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Eigenfrequencies of a composite ring resonator

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***Abstract*. An inhomogeneous ring optical resonator of a special type is considered, which contains two identical dielectric plates of finite thickness separated by an arbitrary distance. The refraction coefficient of these plates is significantly higher than that of the medium filling the rest of the resonator. This system can be treated as a ring resonator formed by two linear resonators coupled through the plates confining them. The classical spectral problem for such a resonator is solved in the plane-wave approximation. It is shown that in the case of a comparatively low reflectance from the plates, it is possible to obtain analytically a physically acceptable description of the spectrum of eigenfrequencies and modes. The method for solving the spectral problem is proposed in which the analytic approach is combined with the numerical experiment. It is shown that the resonator spectrum is simple and is formed by a sequence of doublets. The modes corresponding to these doublets are real and orthogonal. Conditions are found under which the splitting of eigenfrequencies in doublets disappears.**

Keywords: *inhomogeneous ring optical resonator, splitting of eigenfrequencies, coupled resonators.*

1. Introduction

The study an inhomogeneously filled ring resonator attracts attention for several reasons. On the one hand, the main reasons for the appearance of internal synchronisation zones in ring lasers have not been revealed so far (it is obvious that synchronisation is caused by the inhomogeneity but a detailed study of the role of the inhomogeneity structure is absent). On the other hand, no definite answer is given to the question whether it is possible to use a specially produced inhomogeneity (which inevitably causes backscattering) to eliminate (or partially ‘reduce) this synchronisation.

It is difficult to estimate the role of natural inhomogeneities, because, in fact, they are residual uncontrollable defects. Such inhomogeneities, producing radiation backscattering, play an undesirable role. They are studied, as a rule, by using some models (models of point or multipoint reflectors, surface or bulk distributed scatterers). Based on these particular investigations, attempts are made to draw generalised conclusions (see, for example, [1–4]). Quite a different matter is specially produced structural inhomogeneities that are fully determined and functionally substantiated. Such inhomogeneities can cause forward and backward scattering in several cross sections of the resonator and the waves appearing in this case can interfere in a required way. There is nothing unexpected here; it is enough to recall multilayer optical heterostructures in which bleaching is observed. Investigations in this direction are undoubtedly reasonable; however, we are not aware of any works dedicated to this problem. We can, however, mention paper [5], but the approach it uses is specialised taking into account the type of the inhomogeneities being considered.

The study of structural inhomogeneities is difficult *a fortiori* because it involves the solution of the spectral problem for a differential operator on an interval with periodic boundary conditions, and this problem in mathematics belongs to the most complicated one. The aim of this paper is to solve approximately this problem for one particular type of the inhomogeneity by using a special investigation technique (allowing, however, generalisation) proposed for this purpose. This technique can be also used to study structural inhomogeneities of general type.

Let us specify the problem. Consider a ring resonator with a perimeter L containing a linear low-Q resonator of length L1 formed by two identical dielectric plates (forming plates) of thickness L_0 each with the refractive index n. The refractive index of the other part of the resonator medium is $n_0 < n$ and its length is $L_2 = L - 2L_0 - L_1$. Thus, the ring resonator is formed by two coupled linear resonators of different lengths. The forming plates scatter radiation backward with the reflection coefficient R and forward with the transmission coefficient T. If $n/n_0 \to 1$, we have $R \to 0$, $T \to 0$. If $n/n_0 \gg 1$, we have $|R| \to 1$, $T \to 0$. Let us determine the eigenfrequency and mode spectrum of this structure.

Despite the simplicity of the inhomogeneity under study, the solution of the spectral problem for it is unknown. The general approach presented in the literature is based on the use of the Schrödinger equation with the asymptotic of solutions fundamentally different from that of solutions of the wave equation, which makes it impossible to adapt known general results to this particular case [6].

2. Reflection and transmission coefficients of forming plates

Consider these coefficients in the form: $R = \rho \exp(-\mathrm{i}\alpha)$, $T = \tau \exp(-\mathrm{i}\beta)$. Because the losses are absent, $\rho^2 + \tau^2 = 1$.

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The forming plates are of finite thickness, optically homogeneous, and placed between identical optically homogeneous media. In this situation, R and T can be found exactly in the explicit form. The corresponding expressions are presented, for example, in [1]:

$$\rho^2(\theta) = \sin^2\theta\left[\frac{4n^2n_0^2}{\left(n^2-n_0^2\right)^2}+\sin^2\theta\right]^{-1},$$
$$\tau^2(\theta) = \left[1+\left(\frac{n^2-n_0^2}{2nn_0}\right)^2\sin^2\theta\right]^{-1}, \tag{1}$$

$$\alpha(\theta) = \arctan\left(-\frac{2n_0n}{n^2+n_0^2}\cot\theta\right),$$
$$\beta(\theta) = \arctan\left[\frac{0.5(n_0+n)^2\sin 2\theta}{2n^2n_0\cos^2\theta-\left(n^2+n_0^2\right)\sin^2\theta}\right], \tag{2}$$

where $\theta = kL_0n$ is the reduced frequency (the phase incursion on a forming plate); k is the wave number of radiation in vacuum.

Figure 1a shows the change in the moduli of reflection and transmission coefficients and Fig. 1b – the change in their phases as a function of the reduced frequency.

For further calculations we will also need the function

$$F(\theta) = -R^2(\theta) + T^2(\theta)\exp(-\mathrm{i}2\theta). \tag{3}$$

Let us calculate this function and present the results in Fig. 2.

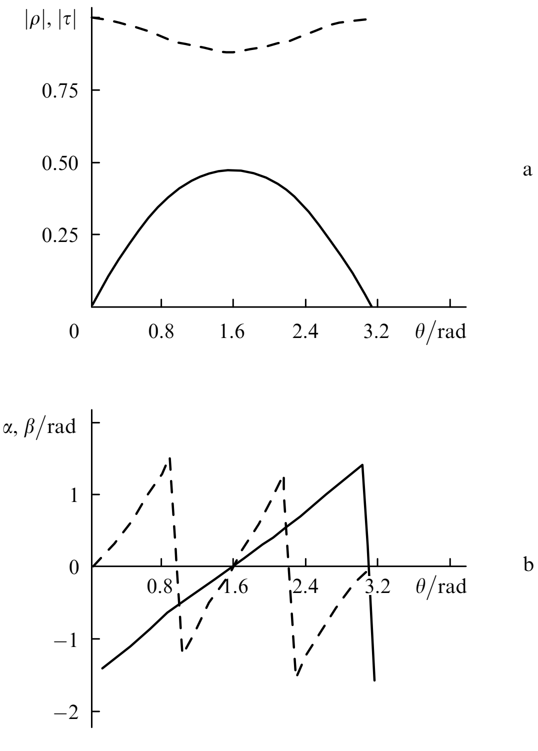

Figure 1. Dependences of the moduli (a) and phases (b) of reflection $R = \rho\exp(-\mathrm{i}\alpha)$ (solid curves) and transmission $T = \tau\exp(-\mathrm{i}\beta)$ (dashed curves) coefficients of forming plates on the reduced frequency θ at $n/n_0 = 5/3$.

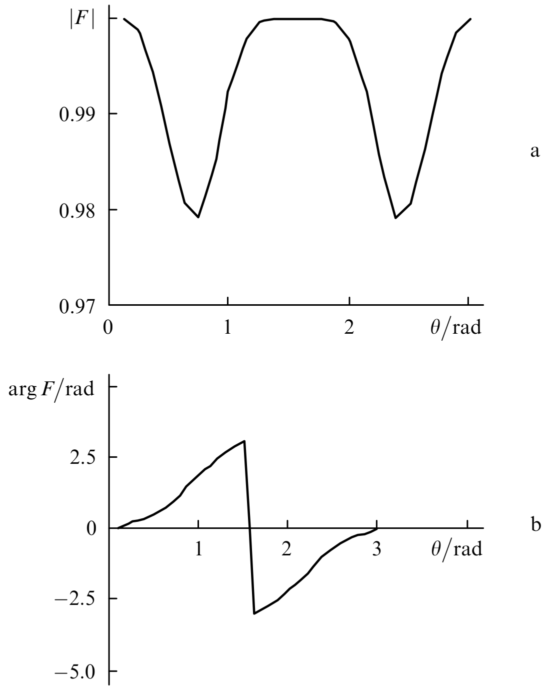

Figure 2. Dependences of the modulus (a) and phase (b) of the function F on the reduced frequency θ at $n_0 = 1$, $n/n_0 = 5/3$.

3. Mathematical formulation of the spectral problem

Consider the intervals $0 \leqslant x \leqslant L_1$ and $0 \leqslant x \leqslant L_2$ of a change in the coordinate x within the resonator perimeter. Let us represent the field in the first interval in the form

$$u^{(1)}(x) = E_-^{(1)}\exp(\mathrm{i}kn_0x) + E_+^{(1)}\exp(-\mathrm{i}kn_0x), \tag{4}$$

and in the second – in the form

$$u^{(2)}(x) = E_-^{(2)}\exp[\mathrm{i}kn_0(x-L_1-L_0)]$$
$$+ E_+^{(2)}\exp[\mathrm{i}kn_0(x-L_1-L_0)]. \tag{5}$$

The amplitudes entering the expressions for fields (4) and (5) satisfy the relations

$$\begin{pmatrix} E_+^{(1)} \\ E_-^{(2)}\exp(\mathrm{i}kn_0L_2)\end{pmatrix} = \begin{pmatrix} T\exp(-\mathrm{i}knL_0) & R \\ R & T\exp(-\mathrm{i}knL_0)\end{pmatrix}$$
$$\times\begin{pmatrix} E_+^{(2)}\exp(-\mathrm{i}kn_0L_2) \\ E_-^{(1)}\end{pmatrix}, \tag{6}$$

$$\begin{pmatrix} E_+^{(2)} \\ E_-^{(1)}\exp(\mathrm{i}kn_0L_1)\end{pmatrix} = \begin{pmatrix} T\exp(-\mathrm{i}knL_0) & R \\ R & T\exp(-\mathrm{i}knL_0)\end{pmatrix}\times$$
$$\times\begin{pmatrix} E_+^{(1)}\exp(-\mathrm{i}kn_0L_1) \\ E_-^{(2)}\end{pmatrix}. \tag{7}$$

We use here the reflection and transmission conditions for forming plates and the periodicity condition. We can obtain from vector relations (6) and (7) that

$$T^2\begin{pmatrix}E_+^{(2)}\\E_-^{(2)}\end{pmatrix} = \begin{pmatrix}\{\exp[ikn_0(L_2+L_1)]+R^2\}\exp(i2knL_0) & -R\exp(i2knL_0)\exp(ikn_0L_2)[\exp(ikn_0L_2)+\exp(ikn_0L_1)]\\ -R\exp(i2knL_0)\exp(-ikn_0L_2)[\exp(ikn_0L_2)+\exp(ikn_0L_1)] & \{\exp[ikn_0(L_2+L_1)]+R^2\}\exp(i2knL_0)\end{pmatrix} \times\begin{pmatrix}E_+^{(2)}\\E_-^{(2)}\end{pmatrix}.$$

This equation is homogeneous and, hence, only at eigenfrequencies k there exists a nonzero vector

$$\begin{pmatrix}E_+^{(2)}\\E_-^{(2)}\end{pmatrix},$$

satisfying this equation. The eigenfrequencies, obviously, should be the roots of the matrix determinant

$$\begin{pmatrix}\{\exp[ikn_0(L_2+L_1)]+R^2\}\exp(i2knL_0)-T^2 & -R\exp(i2knL_0)\exp(ikn_0L_2)[\exp(ikn_0L_2)+\exp(ikn_0L_1)]\\ -R\exp(i2knL_0)\exp(-ikn_0L_2)[\exp(ikn_0L_2)+\exp(ikn_0L_1)] & \{\exp[ikn_0(L_2+L_1)]+R^2\}\exp(i2knL_0)-T^2\end{pmatrix}. \quad (8)$$

After finding the eigenfrequency, we obtain the ratio $E_+^{(2)}/E_-^{(2)}$, and then by using (5), we determine the field (mode) with an accuracy to an arbitrary factor in the interval $0 \leqslant x \leqslant L_2$.

4. Approximate estimate of eigenfrequencies

The eigenfrequencies k should be the roots of the determinant of the above-presented matrix, i.e. satisfy the equation

$$\left\{\left[\exp[ikn_0(L_2+L_1)]+R^2\right]-T^2\exp(-i2knL_0)\right\}^2 - R^2[\exp(ikn_0L_2)+\exp(ikn_0L_1)]^2 = 0. \quad (9)$$

In the limiting case $R = 1$, $T = 0$, when the ring resonator is decomposed into two linearly uncoupled resonators, this equation degenerates into a couple of equations

$$\exp[ikn_0(L_2+L_1)]+1 = \pm[\exp(ikn_0L_2)+\exp(ikn_0L_1)],$$

which have a countable sequence of eigenfrequencies. It is obvious that the frequencies coincide with the roots of equations $\exp(ikn_0L_2) = \pm 1$ and $\exp(ikn_0L_1) = \mp 1$. It is easy to show that other real eigenfrequencies are absent. The set of these eigenfrequencies is not empty if only the lengths L_1 and L_2 are multiple of each other. From the physical point of view, this assumption does not introduce limitations. Hereafter, without the loss of generality, we will assume that $L_1 = m_1L_0$ and $L_2 = m_2L_0$, where m_1 and m_2 are integers.

We can obtain from (9) the right values of eigenfrequencies in another limiting case: $R = 0$, $T = 1$ (the ring resonator is homogeneous). Indeed, Eqn (9) then degenerates into the expression

$$\{\exp[ikn_0(L_2+L_1)]-\exp(-i2knL_0)\}^2 = 0, \quad (10)$$

which allows one to obtain an equidistant sequence of the eigenfrequencies of a homogeneous ring resonator satisfying the equation $\exp[ikn_0(L_2+L_1+2L_0)] = 1$ (we took into account that $n = n_0$ at $R = 0$). It follows from the structure of expression (10) that these eigenfrequencies are double degenerate.

At small enough values of $|R|$, the last term in (9) can be treated as perturbation. We will consider below only small $|R|$, which is quite realistic (see Fig. 1) at least for $1 < n/n_0 \leqslant 5/3$. By neglecting temporarily the last term in (9), we obtain a truncated equation

$$\left\{\left[\exp[ikn_0(L_2+L_1)]+R^2\right]-T^2\exp(-i2knL_0)\right\}^2 = 0. \quad (11)$$

We will seek for the eigenfrequencies in the form $k = k_0 + \delta k$, where k_0 is any root (they form a countable set) of truncated equation (11) and δk is an addition of the order $|R|$. Because an inhomogeneous resonator has no losses, the numbers k, k_0, and, hence, δk should be real. This circumstance will be used below. It is obvious that the root k_0 of equation (10) is doubly degenerate and satisfies the equation

$$\exp[ik_0n_0(L_2+L_1)] = -R^2+T^2\exp(-i2k_0nL_0). \quad (12)$$

Function $F(\theta)$ (3) has a period π. As follows from Fig. 2, its modulus in the interval $0 < \theta \leqslant \pi$ is exactly equal to 1 at $\theta = \pi$ and $1.25 \leqslant \theta \leqslant 1.83$. The dependence $\arg F(\theta)$ is discontinuous and monotonically increases in the domain of continuity. At $\theta = 0$, the modulus and the phase are not defined. The numerical experiment by changing n and n_0 in broad enough (experimentally achievable) ranges showed that the curve in Fig. 2a did not change at all, while the curve in Fig. 2b remained qualitatively invariable, its shape changing insignificantly only in the regions of a monotonic increase. Therefore, we can conclude that the real roots of equation (12) are only those k_0 that for $1.25 \leqslant \theta \leqslant 1.83$ or $\theta = \pi$ simultaneously satisfy the conditions

$$k_0(\theta) = \frac{2\pi q_1 + \arg F(\theta)}{n_0(L_1+L_2)}, \quad k_0(\theta) = \frac{\theta + \pi q_2}{n_0L_0} \quad (13)$$

(q_1 and q_2 are integers).

The eigenfrequencies of the initial inhomogeneous resonator can be found from equation (9), which can be written in the form:

$$\exp[ikn_0(L_2+L_1)] + F(\theta) = \pm R[\exp(ikn_0L_2)+\exp(ikn_0L_1)] \quad (14)$$

The two signs in (14) correspond to a couple of eigenfrequencies $k_0 \pm \delta k$. This means that the frequency spectrum of the inhomogeneous resonator is simple (degeneracy is lifted).

We will find the splitting (of the double root k_0) by the perturbation method. To do this, we represent the left-hand side of (14) as a linear function of δk by using expression (12), and replace the right-hand side of (14) by its value at $k = k_0$:

$$\left[\mathrm{i}n_0(L_2 + L_1)\exp[\mathrm{i}k_0 n_0(L_2 + L_1)] - \left.\frac{\mathrm{d}F(\theta)}{\mathrm{d}k}\right|_{k=k_0} \right]\delta k$$

$$= \pm R|_{k=k_0}[\exp(\mathrm{i}k_0 n_0 L_2) + \exp(\mathrm{i}k_0 n_0 L_1)]. \tag{15}$$

Here

$$\left.\frac{\mathrm{d}F(\theta)}{\mathrm{d}k}\right|_{k=k_0} = \left[-2R\frac{\mathrm{d}R}{\mathrm{d}\theta} + 2T\frac{\mathrm{d}T}{\mathrm{d}\theta}\exp(-\mathrm{i}2\theta) - 2\mathrm{i}T^2\exp(-\mathrm{i}2\theta)\right] nL_0.$$

We obtain from (15) that

$$\delta k = \pm R\left\{ \mathrm{i}n_0(L_2 + L_1)\exp[\mathrm{i}k_0 n_0(L_2 + L_1)] - \left[-2R\frac{\mathrm{d}R}{\mathrm{d}\theta} + 2T\frac{\mathrm{d}T}{\mathrm{d}\theta}\exp(-\mathrm{i}2\theta) - 2\mathrm{i}T^2 \times \exp(-\mathrm{i}2\theta)\right] nL_0 \right\}^{-1}\Bigg|_{k=k_0} [\exp(\mathrm{i}k_0 n_0 L_2) + \exp(\mathrm{i}k_0 n_0 L_1)].$$

This expression, taking equation (12) into account, will take the final form:

$$\delta k(\theta) = \pm\frac{2}{nL_0}\Delta(\theta)\cos\left(\theta\frac{n_0}{n}\frac{L_2 - L_1}{2L_0}\right), \tag{16}$$

where $\Delta(\theta)$ is a periodic function of θ

$$\Delta(\theta) = R\left[-R^2 + T^2\exp(-\mathrm{i}2\theta)\right]^{1/2}\left\{ \mathrm{i}\frac{L_2 + L_1}{L_0} \times \left[-R^2 + T^2\exp(-\mathrm{i}2\theta)\right] + \left[2R\frac{\mathrm{d}R}{\mathrm{d}\theta} - 2T\frac{\mathrm{d}T}{\mathrm{d}\theta}\exp(-\mathrm{i}2\theta) + 2\mathrm{i}\frac{n}{n_0}T^2\exp(-\mathrm{i}2\theta)\right]\right\}^{-1}\Bigg|_{k=k_0}. \tag{17}$$

As was shown above, the additions δk should be real. It follows from (17) that this requirement is equivalent to the condition $\arg\Delta(\theta) = 0, \pm\pi$ for a period of change of θ. Dependence (17) is presented in Fig. 3.

One can see from Fig. 3b that at $\theta = \pi/2$ and $\theta = \pi$, the phase of function (17) is equal to $-\pi$. The exact calculation shows that for none of the values of θ the phase vanishes. Modulus (17) at $\theta = \pi$ is exactly equal to zero, i.e. the splitting is absent. Therefore, the only possible values, at which the splitting is real, are $\theta = \pi/2 \pm \pi q_2$. It is important that the curves in Fig. 3, as follows from the numerical experiment, very weakly change their shape (qualitatively invariable) in extremely broad ranges of the parameters $(L_1 + L_2)/L_0$ and n/n_0. However, for any values of these parameters, the relations $\arg\Delta(\pi/2 + \pi q_2)] = -\pi$ and $|\Delta(\pi + \pi q_2)| = 0$ are exactly fulfilled. As shown above, the values of $k_0(\pi/2)$ calculated by expression (13) are also real. Conditions (13) can be reduced to the condition

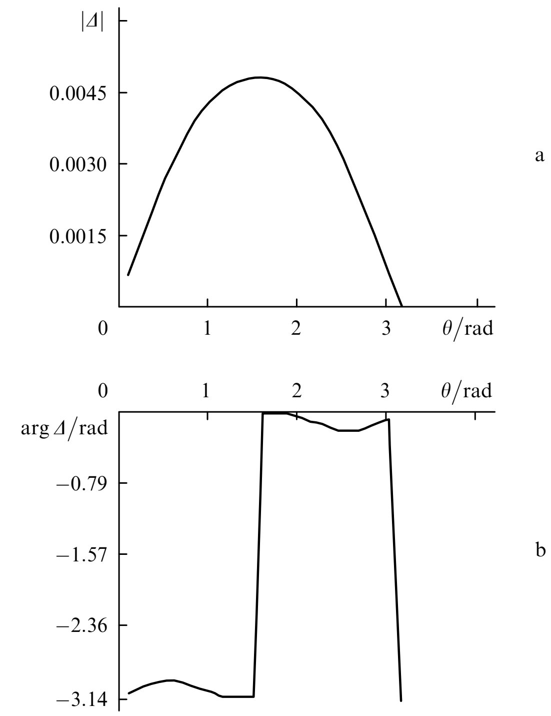

Figure 3. Dependences of the moduli (a) and phase (b) of the function Δ on the reduced frequency θ at $(L_1 + L_2)/L_0 = 110$ and $n/n_0 = 5/3$.

$$k_0\left(\frac{\pi}{2}\right) = \frac{2\pi q_1 + 1}{n_0(L_1 + L_2)}, \tag{18}$$

if the relation

$$\frac{n_0}{n}\frac{L_1 + L_2}{L_0} = 2\frac{2q_1 + 1}{2q_2 + 1} \tag{19}$$

between the inhomogeneity parameters is fulfilled. Expressions (18) and (19) are equivalent to (13).

Thus, if condition (19) is fulfilled, two sequences

$$k_0\left(\frac{\pi}{2}\right) \pm \delta k\left(\frac{\pi}{2} + \pi q_2\right) \tag{20}$$

are the real eigenfrequencies of the inhomogeneous ring resonator.

5. Splitting of eigenfrequencies

Of special interest is the splitting $2|\delta k(\pi/2)|$ of the eigenfrequency doublets. Let us analyse it numerically by using expressions (15) and (18), (19). Below we present the calculation algorithm for the radiation wavelength $\lambda_0 = 0.638$ μm at $L_0 = 3 \times 10^{-3}$ m, $(L_1 + L_2)/L_0 = 110$ and $n_0 = 1$.

Under these conditions, $k_0(\pi/2) \approx 2\pi/\lambda_0$. In accordance with (18), we find the nearest integer q_1 satisfying the relation

$$q_1 = \frac{k_0(\pi/2)n_0(L_1 + L_2) - 1}{2\pi} \approx \frac{(2\pi/\lambda_0)n_0(L_1 + L_2) - 1}{2\pi}.$$

This is a rather large integer of the order of $Q_1 = n_0 \times (L_1 + L_2)/\lambda_0$. Let n_0/n be 3/5; then, in view of Eqn (13)

$$q_2 = 2q_1 \left(\frac{n_0}{n} \frac{L_1 + L_2}{L_0} \right)^{-1},$$

where the value of q_2 should be rounded off to the nearest integer. The number q_2 is of the order $Q_2 = 2nL_0/\lambda_0$, i.e. is also very large. The calculation by expression (17) yields $\Delta(\pi/2)|_{k=k_0(\pi/2)} = -4.781 \times 10^{-3}$. Expression (15) makes it possible to find the splitting $2|\delta k(\pi/2)|$. It depends on the normalised difference $(L_1 - L_2)/L_0$ and is maximal at $L_1 = L_2$. This maximum value of the splitting is equal to 1.1×10^{-2} m^{-1}. Of great interest is the dependence of the splitting on the normalised difference $(L_1 - L_2)/L_0$ for other parameters of the inhomogeneity being fixed (Fig. 4).

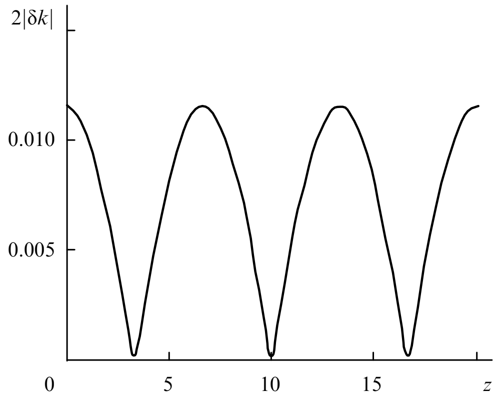

Figure 4. Dependence of the splitting $2|\delta k(\pi/2)|$ in the eigenfrequency doublet on the parameter $z = (L_2 - L_1)/L_0$.

It follows from Fig. 4 that by displacing the dielectric plates with the refractive index $n > n_0$ with respect to each other, we can obtain substantially different splittings of the eigenfrequency doublets. At $(L_1 - L_2)/L_0 = (2q + 1)2n/n_0$ $(q = 0, 1, 2, \ldots)$, the splitting virtually vanishes, i.e. the 'bleaching of the ring resonator takes place. Because of the periodicity of the dependence $\Delta(\theta)$, the curve in Fig. 4 does not change after the substitution $\Delta(\pi/2) \to \Delta(\pi/2 + \pi q_2)$, i.e. when the frequency $\theta = \pi/2 + \pi q_2$ corresponding to the real value $k_0(\pi/2) \approx 2\pi/\lambda_0$ is used. Therefore, the eigenfrequency doublet splitting is independent of the position of the doublet centre but is determined by the type of the resonator inhomogeneity.

6. Resonator modes

Let us estimate the modes of the resonator under study in the region $0 \leqslant x \leqslant L_2$. As is known, they are characterised by the components of the vector

$$\begin{pmatrix} E_+^{(2)} \\ E_+^{(2)} \end{pmatrix}.$$

This ratio can be obtained from matrix determinant (8)

$$\frac{E_+^{(2)}}{E_-^{(2)}} = \tag{21}$$

$$\left. \frac{-R\exp(\mathrm{i}2knL_0)\exp(\mathrm{i}kn_0L_2)[\exp(\mathrm{i}kn_0L_2)+\exp(\mathrm{i}kn_0L_2)]}{\{\exp[\mathrm{i}kn_0(L_2+L_1)] + R^2\}\exp(\mathrm{i}2knL_0) - T^2} \right|_{k=k_0\pm\delta k},$$

at k equal to one of the eigenvalues. According to (14),

$$\left. \frac{R[\exp(\mathrm{i}kn_0L_2) + \exp(\mathrm{i}kn_0L_1)]}{\exp[\mathrm{i}kn_0(L_2+L_1)] + R^2 - T^2\exp(-\mathrm{i}2knL_0)} \right|_{k=k_0\pm\delta k} = \pm 1.$$

The we obtain from (21) that

$$\frac{E_+^{(2)}}{E_-^{(2)}} = \pm \exp(\mathrm{i}2knL_0)\exp(\mathrm{i}kn_0L_2)\Big|_{k=k_0\pm\delta k}. \tag{22}$$

Thus, the modes corresponding to the doublet of eigenvalues $k_0 \pm \delta k$ are determined by the expression

$$u^{(2)}(x)\big|_{k=k_0\pm\delta k} = E_-^{(2)}\exp(\mathrm{i}knL_0)\exp\left(\mathrm{i}kn_0\frac{L_2}{2}\right)$$

$$\times \left[\exp(-\mathrm{i}knL_0)\exp\left(-\mathrm{i}kn_0\frac{L_2}{2}\right)\exp[\mathrm{i}kn_0(x - L_1 - L_0)] \right.$$

$$\left. \pm \exp(\mathrm{i}knL_0)\exp\left(\mathrm{i}kn_0\frac{L_2}{2}\right)\exp[-\mathrm{i}kn_0(x - L_1 - L_0)] \right]_{k=k_0\pm\delta k}$$

with an accuracy to an arbitrary constant.

The above modes are the standing waves

$$u^{(2)}(x) \equiv \cos\left\{(k_0 + \delta k)\left[n_0(x - L_1 - L_0) - nL_0 - n_0\frac{L_2}{2}\right]\right\},$$

$$u^{(2)}(x) \equiv \sin\left\{(k_0 - \delta k)\left[n_0(x - L_1 - L_0) - nL_0 - n_0\frac{L_2}{2}\right]\right\}.$$

If the resonator has a vanishing inhomogeneity, i.e. $n \to n_0$, $\delta k \to 0$, and $R \to 0$, the doublet eigenfrequencies merge into a doubly degenerate eigenvalue to which two orthogonal modes

$$u^{(2)}(x) \equiv \cos\left[k_0n_0\left(x - L_1 - 2L_0 - \frac{L_2}{2}\right)\right],$$

$$u^{(2)}(x) \equiv \sin\left[k_0n_0\left(x - L_1 - 2L_0 - \frac{L_2}{2}\right)\right]$$

correspond.

The corresponding pair of counterpropagating travelling waves (two linear combinations of orthogonal standing waves) is also an admissible pair of modes for the same degenerate eigenfrequency.

7. Conclusions

The eigenfrequencies of the ring resonator formed by a cascade combination of two linear low-Q resonators coupled through dielectric plates are real (in this system losses are absent) and form a countable sequence of doublets. The centres of the doublets correspond to the frequencies $k_0(\pi/2) = (2\pi q_1 + 1)/[n_0(L_1 + L_2)]$. The doublet splitting is determined by the expression

$$\delta k\left(\frac{\pi}{2} + \pi q_2\right) = \frac{2}{nL_0}\Delta\left(\frac{\pi}{2}\right)\cos\left[\left(\frac{\pi}{2} + \pi q_2\right)\frac{n_0}{n}\frac{L_2 - L_1}{2L_0}\right]$$

for the ratio between the inhomogeneity parameters described by expression (19). At any q_2, there exist such values $(n_0/n)(L_2 - L_1)/(2L_0)$ for which splitting becomes negligibly small. This phenomenon can be treated as ‘bleaching’ of the system of two coupled (through forming plates) linear resonators. In other words, because of the interference of forward and backward waves, backscattering ‘disappears’ despite the inhomogeneity (multilayer type) of the ring resonator.

The modes of the resonator under study are real, i.e. they are orthogonal standing waves. It is typical that for a doublet of weakly split eigenfrequencies, the mode distributions are significantly different (phase shifted by $\pi/2$), and this difference is preserved up to passing to the homogeneous resonator.

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