PROPAGATION OF LASER RADIATION IN MATTER

PACS numbers: 42.68.Ay; 42.68.Bz; 42.68.Ge
DOI: 10.1070/QE2009v039n09ABEH013789

A phase screen model for simulating numerically the propagation of a laser beam in rain

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***Abstract*. The method based on the generalisation of the phase screen method for a continuous random medium is proposed for simulating numerically the propagation of laser radiation in a turbulent atmosphere with precipitation. In the phase screen model for a discrete component of a heterogeneous 'air – rain droplet' medium, the amplitude screen describing the scattering of an optical field by discrete particles of the medium is replaced by an equivalent phase screen with a spectrum of the correlation function of the effective dielectric constant fluctuations that is similar to the spectrum of a discrete scattering component – water droplets in air. The 'turbulent' phase screen is constructed on the basis of the Kolmogorov model, while the 'rain' screen model utilises the exponential distribution of the number of rain drops with respect to their radii as a function of the rain intensity. The results of the numerical simulation are compared with the known theoretical estimates for a large-scale discrete scattering medium.**

Keywords: *laser beam, radiation scattering in rain, phase screen, numerical simulation.*

1. Introduction

Laser beams propagating in atmosphere experience phase distortions due to fluctuations of the dielectric constant of the medium. These fluctuations are caused not only by the turbulent inhomogeneities of the air density but also by the presence of a discrete component in air: aerosol particles, fog, and precipitation [1 – 4]. Laser radiation scattering from rain droplets leads, as in the case of a turbulent atmosphere, to the broadening of the beam and to the appearance of intensity fluctuations in it [4]. In addition, the radiation intensity is attenuated due to absorption by discrete scatterers [4, 5].

Earlier, the statistic characteristics of the field of a laser beam propagating in rain were studied experimentally and theoretically [4 – 12]. As in the case of a turbulent atmosphere, the propagation of laser beams in rain can be described in the parabolic approximation of the scalar wave equation [13]. The combined action of turbulent and discrete (water drops) atmospheric inhomogeneities on the laser beam field is scarcely investigated both because of the mathematical difficulties arising in studying the beam properties in the transition region of the scintillation index values of a plane wave [11], and because of the lack of experimental data, which is caused by the complexity of separation of the influence of turbulence and scattering in rain on the radiation parameters [7, 8]. At present, the methods of numerical simulation are being developed [14 – 17] and the method of phase screens is widely used [14 – 16]. The authors of paper [17] considered the corpuscular and wave approaches to the statistical simulation of propagation of laser radiation in discrete scattering media. One of the problems to be solved in realising the splitting method is the determination of the function of radiation scattering by a single particle [17].

In this paper, we propose a phase screen model taking into account scattering from discrete scatterers in a turbulent medium. Based on the exponential law of distribution of the number of water drops by their size with allowance for its dependence on the rain intensity [1], we derived expressions for the spectrum of the correlation function of the effective dielectric constant fluctuations of the discrete component of a turbulent atmosphere with precipitation. For a turbulent component, we used the Kolmogorov model taking into account the effect of the inner scale of inhomogeneities [18]. The results of numerical simulation of a laser beam propagating along the path in a turbulent atmosphere with precipitation are presented and compared with the known theoretical estimates.

2. Spectrum of the correlation function of the effective dielectric constant fluctuations of atmosphere with precipitation

The conditions, when the radiation wavelength λ is much smaller than the characteristic scale of discrete inhomogeneities of a medium – water drops, are always fulfilled in drizzle and rain. Propagation of monochromatic radiation (a laser beam) in a large-scale randomly inhomogeneous medium can be described by a parabolic equation [4, 18]

$$2ik\frac{\partial U(\boldsymbol{r})}{\partial z} + \Delta_{\perp} U(\boldsymbol{r}) + k^2\varepsilon(\boldsymbol{r})U(\boldsymbol{r}) = 0. \quad (1)$$

Here, $k = 2\pi/\lambda$ is a wave number; $\varepsilon(\boldsymbol{r})$ is the dielectric constant of the medium; z is the propagation direction;

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Received 25 December 2007; revision received 31 March 2009
Kvantovaya Elektronika **39** (9) 863 – 868 (2009)
Translated by I.A. Ulitkin

$(x, y) \equiv \boldsymbol{\rho}$ are transverse coordinates. The quantity $U(\boldsymbol{r})$ is a complex field amplitude $E(\boldsymbol{r})$ of a laser beam:

$$E(\boldsymbol{r}) = U(\boldsymbol{r})\mathrm{e}^{\mathrm{i}kz},$$

$$U(\boldsymbol{\rho}, z = 0) = U_0 \exp\left(-\frac{\rho^2}{2a_0^2} - \frac{\mathrm{i}k\rho^2}{2F}\right), \tag{2}$$

where U_0 is the amplitude of initial distribution of a Gaussian beam; a_0 is the beam radius; F is the radius of curvature of the wave front.

The effective dielectric constant $\varepsilon(\boldsymbol{r})$ of the heterogeneous 'air – water drops' medium is a complex quantity [9, 10]. Its real part is associated with the turbulence, while the imaginary one – with discrete scatterers:

$$\varepsilon(\boldsymbol{r}) = \langle\varepsilon(\boldsymbol{r})\rangle + \tilde{\varepsilon}(\boldsymbol{r}),$$

$$\langle\varepsilon(\boldsymbol{r})\rangle = \mathrm{Re}\langle\varepsilon(\boldsymbol{r})\rangle + \mathrm{i}\,\mathrm{Im}\langle\varepsilon(\boldsymbol{r})\rangle \cong 1 + \mathrm{i}\langle\varepsilon_{\mathrm{p}}(\boldsymbol{r})\rangle, \tag{3}$$

$$\tilde{\varepsilon}(\boldsymbol{r}) = \mathrm{Re}\,\tilde{\varepsilon}(\boldsymbol{r}) + \mathrm{i}\,\mathrm{Im}\,\tilde{\varepsilon}(\boldsymbol{r}) = \tilde{\varepsilon}_{\mathrm{t}}(\boldsymbol{r}) + \mathrm{i}\tilde{\varepsilon}_{\mathrm{p}}(\boldsymbol{r}).$$

The spectrum $\Phi_{\tilde{\varepsilon}_{\mathrm{t}}}(\boldsymbol{\kappa}_\perp, \kappa_z)$ of the correlation function of the dielectric constant fluctuations is determined according to the Kolmogorov – Obukhov model [18]:

$$\Phi_{\tilde{\varepsilon}_{\mathrm{t}}}(\boldsymbol{\kappa}_\perp, \kappa_z) = A_{\mathrm{t}} C_\varepsilon^2 (\kappa_0^2 + \kappa_z^2 + \kappa_\perp^2)^{-11/6}$$
$$\times \exp\{-[(\kappa_\perp^2 + \kappa_z^2)/\kappa_{\mathrm{m}}^2]\}, \tag{4}$$

where C_ε^2 is the structure constant of the field of the dielectric constant fluctuations of air; $\boldsymbol{\kappa}_\perp, \kappa_z$ are spectral coordinates; $\kappa_0 = 2\pi/L_0$; $\kappa_{\mathrm{m}} = 5.92/l_0$; L_0 and l_0 are the outer and inner scale of the medium inhomogeneities. The constant A_{t} will be discussed below.

The dielectric constant component $\tilde{\varepsilon}_{\mathrm{p}}(\boldsymbol{r})$ of the 'air – water drops' system related to the discrete scatterers is determined, first of all, by the distribution $p\,(a_{\mathrm{p}}, J)$ of the drop radii a_{p} as a function of the rain intensity J according to the Laws – Parsons law [1 – 4]:

$$p\,(a_{\mathrm{p}}, J) = M_0(J)\exp[-\Lambda(J)a_{\mathrm{p}}], \tag{5}$$

where $M_0(J) = 4.382{\times}10^6 J^{0.112}\ \mathrm{m}^{-4}$; $\Lambda(J) = 5932 J^{-0.182}\ \mathrm{m}^{-1}$. On this basis it is convenient to study statistical characteristics of the field of a laser beam propagating in rain as a function of J. The spectrum of the correlation function $\Phi_{\tilde{\varepsilon}_{\mathrm{p}}}(\boldsymbol{\kappa}, J)$ of fluctuations $\varepsilon_{\mathrm{p}}(\boldsymbol{r})$ according to [10] has the form:

$$\Phi_{\tilde{\varepsilon}_{\mathrm{p}}}(\boldsymbol{\kappa}, J) = \frac{2}{\pi k^4}\int_0^\infty \mathrm{d}a_{\mathrm{p}}\, p\,(a_{\mathrm{p}}, J)\left|f_0(\boldsymbol{\kappa}, a_{\mathrm{p}})\right|^2, \tag{6}$$

where $f_0(\boldsymbol{\kappa}, a_{\mathrm{p}})$ is the amplitude of wave scattering from a separate particle of radius a_{p}. Because the drizzle and rain drops are large compared to the wavelength λ, as the scattering amplitude we can take its diffraction part [2, 3] and approximate (6) by the quadratic exponent

$$\Phi_{\tilde{\varepsilon}_{\mathrm{p}}}(\boldsymbol{\kappa}, J) = A_{\mathrm{p}} C_{\mathrm{p}}^2(J)\exp(-\kappa^2 a_{\mathrm{m}}^2/4), \tag{7}$$

where

$$C_{\mathrm{p}}^2(J) \cong 1.28 \times 10^{-12} J^{1.822} k^{-2} \tag{8}$$

is the constant analogous to C_ε^2; a_{m} is the scale of the inhomogeneities of the medium equivalent to the discrete scattering medium with the average drop radii a_{m}. We can show that fluctuations $\tilde{\varepsilon}_{\mathrm{p}}(\boldsymbol{r})$ are related to the volume median radius of the drops [1, 4]

$$a_{\mathrm{m}} = 6.19 \times 10^{-4} J^{0.182}, \tag{9}$$

and not to the average or root-mean-square radius. The average value of $\langle\varepsilon_{\mathrm{p}}(\boldsymbol{r})\rangle$ determines the quantity of the optical thickness

$$\tau = 2\pi z \int_0^\infty \mathrm{d}a\, a^2 p(a, J), \tag{10}$$

which attenuates the radiation intensity in the discrete scattering medium. Within the framework of approximations taken with respect to the spectrum $\Phi_{\tilde{\varepsilon}_{\mathrm{p}}}(\boldsymbol{\kappa}, J)$, the quantity τ is

$$kz\langle\varepsilon_{\mathrm{p}}(\boldsymbol{r})\rangle \cong \tau = 2.638 \times 10^{-4} z J^{0.658}. \tag{11}$$

The spectrum of the correlation function of the effective dielectric constants fluctuations of the 'air – water drops' medium is a sum of spectra (4) and (7):

$$\Phi_{\tilde{\varepsilon}}(\boldsymbol{\kappa}_\perp, \kappa_z, J) = \Phi_{\tilde{\varepsilon}_{\mathrm{t}}}(\boldsymbol{\kappa}_\perp, \kappa_z) + \Phi_{\tilde{\varepsilon}_{\mathrm{p}}}(\boldsymbol{\kappa}_\perp, \kappa_z, J). \tag{12}$$

Thus, we replaced the 'air – water drops' medium by the equivalent continuous medium with the characteristic scale a_{m}, l_0, and L_0 in which scattering of radiation is accompanied by its attenuation determined by the quantity τ. This replacement makes it possible to assume below that in the case of precipitation the laser beam propagates in a turbulent atmosphere as in a continuous medium, and to apply the splitting method [15] to equation (1) while developing the algorithm of numerical (statistical) simulation.

3. Algorithm for simulating the propagation of a laser beam in rain

Propagation equation (1) in a heterogeneous 'air – water drops' medium with the spectrum of correlation function (12) of fluctuations $\tilde{\varepsilon}(\boldsymbol{r})$ (3) has the form

$$2\mathrm{i}k\frac{\partial U(z, \boldsymbol{\rho})}{\partial z} + \Delta_\perp U(z, \boldsymbol{\rho})$$
$$= -k^2 U(z, \boldsymbol{\rho})\left[\mathrm{i}\frac{\tau(z)}{kz} + \tilde{\varepsilon}_{\mathrm{t}}(\boldsymbol{\rho}) + \tilde{\varepsilon}_{\mathrm{p}}(\boldsymbol{\rho})\right]. \tag{13}$$

Application of the method of splitting over the physical parameters [15] to solve equation (13) leads to decomposition of the path of a specified length L into layers whose length Δz satisfies the conditions of a weak (thin) screen:

$$\Delta z \ll 1,\ \beta_{0,\mathrm{pw}}^2(\Delta z) \ll 1, \tag{14}$$

where $\beta_{0,\mathrm{pw}}^2$ is the dispersion of the intensity fluctuations of a plane wave in the first approximation of the smooth perturbation method for a turbulent medium [18]. In the middle of each layer there is an amplitude – phase screen determining distortions of the field transmitted through this layer:

$$U(\Delta z, \boldsymbol{\rho}) = U(0, \boldsymbol{\rho}) \times \exp\left(\frac{\mathrm{i}k}{2}\int_0^{\Delta z} \frac{\mathrm{i}\tau(z)}{kz}\mathrm{d}z\right)\exp\left[\mathrm{i}\Psi_{\mathrm{t}}(\boldsymbol{\rho}) - \Psi_{\mathrm{p}}(\boldsymbol{\rho})\right], \quad (15)$$

where $U(0,\boldsymbol{\rho})$ is the incident field; $U(\Delta z,\boldsymbol{\rho})$ is the field transmitted through the layer; $\Psi_{\mathrm{t}}(\boldsymbol{\rho})$ is a random phase incursion during scattering from turbulent inhomogeneities; $\exp[-\Psi_{\mathrm{p}}(\boldsymbol{\rho})]$ is a random variation in the field amplitude caused by the discrete scatterers. The quantity

$$\exp\left(-\frac{k}{2}\int_0^{\Delta z}\frac{\tau(z)\mathrm{d}z}{kz}\right) = \exp\left(-\frac{\tau(\Delta z)}{2}\right) \quad (16)$$

determines the field attenuation by a discrete scattering component of the medium. Thus, the effect of the discrete component of a random medium on the laser beam propagated through the layer Δz is given by the amplitude screen

$$\exp\left[-\tau(\Delta z)/2 - \Psi_{\mathrm{p}}(\boldsymbol{\rho})\right]. \quad (17)$$

The authors of paper [12] showed the possibility of replacement of the cofactor $\exp[-\Psi_{\mathrm{p}}(\boldsymbol{\rho})]$ by the equivalent phase screen $\exp[\mathrm{i}\tilde{\Psi}_{\mathrm{p}}(\boldsymbol{\rho})]$, where $\tilde{\Psi}_{\mathrm{p}}(\boldsymbol{\rho})$ is a random phase incursion acquired by the wave propagating through the layer, so that the random displacement $\exp[-\Psi_{\mathrm{p}}(\boldsymbol{\rho})]$ be realised at the following field diffraction behind the screen. This replacement is admissible due to condition (14) for a weak (thin) screen. Because $\tau(\Delta z)$ is independent of the transverse coordinates, the general attenuation of the laser beam, which propagated the path of length L, can be determined by the cofactor $\exp[-\tau(L)/2]$ after performing all the steps Δz along the path consisting of step-by-step multiplication of the field by the phase screen and of the free diffraction of the field between the screens. Generation of a two-dimensional ($N \times N$) phase screen is performed according to the expression

$$\Psi(j\Delta x, l\Delta y, J) = \sum_{n=-\frac{N}{2}}^{\frac{N}{2}-1}\sum_{m=-\frac{N}{2}}^{\frac{N}{2}-1}(\xi_{nm} + \mathrm{i}\zeta_{nm})\exp\left[2\pi\mathrm{i}\left(\frac{jn+lm}{N}\right)\right] \times\left\{\frac{\Phi_{\Psi}(n/N\Delta x, m/N\Delta y, J)}{N^2\Delta x\Delta y}\right\}^{1/2}, \quad (18)$$

where Δx, Δy are the distances between the nodes; N is the number of nodes; (ξ, ζ) are random sequences with the average equal to zero and the dispersion equal to unity; the spectrum of the correlation function of the phase $\Phi_{\Psi}(\boldsymbol{\kappa}_{\perp}, J)$ is related to the spectrum of the correlation function of the dielectric constant fluctuations by the relation [14–16]

$$\Phi_{\Psi}(\boldsymbol{\kappa}_{\perp}, J) = \frac{k^2\Delta z}{4}\Phi_{\tilde{\varepsilon}}(\boldsymbol{\kappa}_{\perp}, 0, J). \quad (19)$$

Let us determine now the constants A_{t} and A_{p} in expressions for spectra (4) and (7) while preserving the coupling between the parameter C_{ε}^2 [or $C_{\mathrm{p}}^2(J)$] on the one hand and the dispersion of the intensity fluctuations σ_I^2 on the other hand in the same form as that in the first approximation of the smooth perturbation method for a plane wave in a turbulent medium without scale (in rain), i.e. assuming that $\sigma_{I,\mathrm{t}}^2 \approx \beta_{0,\mathrm{pw}}^2 \ll 1$ and $\sigma_{I,\mathrm{t}}^2 \approx \beta_{0,\mathrm{pw}}^2 \ll 1$. In this approximation the dispersion of the intensity fluctuations of the wave in a randomly inhomogeneous medium is determined by the expression [18]

$$\sigma_I^2 = \pi k^2 z\int_0^{\infty}\mathrm{d}\kappa_{\perp}\left(1 - k\frac{\sin\kappa_{\perp}z/k}{\kappa_{\perp}z}\right)\Phi_{\varepsilon}(\kappa_{\perp}, 0). \quad (20)$$

By substituting spectrum (5) into expression (20), we calculate the constant A_{t}:

$$A_{\mathrm{t}} = \frac{\beta_{0,\mathrm{pw}}^2}{(2\pi)^2 C_{\varepsilon}^2 k^{7/6} z^{11/6} f_1(\kappa_0, \kappa_{\mathrm{m}})}, \quad (21)$$

where $\beta_{0,\mathrm{pw}}^2 = 0.31 C_{\varepsilon}^2 k^{7/6} z^{11/6}$ [18]; the function

$$f_1(\kappa_0, \kappa_{\mathrm{m}}) = \int_0^{\infty}\mathrm{d}x\left(1 - \frac{\sin x}{x}\right)\left(\tilde{\kappa}_0^2 + x\right)^{-11/6}\exp\left(-\frac{x}{\tilde{\kappa}_{\mathrm{m}}^2}\right)$$

is the scale factor of inhomogeneities;

$$\tilde{\kappa}_0^2 = \kappa_0^2\frac{z}{k} = \frac{2\pi\lambda z}{L_0^2}, \quad \tilde{\kappa}_{\mathrm{m}}^2 = \kappa_{\mathrm{m}}^2\frac{z}{k} = \left(\frac{5.92}{l_0}\right)^2\frac{z}{k}$$

is the normalised scale of inhomogeneities.

Similarly, we determne the constant

$$A_{\mathrm{p}} = \frac{\tau(L, J)}{\pi^2 k^3 C_{\mathrm{p}}^2 f_2(a_{\mathrm{m}})}, \quad (22)$$

where

$$f_2(a_{\mathrm{m}}) = \frac{4L}{ka_{\mathrm{m}}^2} - \arctan\left(\frac{4L}{ka_{\mathrm{m}}^2}\right).$$

Figure 1 shows the random sets of phases on a network with the number of nodes $N = 2^{10}$ and the resolution $\mathrm{d}x = 0.3$ mm generated according to expressions (18)–(22) for the cases of a homogeneous atmosphere with rain, a turbulent atmosphere, and a heterogeneous 'air–water drops' medium.

4. Discussion of the results

During the propagation of radiation the laser beam field is affected, apart from the diffraction broadening, by two factors: broadening due to scattering from water drops and general attenuation upon scattering and absorption by discrete particles according to the Buger law [1–4]. In this case, we can single out two scales in the distribution of the average beam intensity: the first of them is determined by the diffraction [$a_{\mathrm{a}}(L) \cong a_0(1 + \Omega_0^{-2})^{1/2}$, where $\Omega_0 = ka_0^2/z$ is the beam Fresnel number] and the second one results from the size of water drops. Figure 2 presents the results of simulations and calculations of the average beam intensity in the first approximation of the smooth perturbation method, when $\tau < 1$:

$$\frac{\langle I(\boldsymbol{\rho}, z)\rangle}{I_{\mathrm{d}}(\boldsymbol{\rho}, z)}\mathrm{e}^{\tau/2} = 1 - \frac{A_0}{\pi} \times\left\{a_{\mathrm{m}}^{-2} - \int_0^1\mathrm{d}x\exp\left(\frac{\rho^2}{4}\frac{\xi^2(x)}{a_{\mathrm{m}}^2 + \beta(x)}\right)\left(a_{\mathrm{m}}^2 + \beta(x)\right)^{-1}\right\}, \quad (23)$$

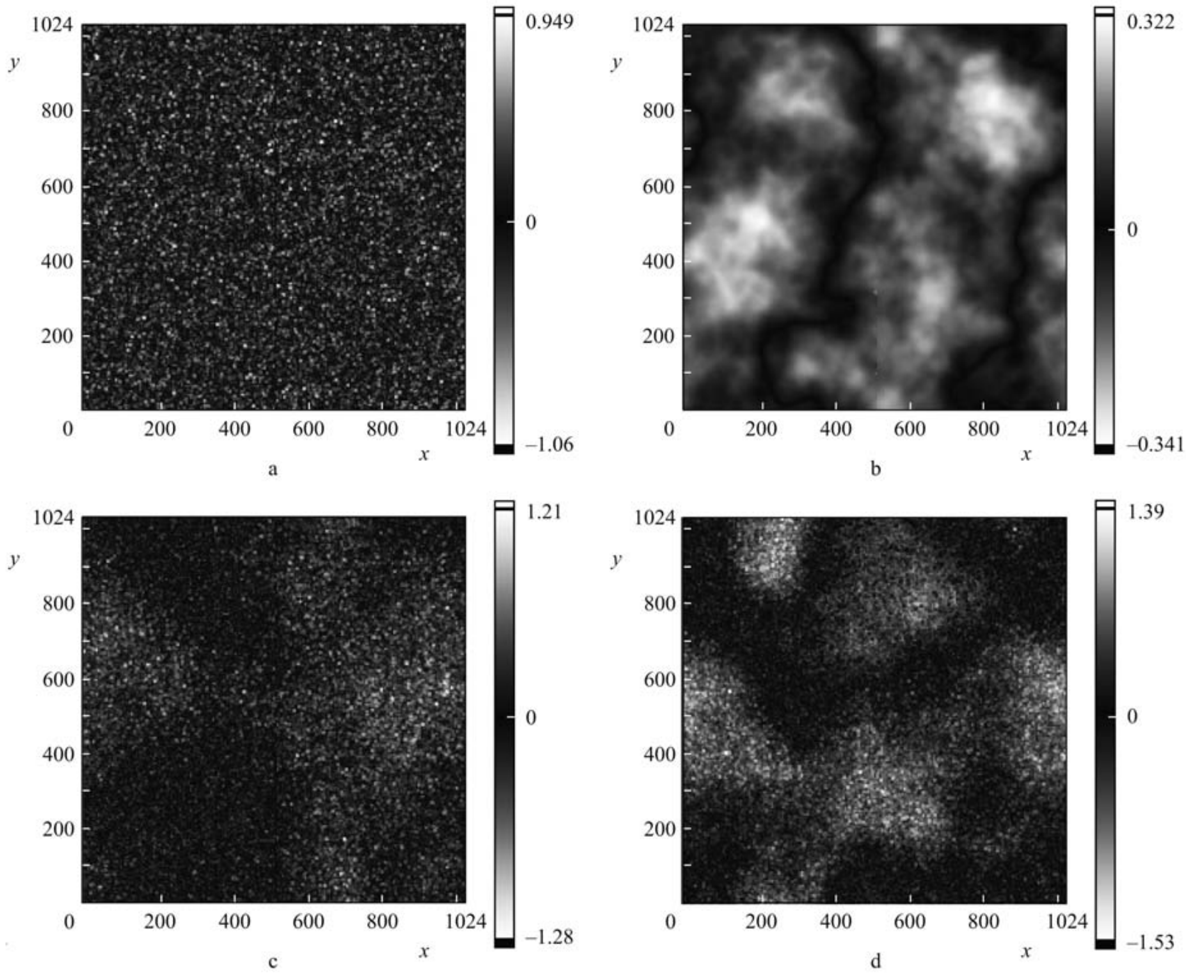

Figure 1. View of the phase screens for a dispersion medium 'raindrops in a homogeneous atmosphere' (a), for a turbulent atmosphere [in accordance with the Kolmogorov–Obukhov model (4)] (b), and for a turbulent atmosphere with rain (c, d). The screens are generated at $\lambda = 10^{-6}$ m, $\Delta z = 30$ m, $J = 40$ mm h^{-1} (a), $C_\varepsilon^2 = 4 \times 10^{-15}$ m$^{-2/3}$, $l_0 = 1.2 \times 10^{-2}$ m, $L_0 = 100$ m (b), the same conditions as in Figs 1a and b (c) as well as at $J = 30$ mm h^{-1}, $C_\varepsilon^2 = 4 \times 10^{-14}$ m$^{-2/3}$, $l_0 = 2 \times 10^{-3}$ m, $L_0 = 100$ m, and $\Delta z = 30$ m (d).

where

$$I_{\rm d}(\boldsymbol{\rho}, z) = \frac{\Omega_0^2}{1+\mu^2\Omega_0^2} \exp\left(-\frac{\rho^2}{a_0^2}\frac{\Omega_0^2}{1+\mu^2\Omega_0^2}\right) \qquad (24)$$

is the beam intensity in the absence of rain (free diffraction);

$$\mu = 1 - z/F;\ x = z/F;\ \beta(x) = (2a_0k)^2(1+\mu^2\Omega_0^2)^{-1};$$

$$\xi(x) = 4\Omega_0 x(1+\mu^2\Omega_0^2)^{-1};$$

$$A_0 = \frac{\lambda\tau(L,J)z/2}{4z/ka_{\rm m}^2 - \arctan(4z/ka_{\rm m}^2)}.$$

The diffraction radius of the beam $a_{\rm d}(L)$ is shown in Fig. 2. This figure also demonstrates the results of the approximate calculation of the average beam intensity by using expressions (14)–(17) for 20 phase screen located uniformly along the path.

At small optical thicknesses, the dispersion of the intensity fluctuations of the plane wave is determined by the quantity τ [7, 9, 11, 12] (Fig. 3),

$$\sigma_{I,\rm pw}^2 \simeq \tau,\ \ \tau \ll 1, \qquad (25)$$

as in the case of the turbulent atmosphere at $\beta_0^2 \ll 1$ [18]:

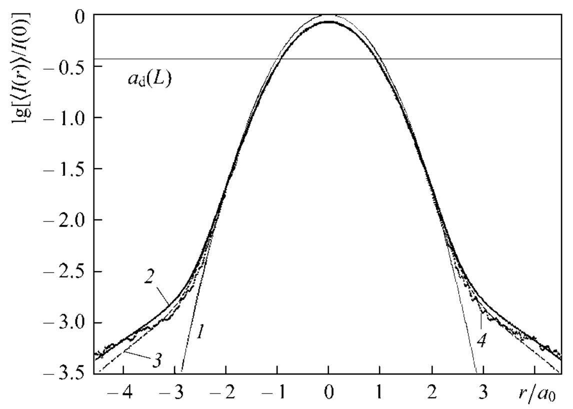

Figure 2. Average intensity of the collimated beam with the radius $a_0 = 3$ cm propagating in rain [$a_{\rm d}(L)$ is the diffraction radius of the beam at the end of the path of length $L = 600$ m; (*1*) is free diffraction (24); (*2*) is expression (23); (*3*) is approximation of solutions (13)–(16) for 20 phase screens in the first approximation of the smooth perturbation method; (*4*) is simulation for $\tau = 0.25$, $a_{\rm m} = 0.8$ mm].

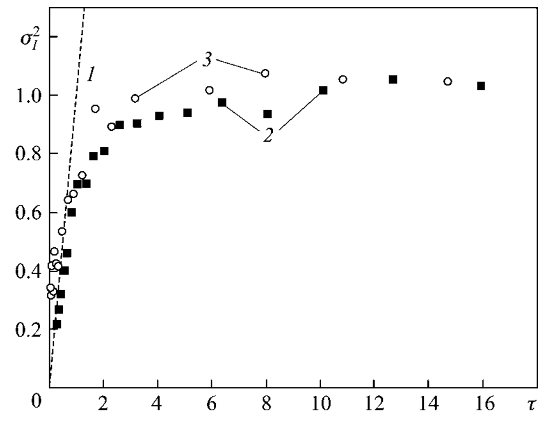

Figure 3. Relative dispersion of intensity fluctuations of the plane wave in the case of the smooth perturbation method (*1*), rain (*2*), and the 'air–water drops' medium at $\beta_0^2 = 0.3$ (*3*).

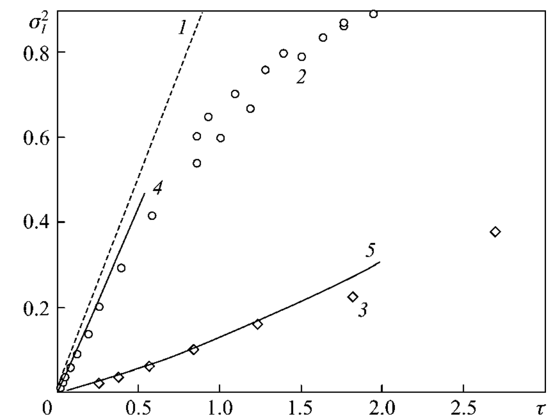

Figure 5. Relative dispersion of intensity fluctuations of the collimated beam of radius $a_0 = 0.3$ cm in rain at $\tau \lesssim 1$: (*1*) – plane wave, smooth perturbation method; (*2, 4*) – beam, $\Omega_0 = 57$; (*3, 5*) – beam, $\Omega_0 = 5.7$ [(*2, 3*) – simulation; (*4, 5*) – smooth perturbation method].

$$\sigma_{I,\mathrm{pw}}^2 \simeq \beta_0^2,\ \beta_{I,\mathrm{pw}}^2 = 0.31 C_\varepsilon^2 k^{7/6} z^{11/6} \leqslant 1. \quad (26)$$

At $\tau > 1$, the quantity $\sigma_{I,\mathrm{pw}}^2$ is comparable with unity and when the optical thickness further increases, it tends to unity [10, 12]:

$$\sigma_{I,\mathrm{pw}}^2 \simeq 1,\ \tau \gg 1. \quad (27)$$

Additional influence of turbulence on the dispersion of intensity fluctuations of the plane wave propagating in rain is illustrated in Fig. 3 as a displacement of dependence (*2*) along the abscissa axis to the origin of coordinates. The contributions of turbulent distortions and fluctuations caused by scattering from water drops to the dispersion of intensity fluctuations are additive in the region $\tau < 1$ both for the plane wave and for a restricted laser beam (Fig. 4).

For a Gaussian beam propagating in rain, the dispersion of intensity fluctuations on its axis in the approximation of the smooth perturbation method is determined by the expression [9, 12]

$$\sigma_I^2(z) \simeq \frac{\tau k a_0 a_\mathrm{m}}{2z}\left(\mu^2 + \Omega_0^{-2}\right)^{1/2} \times \arctan\left[\frac{2z}{k a_0 a_\mathrm{m}}\left(\mu^2 + \Omega_0^{-2}\right)^{1/2}\right], \quad (28)$$

where $\mu = 1 - z/F$; $\tau < 1$. Depending on the beam Fresnel number Ω_0, the domains (with respect to τ) of applicability of expression (28) are different. Figure 5 presents the results of simulation of propagation of beams with $\Omega_0 = 5.7$ and 57. The comparison of dependences (*2*), (*4*) and (*3*), (*5*) in Fig. 5 shows that when the Fresnel number is small, expression (28) can be also used at $\tau > 1$ [Fig. 4, curves (*3, 5*)]. When τ increases, the dispersion of intensity fluctuations of the Gaussian beam has a maximum at large enough Fresnel numbers [Fig. 6, dependences (*1, 3*)]. However, the decrease in Ω_0 leads to the disappearance of the maximum [transition to the regime of a spherical wave, Fig. 6, dependences (*2, 4*)].

As was noted above, the effect of turbulence on the quantity σ_I^2 of the Gaussian beam propagating in rain is

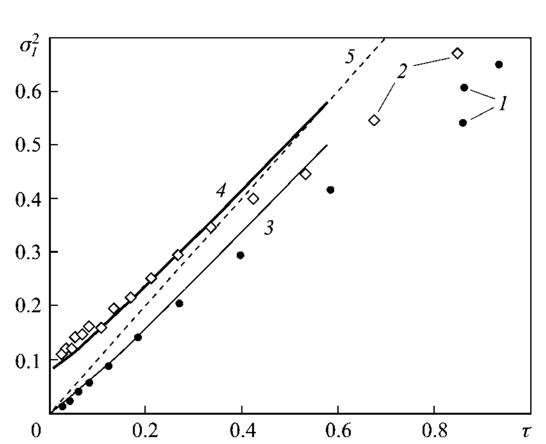

Figure 4. Relative dispersion of intensity fluctuations of the collimated beam of radius $a_0 = 0.3$ cm (the Fresnel number is $\Omega_0 = 57$) in rain at $\tau \lesssim 1$ (*1, 3*) and in the 'air–water drops' medium at $\beta_0^2 = 0.1$ (*2, 4*); (*5*) is rain, plane wave, smooth perturbation method [(*1, 2*) – simulation, (*3, 4*) – smooth perturbation method].

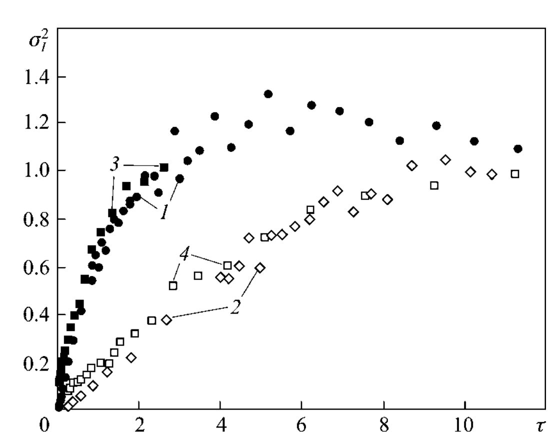

Figure 6. Relative dispersion of intensity fluctuations of the collimated beam of radius $a_0 = 3$ cm in rain at $\tau > 1$ (*1, 2*) and in a heterogeneous 'air–water drops' medium (*3, 4*); $\Omega_0 = 57$ (*1, 3*) and 5.7 (*2, 4*).

manifested in the displacement of the dependences $\sigma_I^2(\tau)$ along the abscissa axis to the origin of coordinates. This leads to a decrease in the values τ of the optical thickness of the path at which the dispersion σ_I^2 of the beam starts saturating (tends to unity). In this case, the dependences $\sigma_I^2(\tau)$, $\sigma_I^2(\tau, \beta_0^2)$ both in the absence of turbulence and at $\beta_0^2 \sim 1$ merge (Fig. 6).

5. Conclusions

We have proposed a model of a phase screen with the Gaussian spectrum of the correlation function of the effective dielectric constant fluctuations, the scale of the spectrum being determined by the volume median radius of the water drops. We have simulated numerically the propagation of the laser beam in rain and turbulent atmosphere with precipita-tion by the phase screen method. The results of simulation agree with the known theoretical estimates of the average intensity and dispersion of the beam intensity fluctuation in the approximation of the smooth perturbation method for both components of the heterogeneous 'air – rain drops' medium.

It has been shown that at small Fresnel number of the laser beam, the domain of applicability of the results of the smooth perturbation method can be applied to the values $\tau > 1$. In the range $\tau < 1$, $\beta_0^2 < 1$, the contributions of turbulent distortions and scattering in rain to the dispersion of intensity fluctuations of the laser beam are additive. At large Fresnel numbers of the beam, the relative dispersion of the beam intensity fluctuation has a maximum in the transition region of the values of $\tau > 1$. For both media ('air – water drops' and rain only), the dispersions σ_I^2 at $\tau \gg 1$ tend to unity and coincide in the case of saturation.

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