PACS numbers: 42.15.–i; 42.55.–f ; 42.60.Jf
DOI: 10.1070/QE2009v039n10ABEH013815

Optimal two-mirror system for laser radiation focusing

A.V. Gitin

***Abstract*. An optical system for laser radiation focusing, which consists of parabolic and elliptic mirrors, is considered. It is shown by the method of elementary reflections that the maximum concentration of laser radiation on the target can be achieved at a certain position of these mirrors.**

Keywords: *off-axis parabolic and elliptic reflectors, method of elementary reflections, laser radiation focusing, optimisation criterion.*

1. Introduction

An urgent problem in modern studies of the interaction of laser radiation with matter is the development of an optical system focusing extremely high-power ultrashort pulses into a spot of minimal diameter. At present, such pulsed radiation is focused by a high-aperture off-axis parabolic mirror and the concentration of radiation in the laser spot is increased by using an electronic adaptive system controlled by a deformable mirror [1 – 3]. Note that upon focusing ultrashort pulses (of duration less than 100 fs), an electronic adaptive system cannot principally operate in the real-time regime because it only selects, by the 'reference' pulse, the shape of a deformable mirror at which the energy concentration in the light spot is maximal.

Investigations by the method of elementary reflections showed [4] that the size of a light spot in the focus of a parabolic mirror depends on the region of incidence of the light beam on the mirror. As a result, the radiation intensity distribution in the light spot in the focus of the off-axis parabolic mirror is asymmetric even in the case of the uniform divergence of the incident laser beam. It was pointed out in [5] that asymmetry in the radiation distribution in the light spot produced by one off-axis mirror can be compensated for by another off-axis mirror. Of course, the shape of this additional mirror can be selected by using an electronic adaptive system with a deformable mirror but it can also be calculated analytically.

In this paper, we propose to supplement the main parabolic off-axis mirror with an additional elliptic off-axis mirror to achieve the maximum concentration of laser radiation on the target, and to optimise their mutual position by using the quasi-axial-symmetry criterion with the help of the method of elementary reflections.

2. Method of elementary reflections

It is known that an ideal lens not only collects all light rays coming from a point A on the optical axis to an optically conjugate point A$'$, but also maps a flat object to a flat image with the same lateral magnification m for central and peripheral rays of a thin light beam producing the image (Fig. 1a). An elliptic reflector also collects all light rays coming from the first focus of the ellipse (point O) in its second focus (point O$'$), but the magnification of this reflector depends on the propagation direction of a thin light beam (Fig. 1b).

It was shown in [4] that it is much simpler to study analytically this specific feature of mirror reflectors by the method of elementary reflections. In this method, each point P, for example, of an elliptic reflector is treated as a centre of two elementary homocentric light beams: incident from a 'spherical' source placed in the front focus O and producing

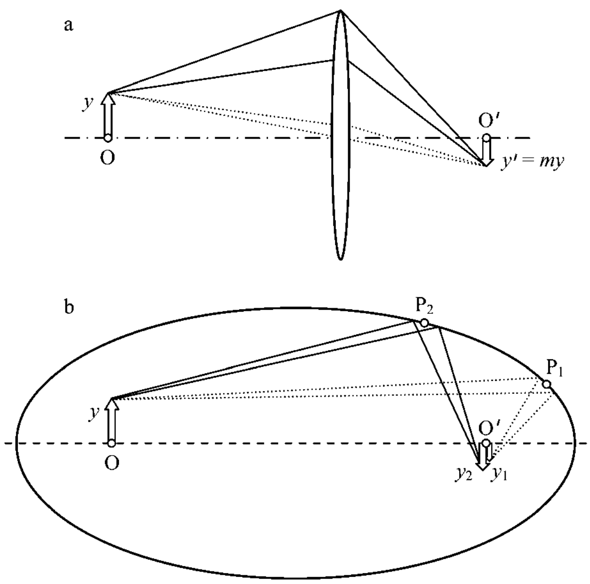

Figure 1. Lateral magnification for an ideal lens (a) and for an elliptic reflector (b) in the case of a thin beam of light rays propagating through the centre and along the periphery of the optical system.

A.V. Gitin Max-Born-Institut für Nichtlineare Optik und Kurzzeitspectroskopie, Max-Born-Str. 2A, 12489 Berlin, Germany; web-site: www.mbi-berlin.de; e-mail: andrey.gitin@gmx.de

Received 17 April 2008; revision received 17 February 2009
Kvantovaya Elektronika **39** (10) 977 – 980 (2009)
Translated by I.A. Ulitkin

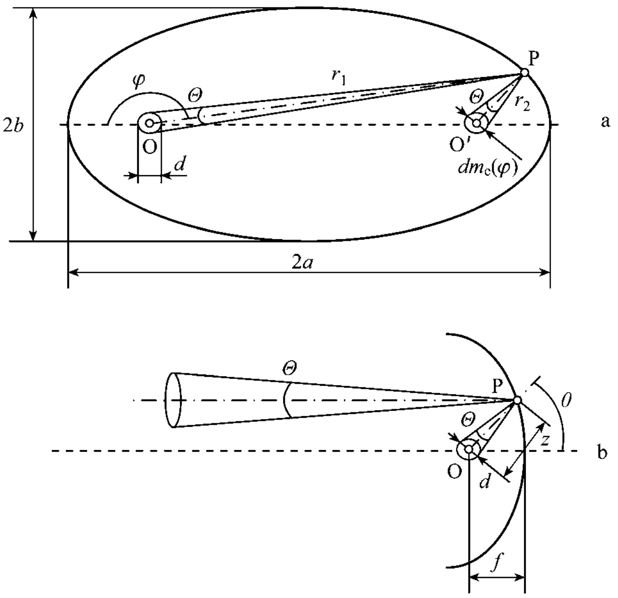

Figure 2. Method of elementary reflections for elliptic (a) and parabolic (b) reflectors.

its 'spherical' image in the vicinity of the back focus O′ (Fig. 2a). According to the reflection law, the solid angles of these homocentric beams are equal; as a result, the diameter of the 'spherical' source image depends on the ratio of distances $|\mathrm{OP}| = |r_1| = a(1-\varepsilon^2)/(1+\varepsilon\cos\varphi)$ and $|\mathrm{PO}'| = |r_2| = 2a - |r_1|$. Here, a is the major semiaxis of the ellipse, ε is its eccentricity ($\varepsilon < 1$), and φ is the angle under which the point P is seen from the front focus O. Note that because the method of elementary reflections uses reflection from a 'spherical' source, the focusing power of the reflector is conveniently characterised by the 'spherical' local magnification $m_e(\varphi)$,which in the case of the elliptic reflector (Fig. 2a) is described by the expresion

$$m_e(\varphi) = \frac{|r_2|}{|r_1|} = \frac{2}{1-\varepsilon^2}(1+\varepsilon\cos\varphi) - 1. \tag{1}$$

When the eccentricity ε tends to unity, the ellipse degenerates to a parabola. A parabolic reflector transforms a thin beam with the angular divergence Θ incident on the parabolic reflector in the vicinity of point $\mathrm{P}(\theta)$ to the focal circular spot (Fig. 2b) with diameter

$$d(\theta) \equiv m_p(\theta)\Theta, \tag{2}$$

i.e. the 'spherical local magnification' of the parabolic reflector is [4]

$$m_p = |\mathrm{OP}| = \frac{2f}{1+\cos\theta}, \tag{3}$$

where θ is the angle under which the point P is seen from the front focus and f is the distance between the focus and the top of the parabolic reflector.

3. Two-mirror focusing system and a lens system equivalent to it

Because an additional reflector should transfer without aberration the image from the point O to the point O′, it is reasonable that this reflector should have the shape of an ellipse with the foci at points O and O′. Consider a two-mirror focusing system consisting of parabolic and elliptic off-axis reflectors, the focus of the parabolic reflector coinciding with the first focus of the ellipsoid (Fig. 3a). In this optical system, a thin beam with the angular divergence Θ, after reflecting from the parabolic mirror in the vicinity of point $\mathrm{P}(\theta)$, passes through the first focus O of the elliptic reflector, reflects from this mirror in the vicinity of point Q, is focused at the optically conjugate point P′ and produces a circular spot of diameter $d(\theta)$ in the vicinity of the second focus O′ of the ellipse. Here, θ is the inclination angle of axis of the beam reflected from the parabolic mirror in the vicinity of point P. One can see from Fig. 3a that this optical focusing system has only one degree of freedom: a compensating ellipsoid reflector with the eccentricity ε can be inclined at an arbitrary angle Δ to the perpendicular to the optical axis of the paraboloid. Thus, the angles of reflection of the light beam in the ellipsoid (φ) and paraboloid (θ) are related by the expression

$$\varphi = \Delta + \pi/2 + \theta. \tag{4}$$

Let us find the 'spherical local magnification' $m_\Sigma(\theta,\Delta)$ of the two-mirror focusing system, i.e. the proportionality coefficient between the divergence Θ of the incident beam of rays

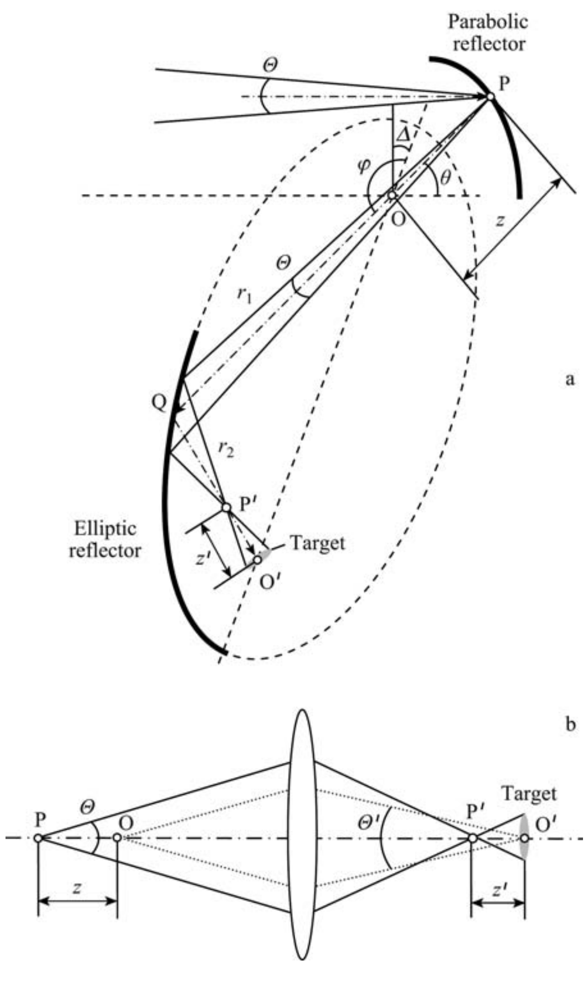

Figure 3. Parabolic–elliptic mirror focusing system (a) and a lens focusing system equivalent to it (b).

and the diameter d of the light spot produced by this system:

$$d = m_{\Sigma}(\theta,\Delta)\Theta. \quad (5)$$

It is often more convenient to replace in theory the mirror optical system by a similar lens optical system and to use the conclusions drawn for the lens system for the mirror system. The described mirror focusing system with a broken optical axis POQP$'$O$'$ (Fig. 3a) is equivalent to the lens focusing system with the direct optical axis POP$'$O$'$ (Fig. 3b) producing a circular spot of diameter d in the vicinity of point O$'$.

Points P$'$ and O$'$ are optically conjugate with points P and O, respectively, and the distances between them $|\mathrm{P'O'}| = z'$ and $|\mathrm{PO}| = z$ are related by the Maxwell formula [6]

$$z' = m m_{\mathrm{o}} z, \quad (6)$$

where m_{o} is the magnification for the optically conjugate points O and O$'$ and m – for optically conjugate points P and P$'$. Because the condition of Abbe sines ($m \sin\Theta' = \sin\Theta$) is fulfilled at points P and P$'$ of the lens system, we have in the small-angle approximation

$$\Theta' \approx \Theta/m. \quad (7)$$

A light beam with the angular divergence Θ coming from point P and propagating through this focusing system is collected in the optically conjugate point P$'$ to produce in the vicinity of point O$'$ a light spot of diameter d (Fig. 3b):

$$d = z'\Theta'. \quad (8)$$

By substituting expressions (6) and (7) into expression (8), we obtain

$$d = (z m_{\mathrm{o}})\Theta, \quad (9)$$

where zm_{o} is the 'spherical total magnification' of the lens focusing system.

Because the conditions $z = m_{\mathrm{p}}(\theta)$ and $m_{\mathrm{o}} = m_{\mathrm{e}}(\varphi)$ are fulfilled for the lens focusing system and its analogous two-mirror focusing system, the comparison of expressions (5) and (9) indicates that the 'spherical local magnification' of the two-mirror focusing system m_{Σ} is equal to the product of 'spherical local magnifications' of parabolic [m_{p} (3)] and elliptic [m_{e} (1)] reflectors, i.e. taking (4) into account, we have

$$m_{\Sigma}(\theta,\Delta) \equiv m_{\mathrm{p}}(\theta)\, m_{\mathrm{e}}(\Delta + \pi/2 + \theta)$$

$$= \left\{ \frac{2}{1-\varepsilon^2}\left[1 - \varepsilon\sin(\theta+\Delta)\right] - 1 \right\} \frac{2\mathrm{f}}{1+\cos\theta}. \quad (10)$$

Thus, the 'spherical local magnification' of the two-mirror focusing system under study and, hence, the illumination distribution in the light spot being produced depend on the angles Δ and θ and on the eccentricity ε of the ellipsoid reflector.

4. Optimisation of a two-mirror focusing system by the quasi-axial-symmetry criterion

For different points P(θ), a parabolic reflector focuses diverging laser radiation into light spots of different diameters, the light spot produced by the parabolic reflector being the larger, the greater is the displacement of point P(θ) from the optical axis of the reflector. Thus, in the case of the off-axis incidence of a light beam, an asymmetric light spot (Fig. 4a) is produced in the focus of the parabolic reflector (Fig. 2b).

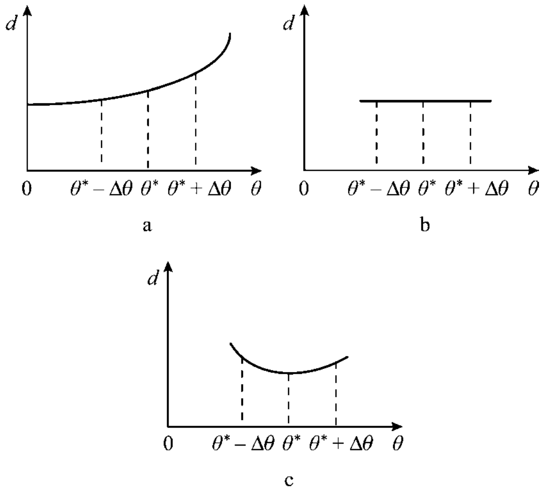

Figure 4. Dependences of the light spot diameter d on the angle θ in the vicinity of the angle θ^* for the off-axis (a), isoplanar (b), and quasi-axially-symmetric (c) focusing systems.

Of course, only an isoplanar focusing system, i.e. a system invariant to the displacement of point P(θ) (Fig. 4b) – can concentrate uniformly the energy in the light spot [7]. However, to provide the the maximum energy concentration in the light spot, it is important first of all to obtain the minimal spot diameter rather than the uniform illumination distribution. The light spot diameter is proportional to the focal distance of the system and it is technologically challenging to fabricate a short-focus isoplanar mirror system.

As shown above, the possible compromise between a rather easily fabricated short-focus off-axis parabolic reflector producing an asymmetric light spot, and a technologically unavailable short-focus isoplanar reflector producing a uniform illumination distribution in the light spot is a two-mirror focusing parabolic – elliptic reflector system. In this system, the change in the 'spherical magnification' of the first off-axis reflector is compensated for by a change in the 'spherical magnification' of the second off-axis reflector ensuring, thereby, the quasi-axially-symmetric illumination distribution in the light spot can be achieved, when the light spot diameter first decreases and then increases (Fig. 4c) upon the monotonic displacement of the centre P(θ) of an elementary light beam along the parabolic reflector of the two-mirror focusing system (Fig. 3).

Let the symmetry axis of the laser beam intersect the parabolic reflector at an angle $\theta = \theta^*$ corresponding to the formation of the light spot of minimal diameter. In this case, because the radiation energy of the Gaussian laser beam is maximal at its axis, the resultant quasi-axially-symmetric focusing system concentrates the radiation energy in the light spot better than the off-axis parabolic mirror (but worse, however, than am imaginary isoplanar focusing system).

We propose to optimise the above-described two-mirror focusing system by the quasi-axial-symmetry criterion. Taking into account the proportionality of the light spot diameter d to the 'spherical magnification' of the two-mirror focusing system $m_\Sigma(\theta, \Delta)$ [see (5)], in the case of arbitrary divergence Θ of an incident light beam, the quasi-axial-symmetry criterion can be formally written in the form

$$\frac{dm_\Sigma(\theta, \Delta)}{d\theta} = 0. \qquad (11)$$

Let the light beam axis intersect the parabolic reflector at point $P(\theta^*)$. Then, according to criterion (11), the parameters of the optimal two-mirror system focusing a diverging light beam into a light spot with a quasi-axially-symmetric illumination distribution can be determined by differentiating expression (10), i.e. by solving the equation

$$\frac{dm_\Sigma(\theta, \Delta)}{d\theta} = \frac{d}{d\theta}\left\{[(1+\varepsilon^2) - 2\varepsilon\sin(\theta+\Delta)]\frac{1}{1+\cos\theta}\right\} = 0. \qquad (12)$$

Thus, at an angle of inclination of an auxiliary elliptic mirror

$$\Delta = \Delta^* = \arccos\left(\frac{1+2\varepsilon^2}{2\varepsilon}\sin\frac{\theta^*}{2}\right) - \frac{\theta^*}{2} \qquad (13)$$

the two-mirror system focuses the diverging light beam in a spot with a quasi-axially-symmetric illumination distribution.

Compared to the conventional off-axis parabolic reflector, the advantage of the quasi-axially-symmetric two-mirror focusing system optimised for the angle θ^* is manifested if the angle θ lies inside the interval $\theta^* \pm \Delta\theta$ (see Fig. 4) upon displacement of the beam axis along the surface of the parabolic reflector.

5. Conclusions

(i) Super-high-peak-power ultrashort pulses can be focused by using a two-mirror system consisting of parabolic and elliptic off-axis reflectors.

(ii) The spherical local magnification of this two-mirror focusing system (7) is equal to the product of spherical local magnifications of parabolic (3) and elliptic (1) reflectors.

(iii) The shape of the illumination distribution in the light spot being produced depends on the angle Δ (the angle between the optical axis of the ellipsoid reflector and the perpendiculat to the optical axis of the parabolic reflector) and on the eccentricity ε of the ellipsoid reflector.

(iv) The two-mirror focusing system can be optimised by using quasi-axial-symmetric criterion (11). The laser radiation energy is better concentrated in the light spot of the quasi-axially-symmetric focusing system than in the light spot of the off-axis focusing system but worse than in the light spot of the isoplanar focusing system.

(v) Expression (13) allows one, at a specified displacement $P(\theta^*)$ of the incident beam axis with respect to the symmetry axis of the parabolic reflector and a at given eccentricity ε of the ellipsoid reflector, to determine the angle Δ^* at which the illumination distribution is quasi-axially-symmetric in the light spot being produced.

References

1. Bahk S.-W., Chvykov V., Kalintchenko G., Maksimchuk A., Mourou G.A., Saleh N., Yanovsky V., in *Ultrafast Opt. IV* (Berlin: Springer-Verlag, 2004) Vol. 95, p. 329.
2. Bahk S.-W., Rousseau P., Planchon T.A., Chvykov V., Kalintchenko G., Maksimchuk A., Mourou G.A., Yanovsky V. *Opt. Lett.*, **29**, 2837 (2004).
3. Bahk S.-W., Rousseau P., Planchon T.A., Chvykov V., Kalintchenko G., Maksimchuk A., Mourou G.A., Yanovsky V. *Appl. Phys. B*, **80**, 823 (2005).
4. Gitin A.V. *Kvantovaya Elektron.*, **37**, 307 (2007) [*Quantum Electron.*, **37**, 307 (2007)].
5. Kingslake R. (Ed.) *Applied Optics and Optical Engineering* (New York: Acad. Press Inc., 1969) Vol. 5, Ch. 2, pp 195–228.
6. Gitin A.V. *Opt Spektrosk.*, **63**, 183 (1987) [*Opt. Spectrosc.*, **63**, 106 (1987)].
7. Gitin A.V. *Opt. Zh.*, **60**, 12 (1993) [*Sov. J. Opt. Technol.*, **60**, 372 (1993)].