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On the hypothetical giant narrowing of radiative atomic and nuclear lines in a Bose – Einstein condensate

L.A. Rivlin

***Abstract*.** **The possibility of existence of ultranarrow atomic and nuclear radiative lines in a 'megaatom' of a Bose – Einstein condensate in a quantum trap is estimated. This phenomenon is caused by the elimination of the inhomogeneous broadening due to suppression of the random motion of atoms in the condensate resulting from the establishment of the higher-order quantum coherence in it.**

Keywords: *Bose – Einstein condensate, quantum coherence, 'megaatom' state, quantum nucleonics, suppression of the excess broadening of radiative lines, metastable states.*

1. Introduction

Absorption and emission transitions involving metastable atomic and nuclear states have extremely narrow natural linewidths, which are inversely proportional to their large spontaneous lifetime. However, a variety of perturbing factors cause the broadening of lines, which exceeds their natural width by many orders of magnitude. At the same time, the problem of obtaining spectral lines with the natural width or nearly natural width is of considerable interest from the general physical point of view, in particular, for precision metrology and quantum nucleonics.

One of the most obvious perturbing factors in the case of gas atoms is their random thermal motion producing the inhomogeneous Doppler broadening, which increases with increasing the transition energy. Attempts to reduce the Doppler broadening by lowering the gas temperature are restricted by the minimal temperatures of the order of hundredths of microkelvin that can be achieved at present.

The hypothetical alternative method for suppressing the random atomic motion can be the formation of an atomic Bose – Einstein condensate (BEC). It can be expected [1, 2] (despite not reliable enough estimates for the practically unrealisable idealised case of an infinite-volume gas [2]) that in such an ensemble of boson atoms with the de Broglie wavelength of the order of the ensemble size and overlapping wave functions, the higher-order quantum coherence will be established and the individual motion of atoms will be considerably suppressed, resulting in the appearance of the so-called hypothetical megaatom [1] and the corresponding decrease in the inhomogeneous Doppler line broadening. The effective quasi-Doppler temperature of the megaatom can be defined by the expression

$$T_{\mathrm{eff}} = T\left(\frac{\Delta\omega_{\mathrm{obs}}}{\Delta\omega_{\mathrm{D}}(T)}\right)^2 \approx 0.18\,\frac{Mc^2}{k_{\mathrm{B}}}\left(\frac{\Delta\omega_{\mathrm{obs}}}{\omega}\right)^2$$

$$\approx 2\times10^{12}A(\Delta\omega_{\mathrm{obs}}/\omega)^2, \qquad (1)$$

This temperature corresponds to the observed linewidth $\Delta\omega_{\mathrm{obs}} \geqslant \Delta\omega_{\mathrm{nat}}$ (assuming that this is the Doppler linewidth) divided by the Doppler linewidth $\Delta\omega_{\mathrm{D}}(T) = 2\omega \times[2\ln 2(k_{\mathrm{B}}T/Mc^2)]^{1/2}$ calculated for the real temperature T (here, ω is the transition frequency; $\Delta\omega_{\mathrm{nat}}$ is the natural linewidth; M is the atom mass; A is the atomic mass number; k_{B} is the Boltzmann constant; and c is the speed of light). The effective temperature T_{eff} is not, of course, the real thermodynamic temperature.

Apart from the Doppler broadening, which can be tentatively eliminated in the megaatom, there also exist other line broadening mechanisms. The influence of these mechanisms, which restrict the narrowing of atomic and nuclear transition lines in the megaatom, is estimated below.

2. Bose condensate in a quantum trap

In experiments, gas, as a rule, is contained in a trap of one or other type. The main features of the behaviour of a BEC under these conditions are as follows [3].

Let a part of the gas of N noninteracting boson atoms be at temperature T in a three-dimensional trap with a harmonic potential confining atoms inside a prolate circular ellipsoid (the so-called cigar). The phase transition and the formation of a Bose condensate by a part of atoms occur under the condition

$$T < T_{\mathrm{c}}, \qquad (2)$$

where the critical phase transition temperature is determined by the expression

$$k_{\mathrm{B}}T_{\mathrm{c}} = 0.94\hbar\Omega_0 N^{1/3}; \qquad (3)$$

$$\hbar\Omega_0 = \hbar(\Omega_r^2\Omega_z)^{1/3} \qquad (4)$$

L.A. Rivlin Applied Physics Laboratory, Moscow State Institute of Radio-Engineering, Electronics and Automation (Technical University), prosp. Vernadskogo 78, 119454 Moscow, Russia;
e-mail: lev_rivlin@mail.ru

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is the energy eigenvalue of the lowest state of the trap with the transverse and longitudinal eigenfrequencies Ω_r and Ω_z.

The effective transverse and longitudinal sizes of the cigar are related to the lowest eigenfrequencies of the trap by expressions

$$l_r \approx \left(\frac{\hbar}{M\Omega_r}\right)^{1/2}, \qquad l_z \approx \left(\frac{\hbar}{M\Omega_z}\right)^{1/2}, \tag{5}$$

while the gas volume V and the averaged concentration N/V in the cigar approximated by a circular cylinder are

$$V \approx \frac{\pi}{4} l_r^2 l_z, \quad \frac{N}{V} \approx \frac{4N}{\pi l_r^2 l_z}. \tag{6}$$

The critical temperature T_c (3) can be conveniently expressed in terms of gas volume (6) in the trap:

$$k_B T_c = 0.8 \frac{\hbar^2}{M} \left(\frac{N}{V^2}\right)^{1/3}. \tag{7}$$

The number of atoms N_{BEC} in the condensed fraction depends on the ratio of the gas temperature to the critical temperature T_c:

$$N_{BEC} = N[1 - (T/T_c)^3]. \tag{8}$$

Despite the cigar shape with $l_r < l_z$ the trap should be reliably described by the three-dimensional model used, which requires the fulfilment of the inequality $k_B T \gg \hbar\Omega_r$ [3]. This restricts the number of atoms from below:

$$N > N_{3D} \equiv \left(\frac{\pi}{4}\frac{l_z}{l_r}\right)^2 \left(\frac{T_c}{T}\right)^3. \tag{9}$$

The phase transition of a part of gas atoms to a condensed fraction usually occurs during the evaporation cooling of gas, which is accompanied by the undesirable decrease in the number of atoms in the trap. However, inequality (2) can be also achieved due to the adiabatic cooling of gas by varying the volume of the potential well of the trap [4–7], as was performed in [7].

3. Doppler width of radiative lines in an elongated quantum trap and the broadening anisotropy

Unlike the ideal case of homogeneous gas in an infinite volume, in which the energy and momentum of an atom in the ground state are exactly zero, the lowest atomic state in a trap has the finite energy and momentum. Therefore, radiative lines are broadened due to oscillations of atoms. The frequencies of the lowest oscillation modes in a strongly rarefied gas of noninteracting atoms [see criterion (16), (17) below] virtually coincide with the fundamental eigenfrequencies Ω_r and Ω_z of a trap with a harmonic potential [3]. These atomic oscillations, appearing in fact due to the uncertainty relation, cause the broadening of radiative lines, which can be considered in a sense as the Doppler effect depending on the observation direction. It is assumed below that the contribution to this broadening from the higher oscillation modes can be neglected.

If oscillating atoms have average velocities $\Delta v_z \approx (\hbar\Omega_z/M)^{1/2}$ and $\Delta v_r \approx (\hbar\Omega_r/M)^{1/2}$ along the longitudinal and transverse directions of the cigar, respectively, a line with the transition energy $\hbar\omega$ observed along the longitudinal axis of the cigar is broadened due to the first- and second-order Doppler effects:

$$\Delta\omega'_D(z) \approx \omega\frac{\Delta v_z}{c} \approx \omega\left(\frac{\hbar\Omega_z}{Mc^2}\right)^{1/2},$$
$$\Delta\omega''_D(z) \approx \frac{\omega}{2}\left(\frac{\Delta v_r}{c}\right)^2 \approx \omega\frac{\hbar\Omega_r}{2Mc^2}, \tag{10}$$

the inequality $\Delta\omega'_D(z)/\Delta\omega''_D(z) \approx 2(Mc^2\hbar\Omega_z)^{1/2}/\hbar\Omega_r \gg 1$, taking place in the cigar, although $l_z \gg l_r$ and $\Omega_z \ll \Omega_r$. At the same time, the Doppler linewidths for observation in the transverse direction are

$$\Delta\omega'_D(r) \approx \omega\frac{\Delta v_r}{c} \approx \omega\left(\frac{\hbar\Omega_r}{Mc^2}\right)^{1/2},$$
$$\Delta\omega''_D(r) \approx \frac{\omega}{2}\left(\frac{\Delta v_z}{c}\right)^2 \approx \omega\frac{\hbar\Omega_z}{2Mc^2}, \tag{11}$$

and here the more so $\Delta\omega'_D(r)/\Delta\omega''_D(r) \approx 2(Mc^2\hbar\Omega_r)^{1/2} \times(\hbar\Omega_z)^{-1} \gg 1$.

Therefore, by neglecting the contribution from the second-order Doppler effect, it is important to have a strong line broadening anisotropy in the cigar:

$$\frac{\Delta\omega_D(z)}{\Delta\omega_D(r)} \approx \frac{\Delta\omega'_D(z)}{\Delta\omega'_D(r)} \approx \left(\frac{\Omega_z}{\Omega_r}\right)^{1/2} = \frac{l_r}{l_z} \ll 1. \tag{12}$$

As a result, by assuming that $\Delta\omega_{obs} \approx \Delta\omega_D(z)$, we see that effective temperature T_{eff} (1) in the cigar depends only on its length and the atom mass:

$$T_{eff} = T\left[\frac{\Delta\omega_D(z)}{\Delta\omega_D(T)}\right]^2 = \frac{\hbar\Omega_z}{8\ln 2 k_B}$$
$$= \frac{(\hbar/l_z)^2}{8\ln 2 k_B M} \approx \frac{8.7\times10^{-16}}{A l_z^2}. \tag{13}$$

The estimate from (13) with $A = 30$ and $l_z = 0.1$ cm gives the amazing result $T_{eff} \approx 3\times10^{-15}$ K.

4. Ratio β of linewidths

It is convenient to introduce another criterion for eliminating the broadening of a radiation line, namely, the ratio of the natural linewidth $\Delta\omega_{nat}$ to the total linewidth $\Delta\omega_{tot}$ determined by all the broadening mechanisms:

$$\beta = \frac{\Delta\omega_{nat}}{\Delta\omega_{tot}} = \frac{2\pi}{\tau\Delta\omega_{tot}}, \tag{14}$$

where $\tau = 2\pi/\Delta\omega_{nat}$ is the excited-state lifetime for spontaneous decay.

The parameter β plays an important role in stimulated emission and resonance absorption of radiation. The line broadening is completely eliminated when $\beta \to 1$.

By assuming that the line broadening is completely determined by the estimate $\Delta\omega'_D(z) \approx \Delta\omega_D(z)$ (10), i.e. that $\Delta\omega_{tot} = \Delta\omega_D(z)$, we obtain

$$\beta = \frac{2\pi}{\omega\tau}\left(\frac{Mc^2}{\hbar\Omega_z}\right)^{1/2} = \frac{M\lambda l_z}{\hbar\tau} \approx 1.6\times10^3 A\frac{\lambda l_z}{\tau}$$

$$\approx 4.8\times10^{13} A l_z \frac{\Delta\omega_{\rm nat}}{\omega} \quad (15)$$

where λ is the wavelength. Thus, for the parameters $A = 30$ and $l_z = 0.1$ cm, the ratio $\beta \to 1$ for $\Delta\omega_{\rm nat}/\omega \approx 7\times10^{-15}$ (i.e., for example, for $\lambda = 10^{-4}$ cm and $\tau \approx 0.5$ s or for $\lambda = 10^{-8}$ cm and $\tau \approx 0.5\times10^{-4}$ s).

5. Other line broadening sources

Attractive estimates (13) and (15) take into account only the suppression of the inhomogeneous Doppler line broadening and assume that this broadening dominates over all other broadening mechanisms in a megaatom. Therefore, the role of these mechanisms should be estimated additionally.

The important criterion of gas rarefaction, which follows from the Gross–Pitaevskii equation [8–10] and excludes the influence of pair interactions of atoms, is established by the requirement that $\varepsilon_{\rm int}/\varepsilon_{\rm kin} \approx N|a|/l_{\rm eff} \ll 1$ ($\varepsilon_{\rm int} \approx 4\pi\hbar^2|a|N^2 \times (MV)^{-1}$ is the total atom–atom interaction energy and $\varepsilon_{\rm kin}$ is the total kinetic energy $\varepsilon_{\rm kin} \approx N\hbar\Omega_0$ in the ground state of the trap [3]), where a is the scattering length of the order of a few nanometres [3] and $l_{\rm eff} = (\hbar/M\Omega_0)^{1/2}$ is the effective trap size. This requirement restricts the total number of atoms in the cigar

$$N \ll N_{\rm a} \cong 0.1 V^{1/3}|a|^{-1} \quad (16)$$

and their concentration

$$N/V \ll (N/V)_{\rm a} \cong 0.1 V^{-2/3}|a|^{-1}. \quad (17)$$

Thus, $N \ll N_{\rm a} \approx 10^3$ and $N/V \ll (N/V)_{\rm a} \approx 10^{10}$ cm^{-3}, if $V = 10^{-7}$ cm^3 and $|a| = 5\times10^{-7}$ cm.

Note that experiments (for example, with Rb [11] and Na [12] atoms) show that a rather strong deviation from inequality (16) not always prevents the formation of a BEC.

It follows from the Gross–Pitaevskii equation that the danger of the BEC collapse [13, 14] appears in the case of attraction between atoms when $a < 0$ and the number N of atoms in the trap approaches the critical value $N_{\rm a}$ (16). This danger is excluded when inequality (16) is strong enough.

One of the main factors preventing the narrowing of a transition line is a finite lifetime of BEC atoms. It can be estimated from the Bose condensation kinetics, in particular, from the stationary solution of the rate equation for the number of BEC atoms. In this case, condensate atoms produced due to stimulated transition should be separated. The latter condition is quite important because only such stimulated atoms belong to the coherent megaatom ensemble (unlike the atoms forming the BEC spontaneously). The condensate is in the dynamic equilibrium with the thermodynamic gas component (i.e. described by the Bose–Einstein distribution with temperature T), which continuously exchanges by atoms with the condensate. Therefore, even in the stationary state with a constant amount of BEC atoms, the lifetime of atoms in the coherent fraction is restricted by the rate of this exchange [15]:

$$\Theta_{\rm BEC} = \left|\frac{{\rm d}N_{\rm BEC}^{\rm coh}}{{\rm d}t}\right|^{-1}. \quad (18)$$

The decrease rate of the number of atoms in the coherent fraction is equal to the rate of their stimulated emission, i.e. to the inverse lifetime $\Theta_{\rm BEC}$ of the BEC.

6. Rate equation for the number of BEC atoms and the estimate of $\Theta_{\rm BEC}$

The parameters of the stationary state of the condensate required for calculating $\Theta_{\rm BEC}$ (18) can be found from the stationary solution of the rate equation.

In the opinion of authors of [16], numerous theoretical studies of the Bose-condensation kinetics ([17–20] and others) are insufficient for obtaining certain quantitative results. This also concerns the productive and clear analogy between the roles of stimulated transitions in the kinetics of massless bosons – photons (in lasers) and massive bosons – atoms [21–24].

The condensation of Bose atoms was considered quantum-kinetically rigorously in [16], where the rate equation for the number $N_{\rm BEC}$ of BEC atoms was derived:

$$\frac{{\rm d}N_{\rm BEC}}{{\rm d}t} = 2\left[\frac{4Ma^2(k_{\rm B}T)^2}{\pi\hbar^3}\right]\left(\frac{\mu_{\rm BEC}}{k_{\rm B}T}K_1\right)\exp\left(\frac{2\mu}{k_{\rm B}T}\right)$$

$$\times\left\{\left[1-\exp\left(\frac{\mu_{\rm BEC}-\mu}{k_{\rm B}T}\right)\right]N_{\rm BEC}+1\right\}, \quad (19)$$

where $\mu_{\rm BEC}$ and μ are the chemical potentials of the condensed and thermodynamic fractions, respectively; $K_1 \equiv K_1(\mu_{\rm BEC}/k_{\rm B}T)$ is the modified Bessel function, and the first factor in brackets is the rate of elastic collisions. Both exponentials in (19) are Bose–Einstein functions truncated to the Boltzmann form, which, in the opinion of authors of [16], does not affect significantly the final result.

Rate equation (19) presents the BEC kinetics as the analogue of processes in a laser [21–24] and describes the spontaneous transition of atoms to the BEC [the last unit in braces in (19)], the stimulated transition to the BEC and backward transition ('resonance evaporation') (the difference $1-\exp[(\mu_{\rm BEC}-\mu)/k_{\rm B}T]$ in brackets). The inequality

$$1-\exp\left(\frac{\mu_{\rm BEC}-\mu}{k_{\rm B}T}\right) > 0, \quad (20)$$

which is valid when $\mu_{\rm BEC} < \mu$, is similar to the population inversion condition in lasers. When condition (20) is fulfilled, the stimulated condensation of atoms dominates over the backward process of 'resonance evaporation'.

Inequality (20) is incompatible with the stability condition ${\rm d}N_{\rm BEC}/{\rm d}t = 0$. Because of this, the stationary solution of rate equation (19) exists only when 'resonance evaporation' exceeds stimulated emission, i.e $\mu_{\rm BEC} > \mu$ and then the stationary number of condensate atoms is

$$N^*_{\rm BEC} = \left(\exp\frac{\mu_{\rm BEC}-\mu}{k_{\rm B}T}-1\right)^{-1}. \quad (21)$$

This equality requires that $N^*_{\rm BEC} \gg 1$ due to the assumption that $(\mu_{\rm BEC}-\mu)/k_{\rm B}T \ll 1$, and then

$$N^*_{\rm BEC} \approx \frac{k_{\rm B}T}{\mu_{\rm BEC}-\mu} \gg 1. \quad (22)$$

As a result, the lifetime of an atom in a stationary BEC (18) is

$$\Theta_{\mathrm{BEC}} = \left|\frac{\mathrm{d}N_{\mathrm{BEC}}^{\mathrm{coh}}}{\mathrm{d}t}\right|^{-1}$$

$$\approx \frac{\pi\hbar^3}{8a^2 M k_{\mathrm{B}} T \mu_{\mathrm{BEC}} K_1 N_{\mathrm{BEC}}^*} \exp\left(-\frac{2\mu}{k_{\mathrm{B}}T}\right). \quad (23)$$

The chemical potential of the BEC in a trap with a harmonic potential is estimated [16] in the Thomas–Fermi approximation as

$$\mu_{\mathrm{BEC}} = (2.65\hbar^2 a \Omega_0^3 M^{1/2} N_{\mathrm{BEC}})^{2/5}. \quad (24)$$

Thus, the BEC lifetime is

$$\Theta_{\mathrm{BEC}} \approx \frac{\hbar^{11/5}}{4K_1 k_{\mathrm{B}} T [a^{12} M^6 \Omega_0^6 (N_{\mathrm{BEC}}^*)^7]^{1/5}}$$

$$\times \exp\left[-\frac{3}{k_{\mathrm{B}}T}(\hbar^2 a M^{1/2} \Omega_0^3 N_{\mathrm{BEC}}^*)^{2/5} + \frac{2}{N_{\mathrm{BEC}}^*}\right] \quad (25)$$

and is a complicated function of the eigenfrequency Ω_0 (4) of the trap and of the stationary number N_{BEC}^* of atoms in the BEC.

The expression for BEC lifetime (25) with numerical coefficients has the form

$$\Theta_{\mathrm{BEC}} \approx \frac{0.35}{[A^6 \Omega_0^6 (N_{\mathrm{BEC}}^*)^7]^{1/5} K_1 T}$$

$$\times \exp\left[-\frac{1.4\times 10^{-13}}{T}(A^{1/2}\Omega_0^3 N_{\mathrm{BEC}}^*)^{2/5} + \frac{2}{N_{\mathrm{BEC}}^*}\right], \quad (26)$$

where $a = 5\times 10^{-7}$ cm and the argument of the function K_1 is

$$\frac{\mu_{\mathrm{BEC}}}{k_{\mathrm{B}}T} \approx \frac{4.6\times 10^{-14}}{T}(A^{1/2}\Omega_0^3 N_{\mathrm{BEC}}^*)^{2/5}. \quad (27)$$

For example, $\Theta_{\mathrm{BEC}} \approx 0.1$ s for $N_{\mathrm{BEC}}^* = 500$, $T = 10^{-6}$ K, $A = 30$ and $\Omega_0 = 10$ s^{-1}.

Note that, according to [16], rate equation (19) does not contain the terms describing the 'nonresonance' loss ('nonresonance evaporation') of BEC atoms because they are negligible compared to other terms (19).

Thus, the rate of three-body recombination accompanied by the formation of molecules is

$$R_{\mathrm{III}} = K_{\mathrm{III}}(N_{\mathrm{BEC}}^*/V)^3 V \quad (28)$$

and, as shown theoretically and confirmed in experiments with ^{87}Rb [26], this rate for condensate atoms is $3! = 6$ times lower than that in thermodynamic gas at the same temperature. This difference is explained by the suppression of the grouping of boson atoms and concentration fluctuations in the BEC, which is, by the way, a feature of the higher-order quantum coherence and is reflected in the corresponding correlation functions [26]. In particular, it was found in experiments with ^{87}Rb [26] that $K_{\mathrm{III}} = 5.8\times 10^{-30}$ cm^6 s^{-1}. This value is 7.4 times smaller than this coefficient for the thermodynamic fraction, which is close to the theoretical value $3! = 6$ [25] and calculated value [27]. Due to the smallness of K_{III}, the three-body recombination can be neglected in (19). Thus, $R_{\mathrm{III}} = 6.25\times 10^{-6}$ s^{-1} if $N_{\mathrm{BEC}}^* = 500, V = 10^{-8}$ cm^3 and $K_{\mathrm{III}} = 5\times 10^{-30}$ cm^6 s^{-1}, which is a few orders of magnitude smaller than the estimate $\Theta_{\mathrm{BEC}}^{-1} \approx 10$ c^{-1} presented above.

As for bipolar relaxation, it seems that it can be neglected at all compared to other factors. At least no contribution of bipolar relaxation to the decrease in the number of BEC atoms was observed in experiments with ^{87}Rb [26].

It is, of course, also assumed that the BEC storage time Δt_{tr} in a trap, which characterises the technological quality of the experiment, is not shorter than Θ_{BEC} :

$$\Delta t_{\mathrm{tr}} \geqslant \Theta_{\mathrm{BEC}}. \quad (29)$$

Thus, the fulfilment of all the restrictions considered above is necessary for the realisation of the potential possibility of giant narrowing of radiative lines of a megaatom in the BEC.

Note that the absorption and emission lines in a megaatom (as in any ensemble of free atoms) are separated by the frequency interval corresponding to the doubled recoil energy $2E_{\mathrm{rec}}/\hbar = \hbar\omega^2/Mc^2$, which can be noticeable for high-energy quanta. In addition, it is not inconceivable that the recoil of an atom, accompanying any radiative transition, can violate partially or completely the quantum coherence of the megaatom. And note finally that the possible influence of fluctuations of the number of BEC atoms [28, 29] on the broadening of radiative lines in the BEC is not studied so far.

7. Pretender atoms and nuclei

The choice of atoms to demonstrate the existence of a megaatom and observation of the anomalous broadening of radiative lines involves great difficulties because a candidate should combine the properties that are not always compatible: it should be a boson with a metastable level with the appropriate lifetime and convenient for deep cooling and formation of a BEC.

Without the attempt to optimise the choice, we can propose, for example, atomic helium $^{4\mathrm{m}}_{2}$He with the 2^3S$_1$ metastable state with energy 19.82 eV and lifetime ~ 8 ms and the isomeric $^{123\mathrm{m}}_{55}$Cs nucleus with the metastable state with energy 156.74 keV and lifetime 1.6 s. The Bose condensate of atomic helium was first observed in [30, 31] with the total number of BEC atoms $N_{\mathrm{BEC}} \approx \times 10^5$ and concentration $(N/V)_{\mathrm{BEC}} \approx 3.8\times 10^{13}$ cm^{-3}. Note that these values considerably exceed limiting values (16) and (17). The possibility of observation of ultranarrow stimulated emission lines in a helium condensate was discussed in [32, 33].

8. Conclusions

It follows from the above discussion that the hypothetical state of a megaatom in a BEC is promising for the radical suppression of the random motion of individual boson atoms due to the establishment of the higher-order quantum coherence. The confirmation of this result can open the possibility to observe ultranarrow radiative lines in a megaatom with the width tending to the natural width of long-lived metastable atomic and nuclear states. Such lines can be used to solve problems in quantum nucleonics, precision metrology, etc.

It is important that the cooling of gas with the thermodynamic distribution of atoms even down to very low temperatures, which can be achieved at present, cannot provide the narrowing of lines by simply reducing the Doppler broadening down to the level estimated by (13) and does not lead to the relation $T_{\rm eff} \ll T$. Indeed, in this case, according to (1), the ratio of the linewidths is equal to unity and $T_{\rm eff} = T$.

Despite the excess optimism of estimates (13) and (15) (which can be, of course, noticeably impaired due to the violation of any of the additional restrictions discussed above or due to weak perturbing factors that were ignored), these extraordinary estimates stimulate the experimental verification of the megaatom hypothesis by using, for example, the scheme based on Mössbauer spectroscopy [34].

Note that, although the conditions for observation of the megaatom contain significant inherent contradictions, experimental attempts seem not hopeless. In particular, one of the problems is the necessity of combining a relatively small number of atoms in a small volume with the reliability of measurement methods. This difficulty can be eliminated by measuring the combined spectrum of a longitudinal row of individual mutually isolated small traps with atomic ensembles that do not interact with each other. The number of atoms in each of the traps is limited, while the total number of atoms in such 'beads' stringed on a common 'optical' axis is quite large. In this case, care should be exercised to prevent the inhomogeneous broadening of such a composite spectral line caused by the possible incomplete identity of individual 'beads'. However, as pointed out above, it is not inconceivable that some contradictions are in fact not so absolute and can be alleviated by the fact that some restrictions imposed on parameters not necessarily should be fulfilled.

Note that hypothetical radiative phenomena in the megaatom discussed above are by no means the analogue of the Mössbauer effect. The interatomic bonding energy in the megaatom is insufficient for the appearance of phenomena of the Mössbauer type, which follows directly from the presence of the frequency shift between the absorption and emission lines. However, the quantum coherence of the megaatom can be sufficient for a strong suppression of the random motion of atoms, resulting in the reduction of the Doppler broadening.

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