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Instability of stationary lasing and self-starting mode locking in external-cavity semiconductor lasers

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Abstract. **Parameters of external-cavity semiconductor lasers, when the stationary lasing becomes unstable, were analysed within the framework of a theoretical model of self-starting mode locking. In this case, a train of ultrashort pulses can be generated due to intrinsic nonlinearities of the laser medium. A decisive role of the transverse optical field nonuniformity, pump rate, and gain spectral bandwidth in the development of the instability of stationary lasing was demonstrated.**

Keywords: *mode locking, ultrashort pulses, laser diodes.*

1. Introduction

Generation of ultrashort light pulses is an important problem of laser physics from both the applied and fundamental points of view. Great progress in the field of femtosecond-pulse generation in solid-state lasers has been made recently, which is mainly due to the use of various types of SESAM saturable absorbers, Kerr-lens mode locking, and intracavity dispersion optimisation. Extremely short pulses of duration approaching the optical oscillation period have been obtained.

Note that despite enormous prospects of numerous applications, semiconductor lasers fall considerably behind the above-mentioned laser systems with respect to the femtosecond-pulse generation [1]. This is due to the relatively low output power of laser diodes, which hampers the use of SESAM absorbers or the Kerr nonlinearity. On the other hand, in high-power laser diodes the power flux density in the active region can reach significant values sufficient for various types of nonlinearities to develop efficiently [2]. It appears rather interesting to use intrinsic nonlinearities of the semiconductor active medium for self-mode locking and femtosecond-pulse generation. From a practical point of view, it is desirable for a laser to pass into the femtosecond-pulse generation regime automatically when the continuous pumping is switched on, i.e. for the mode locking be self-starting without any external action.

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The first step to the development of a mode locking model is the study of the stability of stationary lasing in a continuously-pumped semiconductor laser. Such studies were earlier carried out for fiber and dye lasers [3, 4]. If it turns out that there exists a range of the laser parameters where cw lasing is unstable, then the next step, apparently, will be to study the stability of the femtosecond-pulse generation regime. In the case of crossing the stationary stability region with that of the ultrashort pulse generation, there appears a possibility of the automatic transfer to the mode locking regime as the governing parameter, namely the pump rate, changes under conditions that other attractors of the system phase plane are eliminated. In this paper we restrict ourselves by considering the first part of the problem, namely, the study of the stability of stationary lasing.

We will demonstrate that in the case of the transverse optical field nonuniformity when the pump rate increases, there exists a frequency range where variations in the stationary state are unstable. The physical mechanism of the instability under study is determined by the transverse nonuniformity of the gain saturation due to the mode structure of the optical field. The instability threshold considerably decreases for lasers with a broad optical gain width according to the derived expression for the instability threshold.

We will develop an approach based on the spatiotemporal description of the optical field in the cavity within the framework of the known Haus method [5]. This approach, unlike the method of the field expansion in optical resonator modes, appears to be preferable from the viewpoint of describing ultrashort pulses in the mode-locking regime. Note that, within the framework of the mode approach, the problem of the stability of continuous single-mode operation of laser diodes has been considered in literature, in particular, in papers [6, 7], where the self-stabilisation of the generated mode due to the parametric interaction with neighboring subthreshold modes determined by density beats of the carriers at intermode frequencies, has been investigated. The general theory of stability of quantum oscillators, taking into account the influence of spatial nonuniformity of laser emission, was studied in [8] within the framework of the general mode approach.

In this paper, we consider a laser diode with an external cavity (see Fig. 1) having highly-reflecting mirrors, which provide the maximum power inside the active region. We do not use external nonlinear elements (absorbers, Kerr media, etc.), and nonlinear interactions occur exclusively in the laser active medium. A model with a single cavity is discussed and instabilities caused by the composite cavities due to reflections at the laser chip, are excluded. We also exclude instabilities arising from nonzero transmission coefficients of the cavity mirrors. Therefore, the laser in our case consists of an amplifying medium (a semiconductor crystal with antireflection coatings on its facets) and two external mirrors, one of which is highly reflecting and the other has a small transmission coefficient for the radiation output.

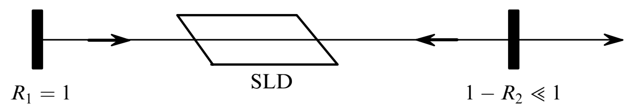

Figure 1. Geometry of an external-cavity laser diode with highly-reflecting mirrors ($R_1 = 1$) and ($1 - R_2 \ll 1$) between which an AR-coated superluminescent diode (SLD) is placed.

2. Theoretical model

To study the stability of the stationary lasing of a laser diode, we will use the spatiotemporal approach developed within the theory of mode locking [5, 9, 10]. Namely, by relating the radiation field amplitudes E_{n+1} and E_n after $n+1$ and n cavity round trips at a certain point (for instant, at one of the mirrors), we will obtain an equation describing the laser generation dynamics within a large time interval $t \gg T_r$, (T_r is a round trip time), and study the stability of its stationary solution.

Consider first the amplification of the field amplitude E per one round trip. The gain g of the medium is determined by the carrier concentration dynamics in the amplifying layer; in the well-known approximations, it can be described by the expression [9]

$$\frac{dg}{dt} = -\frac{g - g_0}{T_0} - \frac{|E|^2}{T_0 |E_s|^2} g, \tag{1}$$

where E_s is the saturation field amplitude; g_0 is the unsaturated gain; and T_0 is a characteristic relaxation time. Due to the inhomogeneity of pumping as well as to the mode structure of the field in the cavity, the gain is a function of the transverse coordinate. Solving Eqn (1) together with the wave equation is a nontrivial problem.

Under the stationary conditions of the laser-diode generation, the structure of transverse modes was studied in detail in literature (see, e.g., [11–13]). In the direction perpendicular to the amplifying layer (along the y axis), the field distribution is almost homogeneous, because it is determined mainly by the geometry of the enveloping layers of the heterostructure. But in the plane of the amplifying layer, the transverse field (along the x axis) distribution is well approximated by the Gauss–Hermite modes. In this case, under the condition of a sufficiently small gain per round trip, so that the self-focusing effect can be neglected in the first approximation, the spatial dependence of the field amplitude can be factorised: $E(x, y, z, t) \approx U(z, t) \times \Phi(x)\Psi(y)$. The transverse mode $\Phi(x)$ satisfies the equation $\Phi''(x) + [k_0^2 n(x) - k^2]\Phi = 0$, where $n(x)$ is the linear refractive index, whose square approximation determines the Gauss–Hermite modes; $k_0 = \omega_0/c$; ω_0 is the carrier frequency; and k is the transverse wave number. The numerical analysis showed the proximity of actual field distributions to the Gauss–Hermite modes. The stability of these distributions and their connection with light–current characteristics of the laser diode was thoroughly studied in [11–13].

Within the framework of the slowly-varying amplitude approximation, the radiation field propagation along the z axis in the amplifying medium of a laser diode is described by the well-known equation [14–16]

$$\left\{\frac{\partial}{\partial z} + \frac{1}{2ik_z}\left[\frac{\partial^2}{\partial x^2} + (k_0^2 n(x) - k^2)\right] - \frac{i}{2}\beta_2 \frac{\partial^2}{\partial \tau^2}\right\}E$$

$$= \left\{\frac{1}{2}g(1 + i\alpha) - \frac{\gamma}{2} - i\left.\frac{\partial g}{\partial \omega}\right|_{\omega_0}\frac{\partial}{\partial \tau} - \frac{1}{4}\left.\frac{\partial^2 g}{\partial \omega^2}\right|_{\omega_0}\frac{\partial^2}{\partial \tau^2}\right\}E$$

$$-(\gamma_{2ph}/2 + ib_2)|E|^2 E, \tag{2}$$

where k_z is the wave number along the z axis; β_2 characterises the dispersion of the refractive index in the medium; α is the line enhancement factor; $\tau = t - z/v_g$ is the intrinsic time; and v_g is the group velocity. Distributed losses in the cavity are taken into account by the parameter γ, and the last term is the contribution of the effects of the Kerr nonlinearity and two-photon absorption with coefficients b_2 and γ_{2ph}, respectively.

To derive the propagation equation for the field amplitude $U(z, \tau)$, we should use the factorised field approximation and then average Eqn (2) over the transverse mode (see, e.g., monograph [17]). In the vicinity of the stationary lasing regime, the spatial profile of the gain from Eqn (1) has the form $g \approx g_0/[1 + |E(x, z, \tau)/E_s|^2]$. Note that a similar relation is valid in a classic scheme of mode locking as well [5, 10], when the pulse repetition rate is assumed small compared to T_0 (the 'slow' amplifier). In this case the gain is determined by a ratio of the intensity averaged by the period to the saturation intensity, i.e. $g \approx g_0/(1 + I_{av}/I_s)$. Thus, the spatial gain profile is

$$g(x) \approx g_0\left[1 + \frac{|U|^2\Phi^2(x)}{E_s^2}\right]^{-1}$$

$$\approx \frac{g_0}{1 + |U|^2/E_s^2}\left[1 + \frac{|U|^2}{E_s^2}\frac{1 - \Phi^2(x)}{1 + |U|^2/E_s^2}\right] \tag{3}$$

[in the mode locking regime the ratio of the pulse repetition rate to T_0 should be taken into account (see Section 4)]. The result of averaging is the equation

$$\left(\frac{\partial}{\partial z} - \frac{i}{2}\beta_2\frac{\partial^2}{\partial \tau^2}\right)U = \left[\frac{1}{2}g(1 + i\alpha)\left(1 + \xi\frac{|U|^2}{E_s^2}\right)\right.$$

$$\left. - \frac{\gamma}{2} - i\left.\frac{\partial g}{\partial \omega}\right|_{\omega_0}\frac{\partial}{\partial \tau} - \frac{1}{4}\left.\frac{\partial^2 g}{\partial \omega^2}\right|_{\omega_0}\frac{\partial^2}{\partial \tau^2}\right]U$$

$$-\left(\frac{\gamma_{2ph}}{2} + ib_2\right)\zeta|U|^2 U, \tag{4}$$

where the parameters are introduced as follows:

$$\xi = \frac{\int \Phi^2(x)[1-\Phi^2(x)]\mathrm{d}x}{\int \Phi^2(x)\mathrm{d}x} \ll 1, \quad \zeta = \frac{\int \Phi^4(x)\mathrm{d}x}{\int \Phi^2(x)\mathrm{d}x}. \qquad (5)$$

The integration is performed over the entire transverse cross section of the amplifying layer. Because the parameter $\zeta \sim 1$, we neglect it hereafter and consider the nonlinearity coefficients $\gamma_{2\mathrm{ph}}$ and b_2 to be appropriately renormalised. The parameter ξ characterises the deviation of the real transverse profile of the laser-radiation field from a homogeneous profile. Indeed, if $\Phi(x)$ is a unit step function with the envelope characterising the laser aperture, then $\xi \equiv 0$. In the case of the fundamental Gauss–Hermite mode $[\Phi = \exp(-x^2/2a^2)]$ $\xi < \xi^* = (1-\sqrt{2})/\sqrt{2} \approx 0.29$, where ξ^* corresponds to an infinite width of the active layer. In real semiconductor heterostructures, the transverse-mode wings significantly exceed the size of the pumped layer, and the inhomogeneity parameter is $\xi \leqslant 0.05-0.15$. The resulting correction of the nonlinear gain compensates for the overestimate due to the medium saturation, which emerges as the result of substitution of the real local value of g by its value in the mode maximum. On the other hand, it is equivalent to the efficient absorption saturation, which compensates for the mentioned overestimate of the gain saturation. In this case, the relaxation time of an 'effective absorber' is sufficiently small, because it is fast in the sense of the mode-locking model [5, 10].

Let us use now the Haus procedure [5, 10] and obtain the relation between the laser-field amplitudes at a point $z = 0$ on $n+1$ and n steps of the cavity round trip. Assuming the field change per round trip to be small enough, we represent the field amplitude on nth round trip in the form $U_n(z,\tau) = U_n(\tau) + \Delta U_n(z,\tau)$, where $U_n(\tau) = U_n(z = 0,\tau)$; $\Delta U_n(z,\tau) \ll U_n(\tau)$ is a small change in the amplitude during the cavity round trip. The solution of the Eqn (4) can be formally written in the form:

$$U_n(z,\tau) = U_n(\tau) + \int_0^z \mathrm{d}z \hat{L}(U_n(z,\tau),\tau)U_n(z,\tau), \qquad (6)$$

where $\hat{L}(U(z,\tau),\tau)$ is a nonlinear operator for the field-amplitude evolution, whose form is obvious from Eqn (4). The operator $\hat{L}(U(z,\tau),\tau)$ depends on the z coordinate implicitly through the field amplitude $U(z,\tau)$ and the gain, in turn, depends on the amplitude and the pump rate is assumed uniform along the amplifier length. Because the field change is small, we will expand

$$\hat{L}U_n(z,\tau) = \hat{L}_0 U_n(\tau) + \hat{L}_1 \Delta U_n(z,\tau) + \ldots \qquad (7)$$

and keep the fundamental term, assuming that $|\hat{L}_1 \Delta U_n(z,\tau)| \ll |\hat{L}_0 U_n(\tau)|$ along the entire cavity length. Here, $\hat{L}_0 U_n(\tau) \equiv \hat{L}(U_n(\tau),\tau)U_n(\tau)$ and does not depend on z. Assuming the field amplitude outside the active medium to be constant (the transmission of the cavity mirrors is small enough), we obtain the total change in the amplitude per each round trip $U_{n+1}(\tau) \equiv U_{n+1}(z = 0,\tau) \approx U_n(\tau) + 2l\hat{L}_0 \times U_n(\tau)$, where l is the length of the amplifying medium. Because $U_{n+1}(\tau) - U_n(\tau) \ll U_n(\tau), U_{n+1}(\tau)$, we will consider this difference as a derivative over a new variable $s = (2l/T_{\mathrm{r}})t$, which is virtually a renormalised time (the number of the cavity round trips)*. Therefore, we finally derive the expression

$$\frac{\partial}{\partial s}U = \left[\frac{1}{2}g(1+\mathrm{i}\alpha)\left(1+\xi\frac{|U|^2}{E_{\mathrm{s}}^2}\right) - \frac{\gamma}{2} + \left(\frac{A_2}{4} + \frac{\mathrm{i}}{2}\beta_2\right)\frac{\partial^2}{\partial\tau^2}\right]U - \left(\frac{\gamma_{2\mathrm{ph}}}{2} + \mathrm{i}b_2\right)\zeta|U|^2U, \qquad (8)$$

which describes the slow evolution of the emission field within a long time interval consisting of many round trip times. As we are interested in the case of GaAs, we set

$$\left.\frac{\partial^2 g}{\partial\omega^2}\right|_{\omega_0} = -A_2 \text{ and } \left.\frac{\partial g}{\partial\omega}\right|_{\omega_0} \approx 0$$

i.e., generation is close to the gain maximum [14–16].

Equation (8) is well known in the mode-locking theory and nonlinear optics [17, 18]. Its solutions describe both the stationary lasing regime with a constant amplitude $|U(z,\tau)|^2 = \mathrm{const}$, and generation of soliton-like ultrashort pulses $|U(z,\tau)|^2 \approx \cosh^{-2}(\tau/\tau_{\mathrm{p}})$ in the mode-locking regime, where τ_{p} is the pulse duration. Depending on particular conditions and relations between the parameters, both solutions can be realised.

In the stationary regime $[U(z,\tau) = U_{\mathrm{c}}\exp(\mathrm{i}P_{\mathrm{c}}s)]$, we obtain from Eqn (1) the stationary gain on the axis

$$g_{\mathrm{c}} = \frac{g_0}{1 + U_{\mathrm{c}}^2/E_{\mathrm{s}}^2}, \qquad (9)$$

and from Eqn (8), a system of algebraic equations for the laser-field characteristics

$$P_{\mathrm{c}} = \frac{1}{2}\alpha g_{\mathrm{c}}\left(1 + \frac{\xi U_{\mathrm{c}}^2}{E_{\mathrm{s}}^2}\right) - b_2 U_{\mathrm{c}}^2,$$
$$g_{\mathrm{c}}\left(1 + \frac{\xi U_{\mathrm{c}}^2}{E_{\mathrm{s}}^2}\right) - \gamma_{2\mathrm{ph}}U_{\mathrm{c}}^2 - \gamma = 0. \qquad (10)$$

Relation (9) and the second equation from (10) determine the stationary field amplitude

$$U_{\mathrm{c}}^2 = \frac{1}{2}\{\pm[(E_{\mathrm{s}}^2 + I_0)^2 + 4E_{\mathrm{s}}^2 I_1]^{1/2} - (E_{\mathrm{s}}^2 + I_0)\}, \qquad (11)$$

where the parameters $I_0 = (\gamma - \xi g_0)/\gamma_{2\mathrm{ph}}$ and $I_1 = (g_0 - \gamma)\gamma_{2\mathrm{ph}}^{-1}$ are introduced. The requirement for the solution to be positive is equivalent to the natural threshold condition, when amplification exceeds the losses, $g_0 > \gamma$. In the limiting case $\gamma_{2\mathrm{ph}} = 0$, the solution takes the form

$$\frac{U_{\mathrm{c}}^2}{E_{\mathrm{s}}^2} = \frac{g_0 - \gamma}{\gamma - \xi g_0}, \qquad (12)$$

*Note that when the field variations per round trip are strong, the replacement of the difference in the field by the derivative over a slow time variable s becomes invalid. Then, the discrete Puancare transformation needs to be studied. This problem is of certain interest for a separate investigation. Here, we restrict ourselves by the mentioned simple approximation.

and, correspondingly, the existence condition for the stationary lasing regime is

$$\xi g_0 < \gamma < g_0. \quad (13)$$

To study the stability condition, we consider the evolution of small deviations δg, δu from the stationary solution depending on the increase in the number of cavity round trips s. By substituting $g = g_c + \delta g$ and $U = [U_c + \delta u(z,\tau)] \exp(iP_c z)$, into Eqns (1) and (8), we obtain a system of equations for δg, δu

$$\frac{d}{dt}\delta g = -\frac{\delta g}{T_1} - \varepsilon_c(\delta u + \delta u^*),$$

$$\frac{\partial}{\partial z}\delta u = \frac{1}{2}(1+i\alpha)\left(1+\xi\frac{U_c^2}{E_s^2}\right)U_c\delta g - \left[\frac{\gamma_{2ph}}{2} - \frac{1}{2}g_c\xi(1+i\alpha)\frac{U_c^2}{E_s^2} + ib_2\right] \times (\delta u + \delta u^*)\left(\frac{A_2}{4} + \frac{i\beta_2}{2}\right)\frac{\partial^2}{\partial\tau^2}\delta u, \quad (14)$$

where

$$\varepsilon_c = \frac{g_0 U_c}{T_0 E_s^2}\frac{1+\xi U_c^2/E_s^2}{1+U_c^2/E_s^2}. \quad (15)$$

For a time-periodic perturbation, we look for solutions in the form $\delta g = G e^{i\omega\tau} + \text{c.c.}$, $\delta u = f e^{i\omega\tau} + h e^{-i\omega\tau}$. Then we have

$$G = -\frac{\varepsilon_c T_1}{1+i\omega T_1}, \quad T_1 = \frac{T_0}{1+U_c^2/E_s^2}, \quad (16)$$

For the functions $F = f + h^*$, $H = f - h^*$ we obtain the system of equations:

$$\frac{\partial F}{\partial z} = -\left[\left(1+\xi\frac{U_c^2}{E_s^2}\right)\frac{\varepsilon_c T_1 U_c}{1+i\omega T_1} + \left(\gamma_{2ph} - g_c\xi\frac{U_c^2}{E_s^2}\right) + \frac{A_2}{4}\omega^2\right]F - \frac{i\beta_2}{2}\omega^2 H,$$

$$\frac{\partial H}{\partial z} = -i\left[\alpha\left(1+\xi\frac{U_c^2}{E_s^2}\right)\frac{\varepsilon_c T_1 U_c}{1+i\omega T_1} + \left(2b_2 - g_c\xi\alpha\frac{U_c^2}{E_s^2}\right) + \frac{\beta_2}{2}\omega^2\right]F - \frac{A_2}{4}\omega^2 H. \quad (17)$$

The corresponding dispersion equation for the instability increment λ ($F, H \propto e^{\lambda s}$) has the form

$$\left[\lambda + \left(1+\xi\frac{U_c^2}{E_s^2}\right)\frac{\varepsilon_c T_1 U_c}{1+i\omega T_1} + \frac{A_2}{4}\omega^2 + \gamma_{2ph} - g_c\xi\frac{U_c^2}{E_s^2}\right] \times \left(\lambda + \frac{A_2}{4}\omega^2\right) + \frac{\beta_2\omega^2}{2}\left[\alpha\left(1+\xi\frac{U_c^2}{E_s^2}\right)\frac{\varepsilon_c T_1 U_c}{1+i\omega T_1} + \frac{\beta_2}{2}\omega^2 + 2b_2 - g_c\xi\alpha\frac{U_c^2}{E_s^2}\right] = 0. \quad (18)$$

In the absence of the linear-refractive-index dispersion $\beta_2 = 0$ Eqn (18) splits into two equations, and we obtain two solutions. The first solution with the increment $\lambda_1 = -(A_2/4)\omega^2$ is always stable. The increment of the second solution, depending on the pump level, can become positive in a certain frequency range, so that perturbations at these frequencies become unstable. By neglecting the two-photon absorption $\gamma_{2ph} \to 0$ and taking into account relations (12), (15), and (16), we obtain the dependence of the instability increment on the ratio of the absorption and pump levels $r \equiv \gamma/g_0$:

$$\text{Re}\lambda_2 = g_0\frac{1-r}{1-\xi}\left[\xi - \frac{(1-\xi)^3 r^2}{r-\xi} \times \frac{1}{(1-\xi)^2 + (r-\xi)^2(\omega T_0)^2}\right] - \frac{A_2}{4}\omega^2. \quad (19)$$

One can see from (19) that the instabilities of the stationary lasing are due to the transverse inhomogeneity of the laser mode; in the limit $\xi \to 0$ we have $\text{Re}\lambda_2 \to \lambda_1 < 0$. Near the generation threshold ($r \approx 1$), stationary solution (12) is also stable within the entire range of perturbations. When the pump increases, at certain ratios of the parameters g_0 and A_2, a frequency domain emerges where perturbations increase exponentially. In the high-pump limit ($r \to \xi$), the stationary solution becomes stable again.

3. Results of calculations

Consider now the parameters at which instability domains exist. It is evident that the pump intensity should play an important role in developing instabilities. Figure 2 shows the dependences of the normalised instability increment on the perturbation frequency for four values of the pump intensity. One can clearly see that at small intensities the increment is negative within the entire frequency domain,

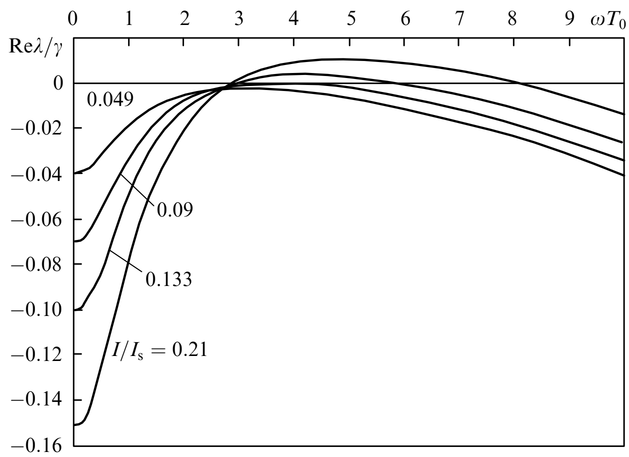

Figure 2. Frequency dependences of the real part of the instability increment for different values of the normalised output power; $A_2 \approx 60$ fs^2 μm^{-1}, $\gamma \approx 25$ cm^{-1}.

and the stationary lasing regime is always stable. However, when the pump increases, one observes the emergence of frequency ranges where the increment becomes positive, and the stationary regime becomes unstable. In an explicit form, this transition is shown in Fig. 3, where the dependences of instabilities on the pump intensity for four frequencies ωT_0 are plotted.

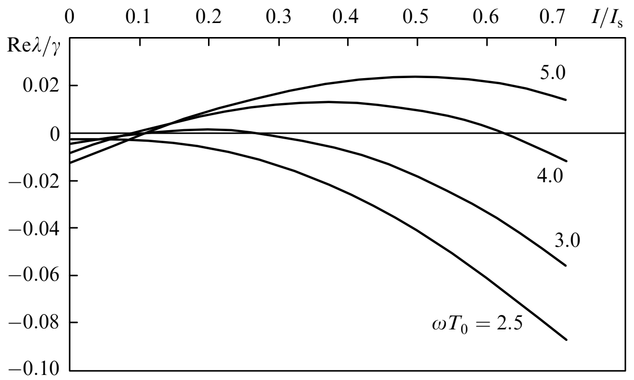

Figure 3. Dependences of the real part of the instability increment on the normalised output power for different values of the frequency; $A_2 \approx 60$ fs^2 μm^{-1}, $\gamma \approx 25$ cm^{-1}.

The threshold instability conditions for the stationary solution are determined by the condition $\max(\mathrm{Re}\lambda)|_{r=\mathrm{const}} = 0$ within the whole domain of perturbation frequencies $\omega T_0 > 0$ at a fixed pump level. For convenience we will use below the parameter $I \equiv U_c^2/E_s^2 = (1-r)/(r-\xi)$, which is a normalised peak intensity in the stationary regime. It is easy to see that under condition

$$\frac{A_2}{4\gamma T_0^2} \geqslant \frac{I(1+\xi I)}{(1+I)^3}$$

the increment of Eqn (15) is a function monotonically decreasing with frequency and its maximum value (achievable at $\omega = 0$) is negative

$$\mathrm{Re}\frac{\lambda_2}{\gamma}\bigg|_{\omega=0} = -\frac{I}{(1+I)(1+\xi I)}(1-\xi+\xi I+\xi^2 I^2). \quad (20)$$

With an increase in the line width, i.e., when the coefficient A_2 decreases, so that

$$\frac{A_2}{4\gamma T_0^2} \leqslant \frac{I(1+\xi I)}{(1+I)^3},$$

the increment of Eqn (19) achieves a maximum at the frequency

$$\omega T_0 = (1+I)\left\{\left[\frac{4\gamma T_0^2}{A_2}\frac{I(1+\xi I)}{(1+I)^3}\right]^{1/2} - 1\right\}^{1/2},$$

and we obtain the following implicit relation for the threshold intensity of the stable stationary regime at a given line width

$$\frac{A_2}{4\gamma T_0^2} \geqslant \frac{I_{\mathrm{th}}(1+\xi I_{\mathrm{th}})}{(1+I_{\mathrm{th}})^3}\left\{1 - \left[1 - \frac{(1+I_{\mathrm{th}})\xi}{(1+\xi I_{\mathrm{th}})^2}\right]^{1/2}\right\}^2. \quad (21)$$

Above the threshold ($I > I_{\mathrm{th}}$), there emerges a frequency range ω, where the increment of Eqn (19) becomes positive, $\mathrm{Re}\lambda_2 > 0$.

We can see that the gain spectrum width plays an important role in developing instabilities, i.e. an increase in the gain spectrum width contributes to developing instabilities. The same dependence was also observed in other laser systems [3, 4]. To obtain the dependence of the threshold of the instability development on the spectral gain width, we approximate the gain by the parabola

$$g(\omega) = g_0\left[1 - \frac{2(\omega-\omega_0)^2}{\Delta\omega^2}\right], \quad (22)$$

where $g_0 = g(\omega_0)$; $\Delta\omega$ is the spectrum width at half amplitude. Then we have

$$A_2 = \frac{g_0}{\Delta\omega^2}, \quad (23)$$

and expression (21) for the stability threshold has the from:

$$\Delta\lambda \leqslant \Delta\lambda_{\mathrm{cr}} = \frac{\lambda_0^2}{4\pi c T_0}\frac{(1+I_{\mathrm{th}})^2}{I_{\mathrm{th}}^{1/2}(1+\xi I_{\mathrm{th}})} \times \left\{1 - \left[1 - \frac{(1+I_{\mathrm{th}})\xi}{(1+\xi I_{\mathrm{th}})^2}\right]^{1/2}\right\}^{-1}. \quad (24)$$

Expression (24) assumes that, at a given output power and the gain spectrum width $\Delta\lambda < \Delta\lambda_{\mathrm{cr}}$, the stationary regime is stable, while at $\Delta\lambda > \Delta\lambda_{\mathrm{cr}}$ the solution becomes unstable. By using expresion (24), it is possible to solve the reverse problem, i.e. at a given gain spectrum width, we can determine the critical output power above which the stationary lasing becomes unstable.

Figure 4 presents the dependences of the critical gain spectrum width $\Delta\lambda_{\mathrm{cr}}$ on the output power P for various values of the parameter ξ and the saturation power of 14 W (the saturation energy is 10 pJ). One can clearly see that, when $\Delta\lambda_{\mathrm{cr}}$ increases, the critical output power, at which the instability emerges, drastically drops. For instance, for $\Delta\lambda_{\mathrm{cr}} = 5$ nm the instability develops at $P \sim 1$ W, whereas for $\Delta\lambda_{\mathrm{cr}} = 30$ nm the stationary regime is unstable at powers of a few tens of mW, depending on the transverse field nonuniformity.

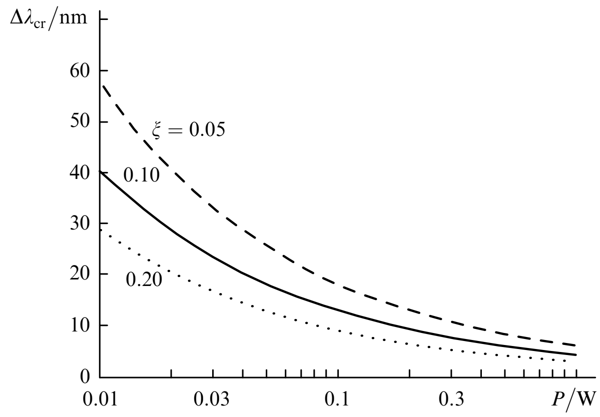

Figure 4. Dependences of the critical gain spectrum width $\Delta\lambda_{\mathrm{cr}}$ on the output power P for different transverse field inhomogeneities; $\lambda_0 = 950$ nm, $T_0 = 700$ fs, the saturation power is 14 W.

4. Mode-locked pulses

In addition to the stationary solution discussed above, evolution equation (8) allows for a mode-locked solution [5, 10]. Generally speaking, both solutions can be realised either separately or simultaneously, depending on given parameters of the problem.

As we saw above, at a certain spectral gain width the stationary lasing regime fails when the pump increases, and the external-cavity semiconductor laser can operate in the self-mode locking regime. The second type of solutions of Eqn (8) corresponds to this regime. This solution is well known [17, 19] and is a train of ultrashort pulses of the form

$$U(s,\tau) = U_{\rm p}[{\rm sech}(\tau/\tau_{\rm p})]^{1+{\rm i}\varphi}\exp({\rm i}\eta s). \qquad (25)$$

Assuming the pulse duration $\tau_{\rm p}$ and the pulse repetition rate $T_m = T_{\rm r}/m$ ($m = 1, 2, 3,...$ is the number of pulses in the cavity) to be small compared to the characteristic gain relaxation time, i.e., $\tau_{\rm p}, T_m \ll T_0$, we can approximately assume [5, 14] that the average value of the gain is

$$g \approx g_{\rm av} = \frac{g_0}{1+\kappa I_{\rm p}/I_{\rm s}}, \qquad (26)$$

where $I_{\rm p}$ is the pulse peak intensity and $\kappa = \tau_{\rm p}/T_m \ll 1$. Herewith, in accordance with the transverse mode averaging procedure, we should replace $\xi \to \kappa\xi$ in Eqn (8).

The substitution of Eqn (25) into Eqn (8) gives the relations for the pulse parameters:

$$\tau^2 = \frac{A_2(1-\varphi^2) - 4\beta_2\varphi}{2(\gamma - g_{\rm av})},$$

$$\eta = \frac{1}{2}\left[\alpha g_{\rm av} + \frac{1}{\tau^2}(A_2\varphi + (1-\varphi^2)\beta_2)\right], \qquad (27)$$

$$U_{\rm p}^2 = \frac{1}{2\tau^2}\frac{3A_2\varphi + 2(2-\varphi^2)\beta_2}{\alpha\kappa g_{\rm av}\xi/E_{\rm s}^2 - 2b_2},$$

$$\frac{2-\varphi^2}{3\varphi} = -\frac{(A_2 + 2\alpha\beta_2)g_{\rm av}\kappa\xi/E_{\rm s}^2 - (A_2\gamma_{\rm 2ph} + 4b_2\beta_2)}{2(b_2A_2 - \beta_2\gamma_{\rm 2ph}) - (\alpha A_2 - 2\beta_2)g_{\rm av}\kappa\xi/E_{\rm s}^2}.$$

In the limit of a small transverse mode inhomogeneity $b_2 \gg \alpha g_{\rm av}\kappa\xi/2E_{\rm s}^2$, the solution corresponds to the solutions known as dissipative solitons [19]. If we neglect the two-photon absorption and dispersion of the linear refractive index β_2, $\gamma_{\rm 2ph} \to 0$, relations (27) take the form

$$\frac{2-\varphi^2}{3\varphi} = -\frac{g_{\rm av}\kappa\xi/E_{\rm s}^2}{2b_2 - \alpha g_{\rm av}\kappa\xi/E_{\rm s}^2}, \quad \tau^2 = \frac{A_2(1-\varphi^2)}{2(\gamma - g_{\rm av})}, \qquad (27a)$$

$$U_{\rm p}^2 = -\frac{1}{2\tau^2}\frac{3A_2\varphi}{2b_2 - \alpha g_{\rm av}\kappa\xi/E_{\rm s}^2}, \quad \eta = \frac{\alpha g_{\rm av}}{2} + \frac{A_2\varphi}{2\tau^2},$$

and for $\xi \to 0$ we have $\varphi \approx -\sqrt{2}$. The pulse duration is $\tau^2 \approx A_2/2(g_{\rm av} - \gamma)$, and its amplitude is determined by the Kerr nonlinearity $U_{\rm p}^2 \approx 3(g_{\rm av} - \gamma)/\sqrt{2}b_2$. In the opposite limit, $b_2 \ll \alpha g_{\rm av}\kappa\xi/2E_{\rm s}^2$, the effect of the transverse field inhomogeneity dominates, and for the average intensity we obtain

$$\frac{I_{\rm p}}{I_{\rm s}} \approx \frac{3\sqrt{2}}{\alpha\xi}\frac{g_{\rm av} - \gamma}{g_{\rm av}}. \qquad (28)$$

Bearing in mind the characteristic values of the parameters $A_2 \approx 60$ fs^2 μm^{-1} and $\gamma \approx 25$ cm^{-1}, we can see that the solutions of Eqns (25) and (27a) give the pulse duration in the range of hundreds of femtoseconds $[(A_2/2\gamma)^{1/2} \approx 110$ fs$]$. The study of the self-mode locking regime and its stability is beyond the scope of this work and will be published elsewhere.

5. Conclusions

We have considered here the development of instabilities of stationary lasing in an external-cavity semiconductor laser for further studying the possibility of obtaining the femtosecond pulse generation. Theoretical calculations have been based on the known Haus approach borrowed from the mode-locked laser theory which is modified for a laser diode active medium. The existence of laser parameters at which stationary lasing becomes unstable has been demonstrated.

The instability development has been shown to be affected mainly by transverse inhomogeneities of the optical field, the pump intensity above the threshold, and the laser spectral gain width. In the limit of the complete transverse field homogeneity and near the lasing threshold, stationary lasing was stable in the entire domain of perturbations. In the case of the field inhomogeneity and with an increase in the pump level, at certain values of the unsaturated gain, a frequency domain emerges where perturbations of the stationary solution increase exponentially. An increase in the spectral gain width of the laser medium leads to a decrease in the pump threshold intensity at which instabilities of stationary lasing develop.

Note that the instability of lasing in laser diodes with continuous pumping can be observed in various forms, including the regime of the stable generation of ultrashort pulses (self-mode locking regime), the dynamic chaos regime, the pulse-repetition-rate doubling (tripling), the unstationary spike regime, etc. Therefore, the next step of our study is the search for laser parameters at which there exists a stable generation regime of a regular train of ultrashort pulses. In other words, it is necessary to investigate the self-starting mode locking regime in external-cavity laser diodes and study their stability.

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