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Parametric reflection upon cascade interaction of focused optical beams

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***Abstract*. The parametric reflection of a signal beam in the waist of the reference pump beam upon mismatched three-frequency interaction in a quadratically nonlinear medium is discussed. The critical angle of total internal reflection from the induced defocusing channel is found as a function of the beam waist parameters. It is shown that when the reference beam is focused, this angle increases and some distortions are introduced into the reflected wave due to a finite length of the waist. The modification of the cross section of a wave reflected from a convex parametric mirror is analysed. The optimal beam focusing geometry is found at which the distortions of the shape and divergence of the reflected wave are minimal. Under certain conditions, the signal wave also flows around a cylindrical inhomogeneity produced by the axially symmetric pump beam. The results of theoretical analysis and numerical simulation are in good agreement.**

Keywords: *nonlinear optics, three-wave interaction, cascade process, diffraction, focusing, refraction, reflection.*

1. Introduction

Photonics studying various methods of manipulating light by light is being recently rapidly developed. A great attention is paid to the problem of controlling light beams by the methods of nonlinear optics [1 – 6]. For example, all-optical control of light beams based on controlling spatial solitons was comprehensively studied in photonics [2, 3]. We proposed and described a fundamentally new parametric mechanism of switching optical beams without frequency change, which uses mismatched noncollinear three-wave interaction [7 – 9]. This method is based on the effects of refraction and total internal reflection in a quadratic medium and its principle is that upon mismatched parametric interaction the high-power reference wave together with the cascade-excited sum-frequency wave produce the effective transverse inhomogeneity of the refractive index of the medium at the signal frequency, which repeats the intensity profile of the reference wave. During the propagation of the signal through the induced inhomogeneity, a peculiar refraction appears. As a result, the beam trajectory is bent and nonlinear total internal reflection of the signal wave from the reference beam can occur (Fig. 1). Note that in this case, the sum-frequency wave is concentrated only in a narrow reflecting layer, where it turns due to refraction.

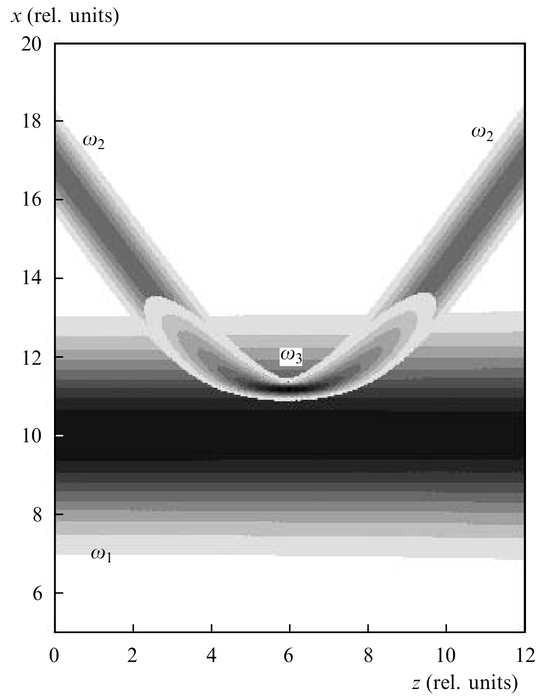

Figure 1. Parametric reflection of the signal beam from the collimated reference beam upon mismatched parametric interaction.

It was shown [7 – 9] that total internal reflection is possible if the initial angle θ_2 at which the signal waves propagates with respect to the reference beam is smaller than the critical angle θ_{cr} determined by the depth of the negative cascade-induced inhomogeneity of the refractive index:

$$\theta_{\mathrm{cr}} = C\left(4\gamma_2\gamma_3 k_3 k_1^{-1} k_2^{-2}\right)^{1/4} E_{1\max}^{1/2}, \qquad (1)$$

where $E_{1\max}$ is the maximum field amplitude on the reference beam axis; $k_j = n_j\omega_j/c$ is the wave vector of the wave at frequency ω_j and $\omega_3 = \omega_2 + \omega_2$; n_j is the refractive

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index; $j = 1 - 3$; c is the speed of light; $\gamma_{2,3}$ is the quadratic nonlinearity coefficient; and C is a numerical parameter depending on the beam envelope shape. Numerical simulation showed that parametric reflection can be observed at angles $\theta_2 \sim 0.5^\circ$, when the nonlinear modulation depth of the refractive index is $\sim 10^{-4}$. In this paper, we study the conditions for increasing the limiting reflection angle by using the optimal focusing of the reference beam.

Note that reflection from a plane parametric mirror – a planar beam, almost does not distort the amplitude profile of the signal wave. At the same time, the axially symmetric reference beam forms a cylindrical inhomogeneity whose shape is determined by the beam intensity profile. In this case, the signal beam reflecting from the cylindrical mirror can experience strong distortion in a plane perpendicular to the plane of incidence. For comparable widths of the beams, the reflected wave becomes diverging and its spatial structure becomes crescent-shaped. These effects can be diminished by focusing preliminarily the signal beam by a cylindrical lens. This problem is also considered in our paper.

2. Reflection from a focused beam

It follows from (1) that to increase the critical angle of total internal reflection from the defocusing reference beam or to decrease the required pump power, it is necessary to increase the peak amplitude $E_{1\,\max}$ of the reference beam. This can be achieved in the simplest way by decreasing the beam diameter. However, the narrowing of the beam enhances the influence of diffraction: the waist in which the signal wave is reflected decreases. In addition, the focusing parameters should be chosen so that the waist is in the region of parametric interaction. Therefore, it is necessary to select optimal focusing parameters taking into account different factors.

We will describe the cascade interaction of the beams by representing the wave amplitudes at the input to a quadratically nonlinear medium in the form

$$A_1(x,0) = E_1 \exp\left(-\frac{x^2}{a_1^2} + \frac{\mathrm{i}k_1 x^2}{2R_1}\right),$$
$$A_2(x,0) = E_2 \exp\left[-\frac{(x-d)^2}{a_2^2} + \mathrm{i}k_2\theta_2 x\right], \tag{2}$$

where R_1 is the initial radius of curvature of the reference beam wavefront; $a_{1,2}$, d, and θ_2 are the radii of the beams, the position of the signal-beam centre, and the signal-beam inclination angle with respect to the reference beam at the input to the nonlinear medium, respectively; x is the transverse coordinate; and $E_2/E_1 \sim 0.01$. It follows from (2) that the intersection region of the beams is located at a distance of $l_{\mathrm{int}} \approx d/\theta_2$, while the reference beam waist – at a distance of $l_{\mathrm{w}} = R_1/(1 + R_1^2/l_{\mathrm{dif}}^2)$, where $l_{\mathrm{dif}} = k_1 a_1^2/2$ is the diffraction length [10]. By equating these two lengths, we obtain the expression for the optimal radius of curvature of the wavefront of the reference beam at the input to the medium:

$$R_1 = l_{\mathrm{dif}}\left[m + \left(m^2 - 1\right)^{1/2}\right], \quad m = \frac{\theta_2 l_{\mathrm{dif}}}{2d}. \tag{3}$$

The maximum amplitude in the waist increases by a factor of $[1 + (l_{\mathrm{dif}}/R_1)^2]^{1/4}$, which, according to (1), allows one either increase the range of admissible inclination angles of the signal beam or to decrease the required power of the reference beam. We performed numerical simulations of the interaction of crossed optical beams, which confirmed the expediency of using optimal focusing (Fig. 2).

Consider some features of the process. First, it should be taken into account that due to parametric refraction, the signal beam propagates along a curve resembling a parabola. The top of the parabola determines a turning point. It approaches the reference beam axis with increasing the inclination angle θ_2 of the signal wave and decreasing the reference beam amplitude $E_{1\,\max}$. As a result, the optimal focal distance R_1 providing the minimal threshold power of the reference beam can insignificantly differ from the value predicted by expression (3).

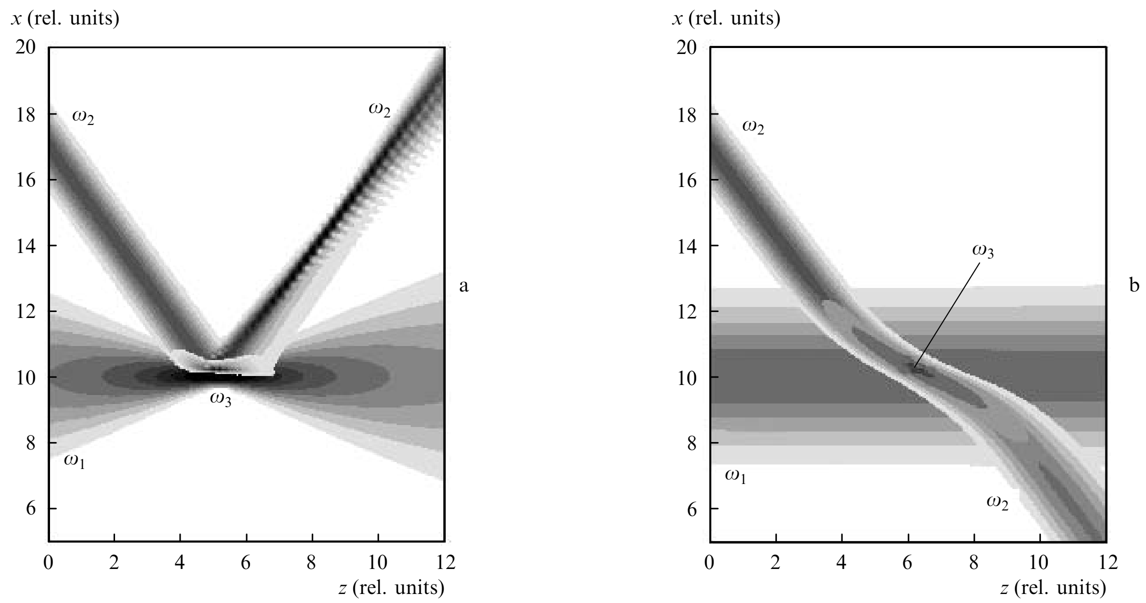

Figure 2. Two regimes of cascade interaction at the same pump power: reflection of the signal beam in the narrow waist of the focused reference beam (a) and its propagation through a broad collimated beam (b).

Second, during the interaction of the beams in the reflected wave, the amplitude of the component at the sum frequency can strongly increase. This can be avoided by introducing a small negative dispersion mismatch $\Delta k_{\text{dis}} = k_1 + k_2 - k_3 < 0$. Note that total internal reflection appears due to mutual repulsion of the beams in a defocusing medium, while cascade nonlinearity becomes defocusing in the case of the negative total mismatch of the wave vectors.

The introduction of the dispersion mismatch Δk_{dis} changes the limiting angle of reflection. This mismatch is added to the vector mismatch to give the effective mismatch

$$\Delta k_{\text{eff}} = \Delta k_{\text{dis}} - \frac{k_1 k_2}{2k_3}\,\theta_2^2. \tag{4}$$

Taking (4) into account, we find from the reflection condition $\theta_2^2 \leqslant 2\gamma_2\gamma_3 E_{1\max}^2/(-\Delta k_{\text{eff}})$ the equation for the limiting angle

$$\theta_{\text{cr}}^4 + \alpha\theta_{\text{cr}}^2 - \theta_{\text{cr}\,0}^4 = 0, \tag{5}$$

where $\theta_{\text{cr}\,0}$ is the limiting angle at the zero dispersion mismatch $\Delta k_{\text{dis}} = 0$ and $\alpha = -2k_3\Delta k_{\text{dis}}/(k_1 k_2) > 0$ is the relative mismatch. By solving equation (5), we obtain

$$\theta_{\text{cr}}^2 = \left(\frac{\alpha^2}{4} + \theta_{\text{cr}\,0}^4\right)^{1/2} - \frac{\alpha}{2}. \tag{6}$$

This means that $\theta_{\text{cr}} < \theta_{\text{cr}\,0}$, i.e. the critical inclination angle decreases after the introduction of the additional negative wave mismatch. However, if the introduced mismatch is small enough ($|\Delta k_{\text{dis}}| \ll k_1 k_2 \theta_{\text{cr}\,0}^2/k_3$) and the parameter α is much smaller than unity, the limiting angle decreases insignificantly and the generation of the sum-frequency wave is efficiently suppressed.

To obtain efficient reflection in the case of strong focusing, it is also necessary to decrease the signal beam width. This is explained by the fact that the stronger the beam focusing, the smaller the waist length and the reflection region length. Upon the reflection of a broad signal beam, a part of the beam falls to the region of the reference wave with a small amplitude and is not reflected (Fig. 3). Indeed, the signal beam interacts with the reference beam at the length $\Delta z_1 \approx 2a_2/\theta_2$ and the length of the reference beam waist at the level $N = E_1/E_{1\max}$ is

$$\Delta z_2 \approx \frac{2l_{\text{dif}}(N^{-4} - 1)^{1/2}}{1 + (l_{\text{dif}}/R_1)^2}.$$

Then, the condition $\Delta z_2 > \Delta z_1$ should be satisfied to obtain efficient reflection.

Another feature of the signal beam reflection from the focused reference beam is the distortion of the reflected signal profile. If the reference beam is not focused, the constant-amplitude curves are the straight lines and reflection occurs as if from a plane surface parallel to the beam pump axis. Upon focusing, the beam waist is formed and the longitudinal curvature of a parametric mirror appears, which changes the profile of the reflected signal wave (see Figs 1 and 2).

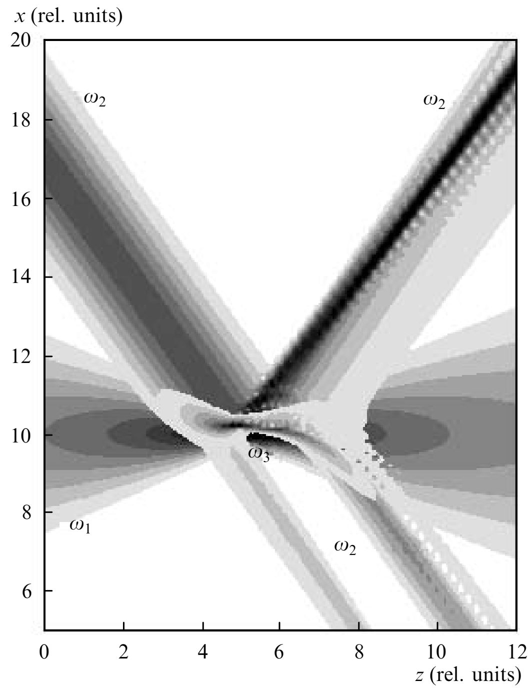

Figure 3. Partial reflection of a broad signal beam from a strongly focused reference beam.

3. Reflection from a cylindrical parametric mirror

If the beams are propagating in a planar waveguide, the amplitude profile of the signal wave reflected from the unfocused reference beam changes insignificantly. A more intricate picture appears in the case of the cylindrical reference beam. Indeed, a two-dimensional beam has the convex cylindrical surface whose curvature is determined by the shape of its cross section. It is known that the profile of a parametric inhomogeneity repeats the pump intensity distribution and, therefore, reflection from a parametric cylindrical mirror can strongly distort the signal beam in a plane perpendicular to the plane of incidence. In the case of comparable widths of the beams, the reflected wave becomes diverging and its spatial structure acquires the characteristic crescent-shaped form (see a frame for a collimated beam in Fig. 4) [9]. These effects can be reduced by focusing preliminarily the signal beam.

This problem was studied by solving numerically the system of equations for slowly varying amplitudes of the interacting waves:

$$\begin{aligned}
&\frac{\partial A_1}{\partial z} + \mathrm{i}D_1\Delta_\perp A_1 = 0,\\
&\frac{\partial A_2}{\partial z} + \mathrm{i}D_2\Delta_\perp A_2 = -\mathrm{i}\gamma_2 A_3 A_1^*,\\
&\frac{\partial A_3}{\partial z} + \mathrm{i}D_3\Delta_\perp A_3 = -\mathrm{i}\gamma_3 A_2 A_1,
\end{aligned} \tag{7}$$

where $\Delta_\perp = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is the Laplacian in the transverse coordinates x and y; and D_j is the diffraction coefficient ($j = 1-3$). We assume that the pump beam at the input to a nonlinear medium has the axially symmetric cross section, i.e.

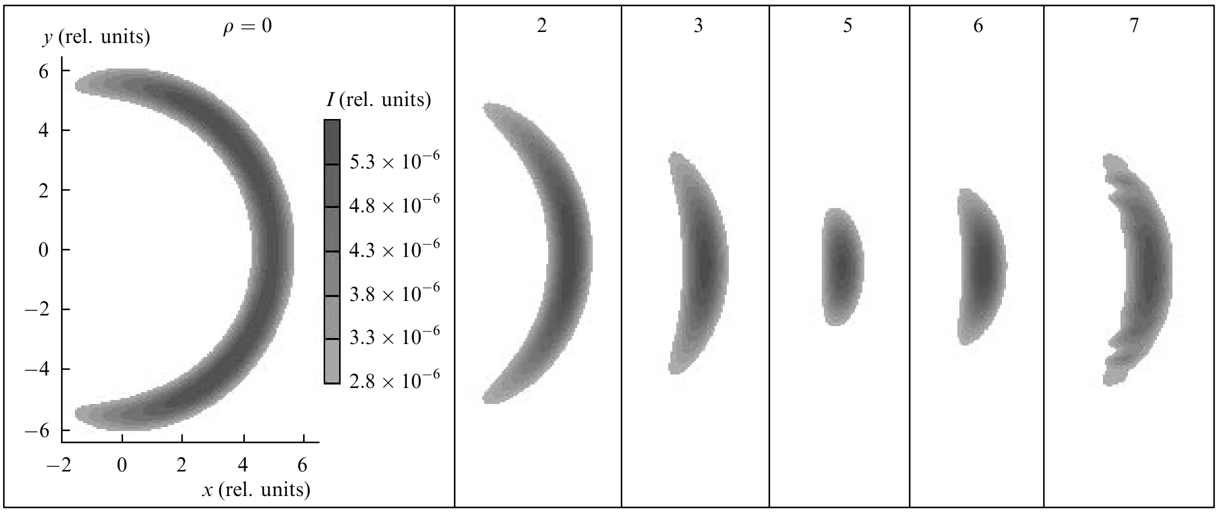

Figure 4. Change in the cross section of the reflected signal beam with increasing the initial curvature of its wavefront $\rho = R_1/l_{\rm dif}$ (I is the signal wave intensity).

$$A_1(x,y,0) = E_1(r) \qquad (8)$$

[where $r = (x^2 + y^2)^{1/2}$], the idler wave is absent, i.e.

$$A_3(x,y,0) = 0, \qquad (9)$$

and the signal beam is displaced with respect to the pump beam by the distance $d \gg a_1$, i.e.

$$A_2(x,y,0) = E_2(x-d,y)\exp({\rm i}\theta_0 x + {\rm i}\rho y^2), \qquad (10)$$

where ρ is the initial wavefront curvature normalised to the inverse diffraction length; θ_0 is the initial inclination angle of the beam normalised to the diffraction divergence angle $\theta_{\rm d} = 1/(k_2 a_2)$. We assume that the axes of the beam lie in the xz plane. A parametric mirror produced by the pump beam has a curvature in the xy plane. The radius of curvature is approximately equal to the pump beam radius. The reflection process as the result of three-particle interaction was simulated by solving numerically equations for the envelopes of three wave beams (7) with boundary conditions (8)–(10). The initial amplitude profiles had a Gaussian shape

$$E_j(x,y,0) = E_{0j}\exp\left(-\frac{x^2+y^2}{a_j^2}\right), \quad j = 1-3,$$

$$E_{02} \sim 0.01E_{01}, \quad E_{03} = 0.$$

The calculation shows that the reflected beam becomes crescent-shaped and strongly diverges (see a frame for a collimated beam in Fig. 4). The characteristic crescent-shape form of the reflected beam appears due to the flowing of the incident signal wave around the pump beam.

The influence of the parametric-mirror convexity can be compensated by focusing preliminarily the signal beam with a cylindrical lens in the yz plane (see frames in Fig. 4). Here, the centres of reflected beams are located at the same distance d from the pump beam axis as the incident beam. Figure 5 shows the change in the transverse dimensions of the reflected beam for different initial focusing (wavefront curvature).

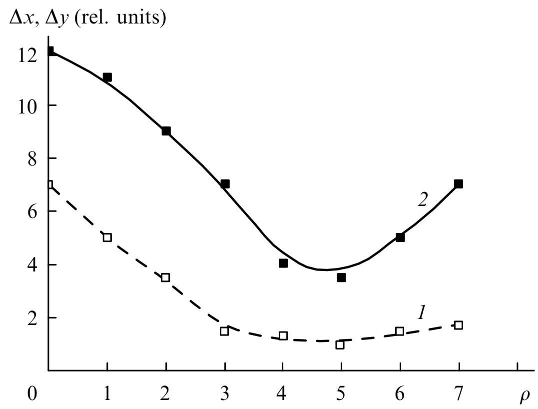

Figure 5. Dependences of the transverse dimensions Δx (1) and Δy (2) of the reflected signal beam on the initial curvature ρ of its wavefront.

The analysis of data presented in Figs 4 and 5 shows that the crescent-shaped cross section gradually decreases with increasing the focusing tightness and transforms to the oval cross section. Minimal distortions (the ratio of the transverse radii of the oval was 1 : 4) were achieved at the optimal focusing with $\rho \approx 5$ by a lens with the focal distance $f = 1/(4D_2\rho) = 4$. A tighter focusing ($\rho = 6$, 7) leads to the increase in the beam cross section. Distortions are reduced due to a decrease in the signal-beam width on the parametric mirror and a decrease in the wavefront curvature after the reflection of a converging wave.

Thus, we can conclude that the focusing of reference beams should be used only in the planar geometry because the influence of a convex mirror in the three-dimensional case will be enhanced and, therefore, the reflected beam will strongly diverge. In the three-dimensional case, it is necessary to focus simultaneously both the reference beam and signal wave to provide interaction in the reference-bream waist and to ensure the width of the interaction region considerably exceeding the signal-beam width.

Note that we studied the effect of a parametric mirror for a Bessel beam as well. In this case, optimal focusing also gives a positive result by reducing the divergence of the reflected wave.

Interesting effects appear when two signal beams are incident on the reference beam from opposite sides. Upon parametric interaction of collimated beams of the same width, a peculiar ring is formed which consists of two reflected crescent-shaped beams flowing around the reference beam (Fig. 6). On the ring a distinct interference pattern of dark and light fringes is formed. The distance between fringes is inversely proportional to the angle between the beams. The total number M of fringes in the ring is approximately equal to the ratio of this angle to the diffraction angle: $M \approx \theta_0$. The ring radius increases with increasing the propagated distance due to the initial inclination of signal beams. The reflected beams do not form a ring when their width decreases and their interference considerably weakens. A great part of the beam with a large initial width bypasses a parametric mirror whose role is played by the reference beam. In other words, we observed the diffraction of signal beams from the induced cylindrical inhomogeneity.

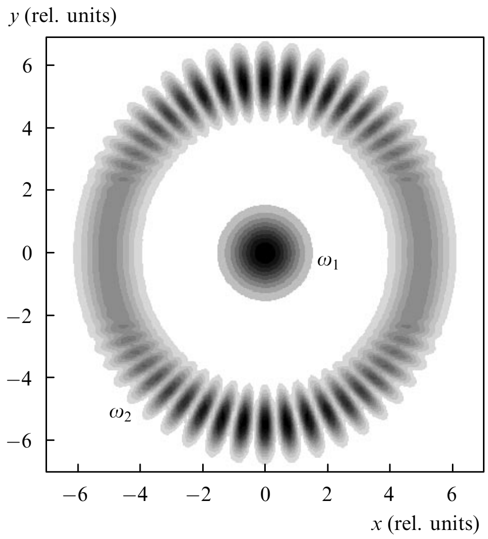

Figure 6. Diffraction from the induced cylindrical inhomogeneity: the interference fringe pattern appearing due to the flowing of two signal beams around the axially symmetric pump beam representing a cylindrical parametric mirror.

If signal beams are focused on the reference beam, the interference pattern disappears because in this case the reflected waves do not overlap. After interaction with the pump wave, they propagate independently of each other.

4. Conclusions

We have shown that the focusing of the reference beam provides the reduction of the power required for parametric reflection and extends the range of admissible inclination angles for observing total internal reflection in a quadratically nonlinear medium. The conditions for the optimal focusing of the reference beam have been found which provide the minimal pump intensity required for parametric reflection of the signal. The presence of a small negative mismatch of the wave vectors leads to a small decrease in the critical angle of reflection, but reduces the efficiency of generation of the sum-frequency wave. A decrease in the waist length in the case of the tight focusing of the main beam restricts the signal-beam width. In the waist the longitudinal curvature of a nonlinear mirror appears, which distorts the profile of the reflected beam. The transformation and divergence of the signal beam parametrically reflected from the cylindrical reference beam can be decreased by several times by focusing preliminarily this beam. In the case of optimal focusing, the crescent-shaped cross section of the reflected beam becomes oval.

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