LASER COOLING

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‘Microwave’ technique for atoms

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***Abstract*. The mechanisms of the transport of cold atomic ensembles and transformation of their parameters in potential channels and wells treated by analogy of electromagnetic microwave waveguides and hollow resonators are considered. The possibility of performing various manipulations with such ensembles, in particular, the isothermal phase transition to a Bose – Einstein condensate is pointed out.**

Keywords: *cold atoms, quantum traps, Bose – Einstein condensation, quantum nucleonic.*

1. Introduction

Experimental advances in deep laser cooling of neutral atoms down to picoelectron-volt energies and correspondingly down to de Broglie wavelengths of the order of micrometer, as well as the development of efficient quantum traps for such atoms (see, for example, [1]) give rise to almost trivial analogy between the behaviour of atomic wave functions in different quantum wells and electromagnetic waves in the elements of microwave technique [2], including various periodic structures [3]. This stimulates the consideration of the basic properties of the corresponding potential devices for atomic wave functions and their possible applications. The experimental development of such an elemental base for matter waves – ‘microwave’ atomic technique, would facilitate the refinement of the methods for manipulating the ensembles of free neutral atoms.

In this connection we can mention various space- and time-separated manipulations performed with atoms such as laser cooling and excitation of atoms (producing the population inversion), laser selection (separation) of atoms by some or other properties, including isotope and nuclear isomer separation, the observation of the Bose condensation, quantum interference and stimulated emission of optical and high-frequency photons, etc. [4, 5].

Unlike electromagnetic technique, where we are dealing practically always with almost monochromatic radiation, atomic ensembles can be treated as monoenergetic only in exclusive cases. The consideration presented below is based on the assumption of the thermodynamic equilibrium described by stationary distribution functions with atomic energies $0 \leqslant E \leqslant \infty$. The total energy E of an atom used in most of the calculations is related to the corresponding separated group.

2. Extended quantum channel (atomic waveguide) and a three-dimensional potential well (atomic resonator)

The basic element of various ‘microwave’ atomic devices is (like a waveguide in electromagnetic technique) a potential well confining atoms along two transverse coordinates x and y and allowing their free movement along the longitudinal axis z. Such a structure can be called a quantum channel or an atomic waveguide.

In experiments [1], the parabolic potential

$$U(r) = \frac{M}{2}(\Omega_x^2 x^2 + \Omega_y^2 y^2) = \frac{M}{2}\Omega_r^2 r^2 \qquad (1)$$

of a transverse well is used most often, where M is the atom mass; Ω_x and Ω_y are the characteristic frequencies of the well; and $r = (x^2 + y^2)^{1/2}$ is the radial coordinate. The second equality in (1) is valid for wells of circular profile with $\Omega_x = \Omega_y \equiv \Omega_r$.

The wave functions of atoms with the total energy E in an extended quantum channel of circular profile with harmonic confining transverse potential (1) have in the cylindrical coordinate system r, ϕ, z the form of waves travelling along the z axis:

$$\Psi_m(r, \phi, z, t) = \psi_m(r, \phi) \exp\left[-\frac{\mathrm{i}}{\hbar}(Et - p_m z)\right], \qquad (2)$$

where the radial factor $\psi_m(r, \phi)$ is the solution of the Schrödinger equation in the cross section [for harmonic potential (1), it is the Hermitean function];

$$p_m = \pm[2M(E - E_m)]^{1/2} \equiv \pm[2ME(1 - \rho_m)]^{1/2}$$

and $\qquad (3)$

$$E_m = \hbar\Omega_r(2m + 1) \equiv \rho_m E$$

are the longitudinal components of the momentum and the transverse (radial) eigenvalues of the atomic energy

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($m = 0$, 1) in the most interesting case of the absence of variations in the angular coordinate ϕ, which is investigated below.

Expressions (2) and (3) concern atoms with the selected energy E in the mth mode of an atomic waveguide. The atomic flux as a whole is described by summation over all the modes with $m = 0, 1, ...,$ taking into account the energy distribution of atoms.

Travelling waves (2) have the dispersion and propagate with the phase and group velocities

$$v_m = \frac{(E/2M)^{1/2}}{(1-\rho_m)^{1/2}}, \tag{4}$$

$$u_m = \frac{\mathrm{d}E}{\mathrm{d}p_m} = \left(\frac{2E}{M}\right)^{1/2}(1-\rho_m)^{1/2}. \tag{5}$$

The travelling waves of atoms with the selected energy E can experience reflections from the section in which two waveguide channels A and B with different phase velocities coaxially join. The coefficient of reflection from this section (for the same indices m) is

$$R = \left[\frac{(1-\rho_{m\mathrm{A}})^{1/2} - (1-\rho_{m\mathrm{B}})^{1/2}}{(1-\rho_{m\mathrm{A}})^{1/2} + (1-\rho_{m\mathrm{B}})^{1/2}}\right]^2. \tag{6}$$

The reflection coefficient for the total flux of atoms with different energies E belonging to modes with different indices m can be obtained by averaging.

If two sections with high enough reflection coefficients R (6) (which allows the approximation of a longitudinal trap by an infinitely deep potential well) are separated by a distance L, then, under the condition $\pi\hbar l/p_m = L$, a longitudinal standing wave with the index m appears in the channel interval between them, which has the energy eigenvalue

$$E_l = (2M)^{-1}(\pi\hbar l/L)^2 \equiv \eta_l E \tag{7}$$

($l = 1$, 2,). In this case, the waveguide channel becomes a three-dimensional trap – an atomic resonator (the cigar-shaped analogue of a hollow microwave resonator) with the total energy eigenvalue

$$E_{ml} = E(\rho_m + \eta_l) \equiv \varepsilon_{ml}E, \tag{8}$$

which for $\varepsilon_{ml} \equiv \rho_m + \eta_l = 1$ proves to be resonant for atoms with the selected energy $E = E_{ml}$.

If the confinement time of atoms in the trap is not infinite and their escape is observed (which in fact always occurs), a width ΔE_{ml} and, similarly to electromagnetic resonators, the Q factor equal to $Q = E_{ml}/\Delta E_{ml}$ can be assigned to the resonance state with energy E_{ml} (8).

Atomic resonators A and B with the energy eigenvalues $E_{ml\mathrm{A}}$ and $E_{ml\mathrm{B}}$ separated by a section with $R < 1$ prove to be coupled, but their energy eigenvalues remain invariable if the condition

$$1 - R < Q^{-1} \tag{9}$$

is fulfilled (weak coupling). In the case of strong coupling

$$1 - R > Q^{-1} \tag{10}$$

a system of two coupled atomic resonators acquires a complicated spectrum of energy eigenvalues, each of the modes with a fixed index forming a doublet with energies

$$E_{m(\pm)} = (2R_m)^{-1/2}(E_{ml\mathrm{A}}^2 + E_{ml\mathrm{B}}^2)^{1/2}$$
$$\times\{1 \pm [1 - 4R(E_{ml\mathrm{A}}E_{ml\mathrm{B}})^2(E_{ml\mathrm{A}}^2 + E_{ml\mathrm{B}}^2)^{-2}]^{1/2}\}^{1/2}. \tag{11}$$

A chain of periodically repeated identical or different intervals of the channel – atomic resonators coupled by sections with $R < 1$ (10) form a grating with the same dispersion characteristic as for known one-dimensional microwave periodic structures of the diaphragmed waveguide type, etc. [3].

If the cigar-shaped trap is approximated by a circular cylinder, we can assign to the lowest radial mode of the trap with $m = 0$ the effective area of the cross section filled with the selected group of atoms of this mode,

$$S_{m=0} = \frac{\pi D_{\mathrm{eff}}^2}{4} = \frac{\pi\hbar}{4M\Omega_r} = \frac{\pi\hbar^2}{4ME\rho_{m=0}}, \tag{12}$$

and the effective volume

$$V_{\mathrm{eff}(m=0,\,l)} = S_{m=0}L = \frac{\pi^2\hbar^3 l}{2^{5/2}M^{3/2}E^{3/2}\rho_{m=0}\eta_l^{1/2}}$$
$$= \frac{\pi^2\hbar^3 l}{2^{5/2}(ME)^{3/2}\rho_{m=0}(1-\rho_{m=0})^{1/2}}. \tag{13}$$

3. Atoms in a quantum channel and a cigar-shaped potential well

During the transport of atoms through a number of different atomic waveguides and cigar-shaped traps coupled with each other under the accepted condition of thermodynamic equilibrium, the parameters of atomic ensembles confined in them are subjected to a variety of transformations.

Thus, in an atomic resonator (trap) with the 'Q_{ml} factors' of energy states loaded with an atomic flux via a coupled atomic waveguide, the accumulation of atoms occurs and their equilibrium number

$$N = \hbar\Phi\sum_{m,l} Q_{ml}/E_{ml} \tag{14}$$

is established.

The 'microwave' technique considered here is close to a quantum conveyer discussed in [4–6], which is also based on the transport of neutral atoms along the extended quantum channel with the potential well whose width is not constant but continuously changes with increasing z. However, there is fear [6] that because of the absence of the longitudinal standing component of the wave function, such a channel with continuously changing parameters quite closely reproduces the properties of one-dimensional structures and therefore many operations mentioned in Introduction can be insignificant. The most illustrative and important example of such impossible operations for boson atoms is the Bose–Einstein condensation in a one-dimensional medium [7], in particular, in a quantum channel with freely longitudinally moving atoms.

Therefore, it is useful to consider the simplest pair of elements of 'microwave' technique consisting of an atomic resonator and a waveguide feeding it (14). As mentioned above, even when the temperature T of the atomic flux Φ in a waveguide representing a one-dimensional medium is low enough, the phase transition to a Bose condensate is impossible. However, after the accumulation of atoms in the resonator at a constant initial temperature T, this prohibition is removed if T is lower than the critical value [7]:

$$T < T_c = \frac{0.94}{k_B} (E_{m=0}^2 E_{l=1} N)^{1/3}, \tag{15}$$

where $E_{m=0} \equiv \hbar\Omega_r$ (3) and $E_{l=1}$ (7) are the energy eigenvalues of the transverse (radial) and longitudinal lowest states of the resonator with $m = 0$ and $l = 1$, respectively, and k_B is the Boltzmann constant. The critical temperature (15) can be conveniently expressed in terms of the effective volume $V_{\mathrm{eff}(m=0,l=1)}$ (13) of the lowest resonator mode:

$$T_c \approx \frac{\hbar^2}{k_B M} \left(\frac{N}{V^2_{\mathrm{eff}(m=0,\,l=1)}} \right)^{1/3}. \tag{16}$$

This means that Bose condensation takes place without any additional cooling of the atomic ensemble below the initial temperature $T = \mathrm{const}$ of the atomic flux Φ when the total number of atoms accumulated in the resonator exceeds the critical value:

$$N > N_c \approx (k_B T M/\hbar^2)^3 V^2_{\mathrm{eff}(m=0,\,l=1)}. \tag{17}$$

In this case, the number of atoms in the condensed fraction is $N_{\mathrm{BEC}} = N - N_c$.

Thus, this opens up attractive experimental possibilities for the formation of a Bose condensate in three stages separated in space and time, which include the preliminary cooling of gas in a remote element of the 'microwave' scheme, the transport of the gas along the atomic waveguide to a three-dimensional trap-resonator, and the *isothermal* Bose-condensation of the gas after the accumulation of the total number of atoms $N > N_c$ in the resonator.

Note also that it is possible to observe the quantum interference pattern of atoms in the Bose condensate in a trap with $l > 1$. This pattern is in fact the manifestation of the longitudinal resonance. In this case, to provide convenient measurements, the pattern period can be arbitrarily increased by choosing large phase velocity (4).

4. Conclusions

The 'microwave' technique considered in the paper can be used to perform the transport of cold atoms along coupled quantum channels (atomic waveguides) and quantum traps (resonators) accompanied by the controllable transformation of gas parameters.

One of the types of such a transformation of the gas ensemble of bosons, described by (17), is the phase transition to a Bose condensate, which occurs *isothermally*, i.e. the condition $T < T_c$ of the phase transition is fulfilled due to the removal of the prohibition related to the dimensionality of the gas ensemble at $T = \mathrm{const}$ rather than due to the decrease in temperature T, and no rapid cooling in the final unit of the 'microwave' scheme is required. This can be convenient in experiments, especially when they include a number of operations of different types, which were mentioned in Introduction.

Thus, the scenario of a complex experiment, which requires the formation of a Bose condensate among other operations, can be constructed based on a chain of coupled atomic waveguides and three-dimensional traps, in which different operations are performed successively in the corresponding units of the chain (as in the quantum conveyer scheme [4–6]).

The 'microwave' technique for atoms considered above can be used, in particular, in experiments with the aim to confirm the appearance of the 'mega atom' state in a Bose–Einstein condensate and the narrowing of radiative lines, as well as to observe the stimulated emission of hard photons, etc. [6].

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