the ray. The polychromatic light ray falls obliquely on the first diffraction grating of the compressor DGC1 and acquires a spatial chirp at the output from the second diffraction grating of the compressor DGC2. Then, this spatial chirp is incident on the second diffraction grating of the stretcher DGS2 (Fig. 4). Because the second diffraction gratings of the compressor DGC2 and stretcher DGS2 are parallel and their surfaces reflecting light face each other, according to the Lemma (Fig. 1c), each of the light rays starting from point F of the compressor corresponds to a parallel light ray starting as if from point P′ of the stretcher. The ideal optical system of the stretcher will collect all these spectrally coloured rays at point P, which is optically conjugate with point P′ Because the optical system has the magnification $m = -1$ and points P and P′ lie on the optical axis, the incident and output polychromatic rays as well as homocentric beams of spectrally coloured rays are mirror

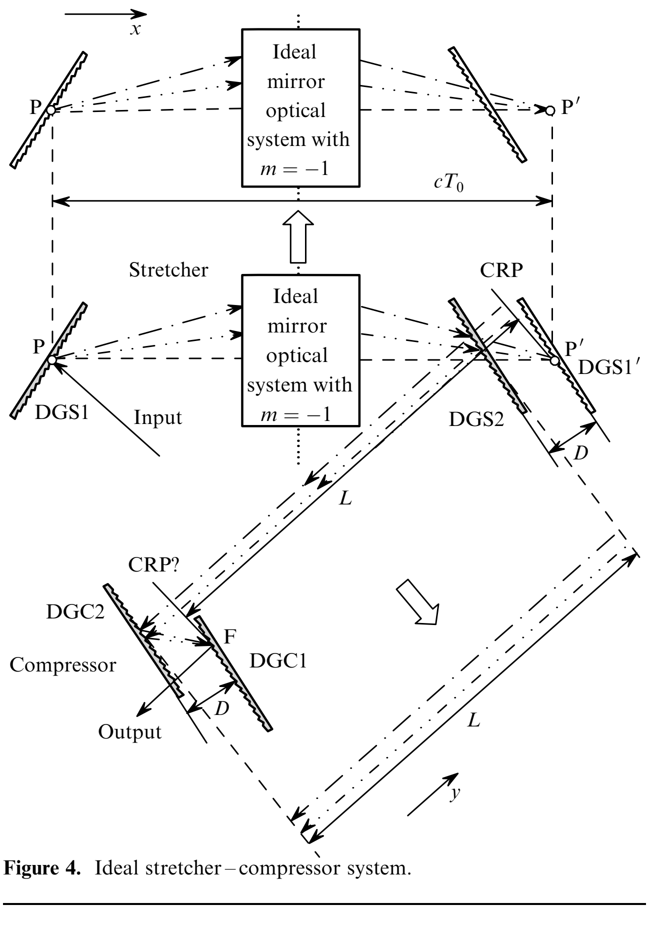

Figure 4. Ideal stretcher–compressor system.

symmetric in the object and image spaces. Because the first diffraction grating of the stretcher DGS1 is located at point P, the beam of coloured rays converging at point P will be again transformed after diffraction from this grating, according to the ray reversibility law, into a polychromatic light ray.

In the compressor – stretcher system, a ray of monochromatic radiation (the pulse spectral component at frequency ω) propagates from point F to point P along a broken trajectory with sections that are parallel either to the optical system axis (the x axis) or to the straight line parallel to the light beam direction at the compressor input (the y axis). Therefore, if the system under study is projected on these axes (Fig. 4), it is easy to notice that the transit time of light from point P$'$ to point P is equal to T_0, and the transit time of light from the second diffraction grating of the compressor DGC2 to the second diffraction grating of the stretcher DGS2, which is parallel to it, is equal to L/c, where L is the distance between these gratings. Thus, the transit time t of light along the entire compressor – stretcher system from point P to point F for all spectral components of the light pulse is the same:

$$t = L/c + T_0. \tag{6}$$

Because all the diffraction gratings in the system under study are parallel, the distance between the reference planes of a real (CRP) and virtual (VCRP) compressors is also equal to L and the transit time t of light along the entire compressor – stretcher system from point P to point F can be presented in the form:

$$t = L/c + T_{\mathrm{s}}(\omega) + T_{\mathrm{c}}(\omega). \tag{7}$$

By equating expressions (6) and (7) and taking (5) into account, we obtain the required relation between the virtual compressor phase delay $T_{\mathrm{vc}}(\omega)$ with the stretcher phase delay $T_{\mathrm{s}}(\omega)$ and the transit time T_0 of the light pulse in an ideal optical system with the magnification $m = -1$:

$$T_{\mathrm{s}}(\omega) = T_0 - T_{\mathrm{vc}}(\omega). \tag{8}$$

Thus, if the optical system of the stretcher is ideal and has a mirror symmetry, and point P of incidence of a polychromatic light beam lies on the optical axis of the system, the stretcher phase delay $T_{\mathrm{s}}(\omega)$ with an accuracy to a constant T_0 will be equal and opposite in sign to the phase delay of its virtual compressor $T_{\mathrm{vc}}(\omega)$.

Note that the simplest ideal mirror optical system with the magnification $m = -1$ is a spherical mirror and, hence, the stretcher with a single spherical mirror (although two- or three-component optical systems are usually used in the stretcher) is also quite efficient [14].

5. Group velocity dispersion of the stretcher with a real optical system

Note that, according to (8), for the specified parameters of diffraction gratings and their tilt with respect to the symmetry axis of an ideal optical system, the stretcher phase delay $T_{\mathrm{s}}(\omega)$ is unambiguously determined by the transit time T_0 of light from point P$'$ to its optically conjugate point P of the optical system with the magnification $m = -1$ and the distance D between the diffraction gratings of the virtual compressor. We will show how the aberrations of the stretcher optical system affect these parameters determining the stretcher dispersion.

If an ideal optical system transforms a broad homocentric light beam emerging from point P into a homocentric beam converging at point P$'$, which is optically conjugate with it, the real optical system transforms a broad diverging homocentric light beam into a converging non-homocentric beam having a 'beak-shaped' envelope with the top at the point of the paraxial image P$'$, the so-called caustic $f(z)$ (Fig. 5). It is known [15] that each point A(z,f) of the caustic in the meridional plane can be interpreted as an image of a point source P produced by an infinitely thin homocentric beam emerging from this source at an angle θ (Fig. 5).

Let the first diffraction grating of the stretcher DGS1 be inclined at angle α to its optical axis (Fig. 5). A polychromatic light ray (or an ultrashort broadband pulse) incident at angle ψ on the first diffraction grating DGS1 will be transformed after diffraction into a homocentric set of spectrally coloured rays with a top at the incidence point P. In this case, the spectral component with the frequency ω is diffracted at angle φ_ω to the normal of the grating, and, hence, at angle $\theta_\omega = \varphi_\omega - \alpha$ to the stretcher optical system (Fig. 5). The optical system of the real stretcher transforms this set of rays into a converging non-homocentric set of spectrally coloured rays, each of them being focused at different points of the caustic (Fig. 5): the red beam is focused at one point of the caustic, the yellow one – at another, the blue one – at a third point, etc. Due to this, the change in the angle θ_ω at which the spectral component with the frequency ω will emerge from point P, will result in the

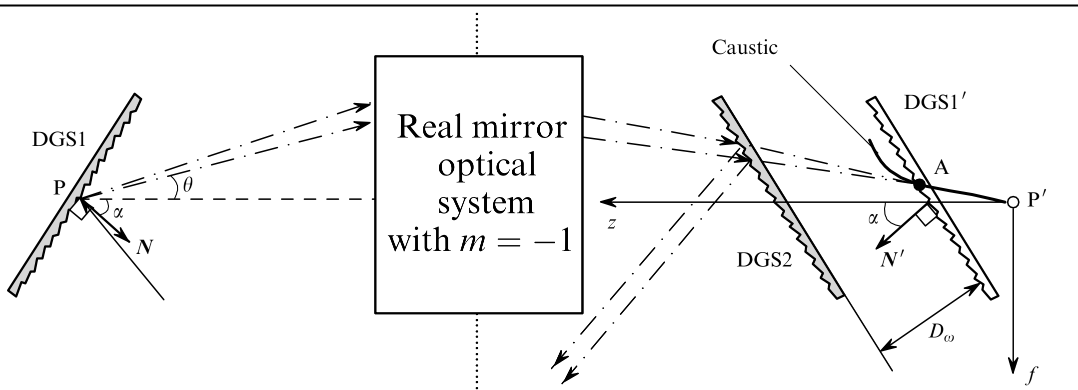

Figure 5. Effect of the optical system caustic on the parameters of the virtual compressor DGS2 – DGS1$'$.

change in the distance D_ω between the image of the first diffraction grating and the second real diffraction grating (the distance between diffraction gratings of the virtual compressor) as well as in the change in the time T_ω during which a light ray propagates the distance between the point source P and its image – the caustic point $A(z,f)$.

One can see from Fig. 5 that the distance D_ω is related to the caustic shape $f(z)$ and the inclination angle α of gratings with respect to the symmetry axis of the stretcher optical system by a simple relation

$$D_\omega = D - \boldsymbol{r}_\omega \boldsymbol{N}. \tag{9}$$

Here, $\boldsymbol{r}_\omega \equiv (z_\omega, f_\omega)$ is the vector representation of the caustic; $\boldsymbol{N} \equiv (\cos\alpha, \sin\alpha)$ is the normal to the image of the first diffraction grating of the stretcher in the image space. Taking into account expression (4), the dependence of the phase delay $T_{\mathrm{vc}}(\omega)$ of the virtual compressor on the caustic $\boldsymbol{r}_\omega$ of the stretcher optical system (for the given parameters of diffraction gratings) is exhaustively described by the expression

$$T_{\mathrm{vc}}(\omega) = (D - \boldsymbol{r}_\omega \boldsymbol{N}) \frac{1 + \cos(\psi + \varphi_\omega)}{c \cos\varphi_\omega}. \tag{10}$$

Thus, expression (8) for the calculation of the stretcher phase delay with the ideal optical system is easily generalised for the case of the real optical system:

$$T_{\mathrm{s}}(\omega) = T[\sin(\varphi_\omega - \alpha)] - (D - \boldsymbol{r}_\omega \boldsymbol{N}) \frac{1 + \cos(\psi + \varphi_\omega)}{c \cos\varphi_\omega}. \tag{11}$$

The caustic shape $\boldsymbol{r}_\omega$ as the function $T[\sin(\varphi_\omega - \alpha)]$ is determined by the aberrations of the optical system. It is especially simple to describe aberrations of an isoplanar (spatially invariant) optical system in which aberrations are the same in the entire field of view (see Appendix). In the stretcher, $m = -1$, $n = n' = 1$, and $p = \sin(\varphi_\omega - \alpha)$, thus taking (A12) into account we obtain

$$T[\sin(\varphi_\omega - \alpha)]c = W[\sin(\varphi_\omega - \alpha)] + \sin(\varphi_\omega - \alpha) \left.\frac{\partial W(\xi)}{\partial \xi}\right|_{\xi=p} - \left[1 - \sin^2(\varphi_\omega - \alpha)\right] \left.\frac{\partial^2 W(\xi)}{\partial \xi^2}\right|_{\xi=p}, \tag{12}$$

where $W(\xi)$ is the wave aberration function.

6. Conclusions

Thus, by using the principles of ray reversibility and tautochronism, we constructed a geometrical model of the stretcher with the ideal optical system with the unit magnification. We have shown that the constructed geometrical model can be generalised to the case of an optical system with caustic, which allows us to take into account the influence of aberrations of the stretcher optical system in the calculations of its group velocity dispersion.

Appendix. Caustic of an isoplanar optical system

According to Luneburg [16], the symmetry axis z of the optical system is conveniently described as an analogue of the time axis in the analytical mechanics. In this case, the change in the distance from the light beam to the optical system in the meridional cross section of the optical system is described by the function $x(z)$, and the propagation direction of the light beam – by the 'velocity'

$$\dot{x} = \frac{\mathrm{d}x}{\mathrm{d}z} = \tan\vartheta \tag{A1}$$

or 'pulse'

$$p \equiv \frac{\partial \mathscr{L}}{\partial \dot{x}} = n(x,z) \frac{\dot{x}}{\sqrt{1 + \dot{x}^2}} = n(x,z) \sin\vartheta. \tag{A2}$$

Here, $\mathscr{L}(\dot{x}, x, z) \equiv n(x,z)\sqrt{1 + \dot{x}^2}$ is the Lagrangian; $n(x,z)$ is the refractive index of the optical medium at point (x,z); ϑ is the inclination angle of the beam to the optical axis of the system.

The parameters x, p fully describe the propagation trajectory of a straight light beam in the object space and the parameters x' and p' – in the image space. The exhaustive characteristic of the optical system aberrations is eikonals, for example, the point–angle eikonal $V(x,p')$, which has interesting differential properties [15–17]:

$$p = -\frac{\partial V(x,p')}{\partial x}, \qquad x' = -\frac{\partial V(x,p')}{\partial p'}. \tag{A3}$$

Note that the point–angle eikonal of the ideal optical system with the linear magnification m is

$$V_{\mathrm{id}}(x,p') = -mxp'. \tag{A4}$$

By substituting it into expression (A3) we obtain

$$p = mp', \qquad x' = mx, \tag{A5}$$

i.e. the ray pulse p in the object space is proportional to the ray pulse p' in the image space and all rays from point x of the object space will be collected at point x' of the image space.

The simplest optical system with aberrations is the isoplanar (invariant with a shift) optical system, whose point–angle eikonal V_{iso} can be obtained by adding to the point–angle eikonal of the ideal optical system (A4) the function of wave aberrations $W(p')$ [17–19]:

$$V_{\mathrm{iso}}(x,p') = -mxp' + W(p'). \tag{A6}$$

By substituting (A6) to expression (A3) we obtain

$$p = mp', \tag{A7a}$$

$$x' + mx = -\left.\frac{\partial W(\xi)}{\partial \xi}\right|_{\xi=p'}. \tag{A7b}$$

The quantity $g = x' + mx$ is called the transverse aberration of the optical system (with a negative magnification m).

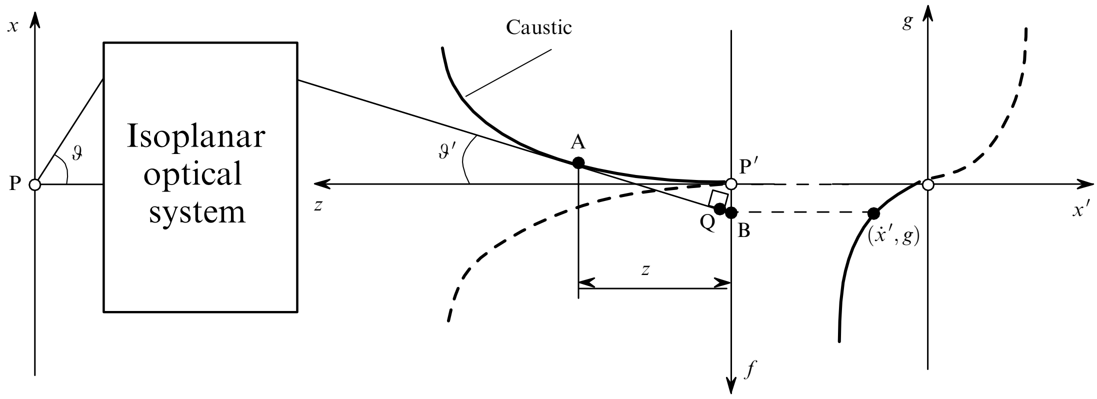

Figure 1A. The light ray path length PQ as a geometrical interpretation of the wave aberration $W(m^{-1}\sin\vartheta)$ and the Legendre transform relating the caustic (z, f) and the transverse spherical aberration $(\dot{x}', g)$.

Note that because the direction of the light ray in the image space of the optical system can be equivalently characterised by the pulse p' and the velocity $\dot{x}'$, the transverse spherical aberration (A7b) can be written as the velocity function:

$$g(\dot{x}') = -\frac{\partial W(\xi)}{\partial \xi}\bigg|_{\xi=n'\dot{x}'/\sqrt{1+\dot{x}'^2}}.$$

It was shown in [20] that the transverse aberration $g(\dot{x}')$ of the optical system and its caustic $f(z)$ are related by the Legendre transformation (Fig. 1A)

$$f(z) = \dot{x}z - g(\dot{x}), \tag{A8}$$

where $z \equiv \mathrm{d}g/\mathrm{d}\dot{x}$. Taking expression (A7b) into account and using (A8), we obtain

$$z = \frac{\partial g}{\partial \dot{x}'} = -\frac{\partial p'}{\partial \dot{x}'}\frac{\partial^2 W(\xi)}{\partial \xi^2}\bigg|_{\xi=p'} = -\left(\frac{\partial \dot{x}'}{\partial p'}\right)^{-1}\frac{\partial^2 W(\xi)}{\partial \xi^2}\bigg|_{\xi=p'}$$
$$= -\frac{(n'^2-p'^2)^{3/2}}{n'^2}\frac{\partial^2 W(\xi)}{\partial \xi^2}\bigg|_{\xi=p'}. \tag{A9}$$

The wave aberration has a simple geometrical interpretation: it is equal to the optical length of the light ray path from point P to the perpendicular Q′ lowered from point P′ on the light ray with the pulse p' in the image space of the optical system (Fig. 1A). One can see from Fig. 1A that the required quality $C(p') = |\mathrm{PA}|$ is related to the wave aberration $|PQ|$ by the relation

$$C(p') \equiv |\mathrm{PA}| = |\mathrm{PQ}| + |\mathrm{QB}| - |\mathrm{AB}| = W(p')$$
$$+ p'g - \frac{z}{\sqrt{n'^2 - p'^2}}, \tag{A10}$$

because according to the geometrical interpretation of the Legendre transforms in the theory of eikonals [17–19], we have $|\mathrm{QB}| = p'g$.

Thus, by combining expressions (A7b), (A8), (A9) and (A10) we obtain

$$C(p') = W(p') - p'\frac{\partial W(\xi)}{\partial \xi}\bigg|_{\xi=p'}$$
$$+ \frac{n'^2 - p'^2}{n'^2}\frac{\partial^2 W(\xi)}{\partial \xi^2}\bigg|_{\xi=p'}. \tag{A11}$$

Taking into account expression (A7a) this expression can be presented as a pulse function p in the object space:

$$C\left(\frac{p}{m}\right) = W\left(\frac{p}{m}\right) - \frac{p}{m}\frac{\partial W(\xi)}{\partial \xi}\bigg|_{\xi=p/m}$$
$$+ \frac{(mn')^2 + p^2}{(mn')^2}\frac{\partial^2 W(\xi)}{\partial \xi^2}\bigg|_{\xi=p/m}. \tag{A12}$$

The sought propagation time is $T(n\sin\vartheta) = C(nm^{-1}\sin\vartheta) \times c^{-1}$.

References

1. Mourou G.A., Barty C.P.J., Perry M.D. *Phys. Today*, **51**, 22 (1998).
2. Gitin A.V. *Kvantovaya Elektron.*, **36**, 376 (2006) [*Quantum Electron.*, **36**, 376 (2006)].
3. Kane S., Squier J. *J. Opt. Soc. Am. B*, **14**, 1237 (1997).
4. Jenkins F.A., White H.E. *Fundamentals of Optics* (New York, Toronto, London: McGraw-Hill, 1957).
5. White W.E., Patterson F.G., Combs R.L., Price D.F., Shepherd R.L. *Opt. Lett.*, **18**, 1343 (1993).
6. Sullivan A., White W.E. *Opt. Lett.*, **20**, 192 (1995).
7. Boyd R.D., Britten J.A., Decker D.E., Shore B.W., Stuart B.C., Perry M.D., Li L. *Appl. Opt.*, **34**, 1697 (1995).
8. Treacy E.B. *Phys. Lett. A*, **28**, 34 (1968).
9. Treacy E.B. *IEEE J. Quantum Electron.*, **5**, 454 (1969).
10. Martinez O.E. *IEEE J. Quantum Electron.*, **23**, 59 (1987).
11. Martinez O.E. *IEEE J. Quantum Electron.*, **23**, 1385 (1987).
12. Landsberg G.S. *Optika* (Optics) (Moscow: Nauka, 1976).
13. Naik P.A., Sharma A.K. *J. Opt. A: Pure Appl. Opt.*, **29** (3), 105 (2000).
14. http:/www.clf.rl.ac.uk/Reports/1999-2000/pdf/87.pdf.
15. Walther A. *The Ray and Wave Theory of Lenses* (New York: Cambridge University Press, 1995).
16. Luneburg R.K. *Mathematical Theory of Optics* (Berkely, Los Angeles: California University Press. 1964).
17. Brouwer W., Walther A., in *Advanced Optical Techniques* (Amsterdam: North Holland Publ. Co., 1967).
18. Gitin A.V. *Trudy GOI*, **70**, 4 (1988).
19. Gitin A.V. *Opt. Zh.*, **60**, 12 (1993) [*Sov. J. Opt. Technol.*, **60**, 372 (1993)].
20. Gitin A.V. *Opt. Commun.*, **281**, 3062 (2008).