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Production of broadband modal gain spectra in asymmetric multiple quantum-well $Ga_{0.47}In_{0.53}As/Ga_{0.18}In_{0.82}As_{0.4}P_{0.6}$ heterostructures

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***Abstract*. The modal gain spectra of asymmetric multiple quantum-well $Ga_{0.47}In_{0.53}As/Ga_{0.18}In_{0.82}As_{0.4}P_{0.6}$ heterostructures are theoretically analysed within the framework of the four-band *kp* method. An efficient procedure for obtaining the broadband and almost flat gain spectrum is proposed. The designs of semiconductor radiation sources with different sets of nonuniformly excited quantum wells producing broadband amplification in spectral ranges from 1.28 to 1.525 μm and from 1.36 to 1.6 μm are calculated.**

Keywords: *quaternary compounds, modal gain, **kp** method, set of quantum wells, nonuniform excitation, broadband spectrum, IR region.*

1. Introduction

The problems of coherent laser spectroscopy, fibreoptic communications, chemical analysis, metrology, and environment monitoring require the use of efficient tunable lasers covering the spectral range from 1.3 to 1.6 μm. Lasing in this region can be obtained in asymmetric multiple heterostructures based on ternary and quaternary $Ga_xIn_{1-x}As/Ga_xIn_{1-x}As_yP_{1-y}$ compounds [1–4] with a set of nonuniformly excited quantum wells of different widths.

The idea of nonuniform excitation of quantum wells in $GaAs/Al_xGa_{1-x}As$ compounds was proposed in [5, 6], where the broadband gain spectrum was obtained in the wavelength range between 0.79 and 0.85 μm. It was shown that the emission wavelength of interband transitions in quantum-well lasers depends on the thickness of active layers. Quantum wells of different widths amplify radiation in different wavelength ranges, and therefore the total gain spectrum of a quantum-well heterostructure with such quantum wells can cover a rather broad spectral range.

The width of the gain spectrum can be increased by using ternary and quaternary compounds with a narrower energy gap lying in the IR region. In this paper, we study the modal gain spectra of asymmetric multiple quantum-well heterostructures based on $Ga_{0.47}In_{0.53}As$ quantum wells and $Ga_{0.18}In_{0.82}As_{0.4}P_{0.6}$ barrier layers matched with an InP substrate.

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2. Theoretical calculation

The calculation of the energy levels and wave functions of $Ga_{0.47}In_{0.53}As/Ga_{0.18}In_{0.82}As_{0.4}P_{0.6}$ compounds should take into account the mixing of the subbands of the valence band. We performed calculations by the four-band ***kp*** method in the Luttinger and Kohn approximation [7–9] by using the finite element method [10, 11]. The parameters of numerical calculations for quaternary $A_xB_{1-x}C_yD_{1-y}$ compounds were approximated by experimental and theoretical data for ternary ($A_xB_{1-x}C$) and binary (AB) compounds by using the expression [12]

$$G_{ABCD}(x,y) = \frac{x(1-x)[yG_{ABC}(x) + (1-y)G_{ABD}(x)]}{x(1-x) + y(1-y)} + \frac{y(1-y)[xG_{ACD}(y) + (1-x)G_{BCD}(y)]}{x(1-x) + y(1-y)}, \quad (1)$$

$$G_{ABC}(x) = xG_{AC} + (1-x)G_{BC} - x(1-x)b_{ABC}.$$

Here, G may be the effective electron mass m_c, Luttinger parameters γ_1, γ_2, and γ_3, the spin–orbital splitting energy ΔS_0, the energy band gap (E_g), and the optical (ε_g) and static (ε) permittivities. These interpolations for $Ga_{0.47}In_{0.53}As$, $Ga_{0.18}In_{0.82}As_{0.4}P_{0.6}$ and InP compounds are presented in Table 1. The band energy diagram, energy levels and wave functions were calculated by assuming that potential well depths in the conduction and valence bands were 0.161 and 0.241 eV, respectively.

The gain $K_j(v)$ at the frequency v for TE and TM polarisations (j = TE or TM) in the model satisfying selection rules for the wave vector for a quantum well of width d was calculated from the expression [1, 9]

$$K_j(v) = \frac{e^2}{c\varepsilon_0 m_e^2 n_a v d}\int \frac{dk_\parallel}{(2\pi)^2}\sum_{n,i}|M_{mn}|_j^2\{f_e[E_{cn}(k_\parallel)] + f_h[E_{vin}(k_\parallel)] - 1\}F(hv - hv_0), \quad (2)$$

where $hv_0 = E_g + E_{cn}(k_\parallel) - E_{vin}(k_\parallel)$; E_g is the energy gap width; $E_{cn}(k_\parallel)$ and $E_{vin}(k_\parallel)$ are the energies of levels

Table 1. Parameters of the quantum-well $Ga_{0.47}In_{0.53}As/Ga_{0.18}In_{0.82}As_{0.4}P_{0.6}/InP$ heterostructure ($T = 300$ K) used in numerical calculations.

Material	m_c/m_e	γ_1	γ_2	γ_3	ΔS_0	E_g/eV	ε	ε_g
$Ga_{0.47}In_{0.53}As$	0.0430	13.8806	5.4732	6.2531	0.3296	0.735	14.09	11.47
$Ga_{0.18}In_{0.82}As_{0.4}P_{0.6}$	0.0646	9.9993	3.7764	4.5782	0.1914	1.137	13.25	10.52
InP	0.0795	5.0800	1.6000	2.1000	0.1080	1.357	12.50	9.61

involved in the radiative recombination of electrons and holes, respectively; f_e and f_h are the Fermi–Dirac distribution functions for electrons and holes, respectively; n_a is the refractive index of the active region of the structure; and $F(h\nu - h\nu_0)$ is the Gaussian profile of a broadened emission line. Apart from the fulfilment of the selection rule for k_x and k_y, expression (2) also assumes that the selection rule for the quantum number n is fulfilled.

The matrix elements $|\boldsymbol{M}_{ln}|_j^2$ of direct interband electronic transitions from the level l to the level n of a light or heavy hole, taking into account band mixing, have the form [1, 9]

$$|\boldsymbol{M}_{ln}|_{\mathrm{TE}}^2 = \frac{1}{4}|\boldsymbol{P}_{\mathrm{cv}}|^2\Big\{\langle\phi_l|f_n^{3/2}\rangle^2 + \langle\phi_l|f_n^{-3/2}\rangle^2 + \frac{1}{3}\Big(\langle\phi_l|f_n^{1/2}\rangle^2 + \langle\phi_l|f_n^{-1/2}\rangle^2\Big) + \frac{2}{\sqrt{3}}\Big(\langle\phi_l|f_n^{3/2}\rangle\langle\phi_l|f_n^{-1/2}\rangle + \langle\phi_l|f_n^{-3/2}\rangle\langle\phi_l|f_n^{1/2}\rangle\Big)\cos 2\varphi\Big\},$$

$$|\boldsymbol{M}_{ln}|_{\mathrm{TM}} = \frac{1}{3}|\boldsymbol{P}_{\mathrm{cv}}|^2\Big\{\langle\phi_l|f_n^{1/2}\rangle^2 + \langle\phi_l|f_n^{-1/2}\rangle^2\Big\}. \quad (3)$$

Here, $|\boldsymbol{P}_{\mathrm{cv}}|^2 = |\langle S|p_x|X\rangle|^2 = |\langle S|p_y|Y\rangle|^2 = |\langle S|p_z|Z\rangle|^2 \approx m_e^2 E_g/2m_c$; $\varphi = \arctan(k_y/k_x)$; ϕ is the envelope wave function in the conduction band; f_n^{υ} is the envelope wave function with the magnetic quantum numbers $\upsilon = 3/2, -1/2, 1/2, -3/2$; and $|S\rangle$ is the wave function corresponding to the s orbital.

Because not the whole electromagnetic wave is localised in a quantum well, it is necessary to calculate the modal gain. The optical confinement factor Γ can be found from the model of a three-layer waveguide [13]. For the ith quantum well and TE and TM polarisations, the optical confinement factors have the form [13]

$$\Gamma_{\mathrm{TE}} = d_i\left[d + \frac{2}{d(\varepsilon_{\mathrm{eff}} - \varepsilon_c)}\left(\frac{\varepsilon_c}{\varepsilon_i}\right)^{1/2}\left(\frac{\lambda}{2\pi}\right)^2\right]^{-1},$$

$$\Gamma_{\mathrm{TM}} = d_i\left[d + \frac{2}{d(\varepsilon_{\mathrm{eff}} - \varepsilon_c)}\left(\frac{\varepsilon_{\mathrm{eff}}}{\varepsilon_c}\right)\left(\frac{\varepsilon_i}{\varepsilon_c}\right)^{3/2}\left(\frac{\lambda}{2\pi}\right)^2\right]^{-1}, \quad (4)$$

where ε_i and ε_c are the permittivities of the ith quantum well and cladding layers, respectively; $\varepsilon_{\mathrm{eff}} = d^{-1}\int\varepsilon(z)\mathrm{d}z$ and $\varepsilon_{\mathrm{eff}}^{-1} = d^{-1}\int\varepsilon^{-1}(z)\mathrm{d}z$ are the effective permittivities for the TE and TM modes, respectively. The refractive indices for $Ga_{0.47}In_{0.53}As$, $Ga_{0.18}In_{0.82}As_{0.4}P_{0.6}$ and InP compounds at a wavelength of 1.5 μm are 3.79, 3.39, and 3.24, respectively. The total modal gain for a system of several quantum wells is

$$g(\nu) = \sum_i g_i(\nu) = \sum_i \Gamma_i K_i(\nu). \quad (5)$$

The electrophysical parameters of a quantum-well heterostructure were calculated from a system of equations including the Poisson equation for the electrostatic potential φ and the continuity equation for the densities of electron (j_e) and hole (j_h) currents [14]:

$$\frac{\partial^2\varphi}{\partial z^2} = -\frac{e}{\varepsilon\varepsilon_0}(p - n + N_d - N_a),$$

$$\frac{\partial j_e}{\partial z} = eR,\ \ j_e = \mu_e n\frac{\partial F_e}{\partial z},$$

$$\frac{\partial j_h}{\partial z} = -eR,\ \ j_h = \mu_h p\frac{\partial F_h}{\partial z}, \quad (6)$$

where n and p are the electron and hole concentrations, respectively; N_a and N_d are the concentrations of ionised acceptors and donors; R is the integrated recombination rate; μ_e and μ_h are the electron and hole mobilities, respectively; and F_e and F_h are quasi-Fermi levels for electrons and holes, respectively.

3. Discussion of results

Modal gain spectra were calculated in several stages. At the first stage, we calculated the energy and spectral characteristics for quantum wells of width 2–15 nm by the four-band ***kp*** method at different excitation levels. The data obtained at the first stage were used at the second stage to optimise the excitation levels of a semiconductor system containing several quantum wells of different widths for obtaining a broad and flat modal gain spectrum in the required wavelength region. At the third stage, based on the system of equations (6), electrophysical characteristics were determined and doping levels and thicknesses of barrier layers were found to obtain the nonuniform excitation levels of quantum wells calculated at the second stage.

Table 2 presents the ground-state energies, transition frequencies and wavelengths calculated for quantum wells of different widths. It follows from this table that, by using the sets of various quantum wells, we can obtain emission in a broad range from 1.3 to 1.6 μm. Figure 1 presents modal gain spectra numerically calculated for a multiple quantum-

Table 2. Ground-state energy levels and the transition frequencies and wavelengths calculated for different quantum-well widths.

d/nm	E_{c1}/meV	E_{v1}/meV	$h\nu_{c1v1}$/meV	λ_{c1v1}/μm
2	129	87	951	1.30
3	107	54	896	1.38
4	89	36	860	1.44
5	74	26	835	1.48
7	53	15	803	1.54
9	40	10	785	1.58
11	31	7	773	1.60
13	25	5.3	765	1.62
14	22	4.6	762	1.628
15	20	4.1	759	1.633

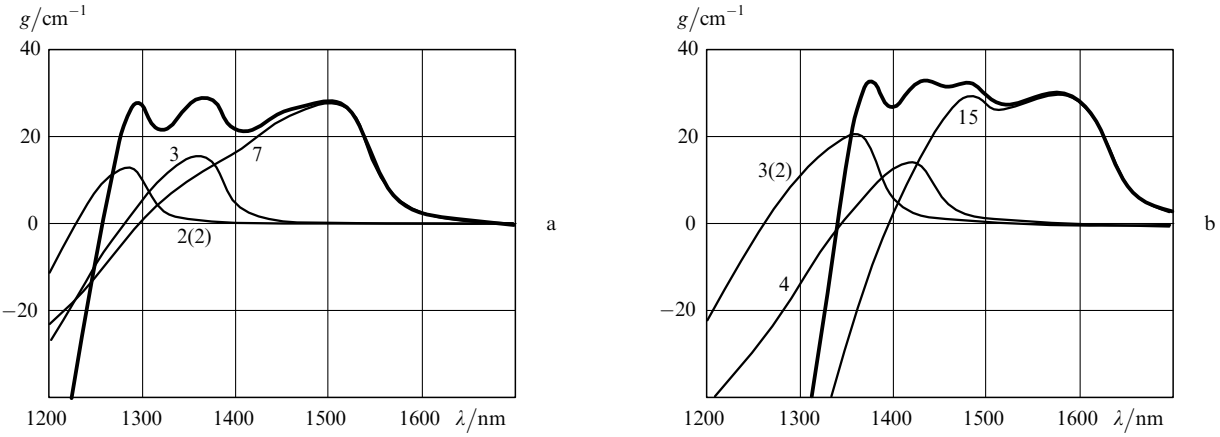

Figure 1. Modal gain spectra of the TE mode for the quantum-well $Ga_{0.47}In_{0.53}As/Ga_{0.18}In_{0.82}As_{0.4}P_{0.6}$ structure with four quantum wells with sets of widths 2(2)–3–7 nm (a) and 3(2)–4–15 nm (b) calculated upon nonuniform excitation of quantum wells and the total modal gain spectrum (thick curve). The numbers at the curves denote quantum-well widths d: (a) two wells with $d = 2$ nm, $\Delta F = 1.01$ eV, $\Gamma = 2.4 \times 10^{-3}$; a well with $d = 3$ nm, $\Delta F = 0.97$ eV, $\Gamma = 3.6 \times 10^{-3}$; a well with $d = 7$ nm, $\Delta F = 0.96$ eV, $\Gamma = 8.4 \times 10^{-3}$ ($U = 1.015$ V, $j = 355$ A cm^{-2}); (b) two wells with $d = 3$ nm, $\Delta F = 0.99$ eV, $\Gamma = 3.9 \times 10^{-3}$; a well with $d = 4$ nm, $\Delta F = 0.92$ eV, $\Gamma = 5.2 \times 10^{-3}$; a well with $d = 15$ nm, $\Delta F = 0.89$ eV, $\Gamma = 19.7 \times 10^{-3}$ ($U = 0.987$ V, $j = 373$ A cm^{-2}).

well heterostructure with two sets of four quantum wells. The spectral range of amplification at the maximum gain level $g_{max} = 25$ cm^{-1} in a system of quantum wells with widths 2(2)–3–7 nm is 1.28–1.525 μm. In this case, two uniformly excited quantum wells of width 2 nm each amplify at half the height in the spectral region from 1.25 to 1.3 μm, a quantum well of width 3 nm amplifies in the region from 1.31 to 1.39 μm, and a quantum of width 7 nm amplifies in a broad region from 1.36 to 1.54 μm.

When a set of quantum wells with widths 3(2)–4–15 nm is used, amplification at the level $g_{max} = 30$ cm^{-1} occurs in the region from 1.36 to 1.60 μm. In this case, two uniformly excited quantum wells of width 3 nm each amplify in the region from 1.29 to 1.39 μm, a quantum well of width 4 nm amplifies in the region between 1.36 and 1.54 μm, and a quantum well of width 15 nm amplifies in the region from 1.43 to 1.63 μm. Note that in both cases, to compensate for absorption by a broad quantum well in the short-wavelength region, two narrow quantum wells are used, and deviations from the specified g_{max} are ~ 3 cm^{-1}. By choosing other quantum wells, it is possible to obtain the higher maximum modal gain with a flat spectrum. For example, for the 4(2)–5–15-nm set, the amplification range for $g_{max} = 40 - 50$ cm^{-1} is 1.42–1.6 μm and deviations from g_{max} do not exceed 1 cm^{-1}.

Figure 2 presents the results of numerical solution of the system of equations (6) for the $Ga_{0.47}In_{0.53}As/Ga_{0.18}In_{0.82}As_{0.4}P_{0.6}$ structure with an InP substrate and a set of quantum wells of widths 2(2)–3–7 nm and 3(2)–4–15 nm. To control excitation levels in quantum wells (according to data in Fig. 1), the $Ga_{0.18}In_{0.82}As_{0.4}P_{0.6}$ and InP barrier layers between the second and third, as well as between the third and fourth quantum wells were doped with donor impurities at a concentration of 5×10^{17} cm^{-3}. In addition, the thickness of the barrier layers was properly selected. The calculated voltage U and injection current density j for the first and second structures were 1.015 V, 355 A cm^{-2} and 0.987 V, 373 A cm^{-2}, respectively.

Note that, as the pump current is increased, the gain spectrum transforms and its maximum shifts in the short-wavelength region from 1.25 to 1.35 μm due to increasing population mainly in narrow quantum wells, so that the gain in the long-wavelength region between 1.5 and 1.6 μm increases much slower. A similar behaviour of the total

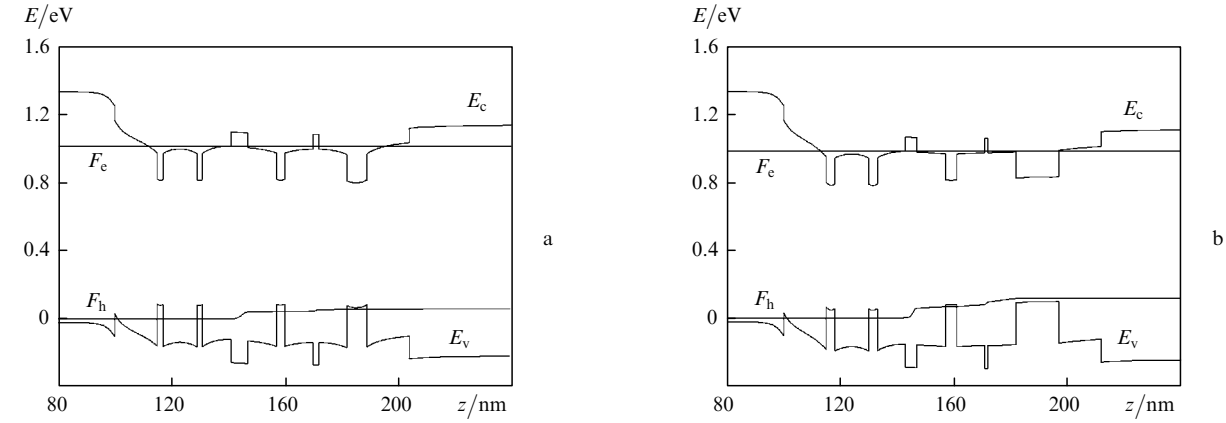

Figure 2. Distributions of the conduction band $E_c(z)$ and valence band $E_v(z)$ for the multiple $InP/Ga_{0.47}In_{0.53}As/Ga_{0.18}In_{0.82}As_{0.4}P_{0.6}/InP$ heterostructure with four quantum wells with sets of widths 2(2)–3–7 nm (a) and 3(2)–4–15 nm (b).

modal gain was observed for asymmetric multiple quantum-well GaAs heterostructures [5].

4. Conclusions

We have calculated the modal gain spectra of quantum-well $Ga_{0.47}In_{0.53}As/Ga_{0.18}In_{0.82}As_{0.4}P_{0.6}$ heterostructures taking into account the mixing of the subbands of the valence band in the four-band ***kp*** method approximation. An almost flat and broad modal gain spectrum is obtained upon nonuniform excitation of quantum wells of different widths. By using different sets of quantum wells, flat regions have been obtained between 1.28 and 1.525 μm and between 1.36 and 1.60 μm. The designs of asymmetric multiple quantum-well $Ga_{0.47}In_{0.53}As/Ga_{0.18}In_{0.82}As_{0.4}P_{0.6}/InP$ hetero-structures proposed in our paper can be used as elements of optoelectronic and photonic integrated circuits.

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