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Nonlinear dynamics of optical pulses in fibres with a travelling refractive-index-change wave

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***Abstract*. Dynamics of soliton-like wave packets in fibres with a travelling refractive-index-change wave is studied. It is shown that both a soliton-like propagation regime of a pulse and a self-compression regime in the region of normal group velocity dispersion are possible. It is also shown that in the case of a copropagating or counterpropagating pulse and optically inhomogeneous wave nonreciprocal effects appear.**

Keywords: *a fibre with a travelling refractive-index wave, soliton-like wave packets, nonreciprocal effects, self-compression.*

1. Introduction

Optical properties of fibres with the refractive indix changing over the fibre length and in time have been actively studied for the last decades [1 – 4]. Interest in them is explained by the fact that fibres with length- and time-modulated parameters find a wide application as highly efficient control systems of optical and, first of all, laser radiation [5]. As a rule, when studying the influence of the travelling refractive-index wave on the optical radiation dynamics, quasi-monochromatic wave packets and effects related to a change in their polarisation and to a shift of the carrier frequency are considered [6, 7]. At the same time, pulsed regimes of radiation propagation in nonlinear fibres with a refractive-index wave have been hardly investigated except paper [8]. As a result, possibilities of the dynamics control of compression, duration and spectral width of pulses interacting with the refractive-index wave as well as the possibility of formation of soliton-like wave packets of the Schrödinger type in such fibres have not been considered.

In this paper, we study the dynamics of parameters of an optical pulse produced by two unidirectional modes and propagating in a fibre with a cubic nonlinearity, in which the refractive index changes periodically in length and time according to the harmonic law. It is shown that the dynamics of the parameters of this pulse depends in a complex way on the parameters of the fibre and radiation coupled into it and is mainly determined by the intermode dispersion realised in these fibres. We discuss the possibility of formation of a soliton-like wave packet in the spectral region with a normal material dispersion of the medium as well as the nonreciprocal character of the pulse compression in the fibre, which is associated with the copropagation or conterpropagation of a pulse and a refractive-index wave.

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2. General relations

Consider a fibre with the refractive index n depending on time and coordinate as

$$n(r,z,t) = n_0(r)[1 + m\cos(2\pi z/\Lambda - \Omega t)]. \qquad (1)$$

Here, $n_0(r)$ is a function determining the radial distribution of the optical inhomogeneity in the fibre; $m \ll 1$ is the modulation depth; Ω and Λ is the frequency and the spatial period of a refractive-index wave. We assume that the perturbation induced in the fibre has a large period (i.e. $\Lambda \gg \lambda$, where λ is the radiation wavelength) and provides the coupling between copropagating waves. In this case, the total field E of the pulse propagating in the length- and time-modulated fibre can be represented in the form

$$\boldsymbol{E}(r,z,t) = \frac{1}{2}\sum_j \{\boldsymbol{e}_j A_j(t,z) U_j(r) \times \exp[\mathrm{i}(\omega_j t - \beta_j z)] + \mathrm{c.c}\}, \qquad (2)$$

where $j = 1, 2$ is the mode number; $\boldsymbol{e}_j$ are the unit vectors of the mode polarisation; $A_j(t,z)$ are time envelops of mode pulses; $U_j(r)$ is the profile functions describing the field distribution of the corresponding mode across the fibre cross section; ω_j and $\beta_j = \beta(\omega_j)$ are the carrier frequencies and wave numbers of wave modes producing a single wave packet and propagating in the fibre. If $\omega_j \gg m\Omega$ and $\beta_j \gg 2\pi m/\Lambda$, the pulse propagation in the fibre under study can be described by a system of equations

$$\frac{\partial A_j}{\partial z} - \frac{\xi_j}{\upsilon}\frac{\partial A_j}{\partial \tau} - \frac{\mathrm{i}d_j}{2}\frac{\partial^2 A_j}{\partial \tau^2} + \mathrm{i}(\gamma_{sj}|A_j|^2 + \gamma_{cj}|A_{3-j}|^2)A_j = \mathrm{i}\sigma A_{3-j}\exp[-\mathrm{i}\xi_j(\delta_\beta z - \delta_\omega \tau)]. \qquad (3)$$

Here, $\xi_j = (-1)^j$; $\tau = t - z/u$ is the time in the running coordinate system; $u = 2u_1u_2/(u_1 + u_2)$; $u_j = (\partial\beta_j/\partial\omega|_{\omega=\omega_j})^{-1}$ are the group velocities of mode pulses; $\upsilon^{-1} = (u_1 - u_2) \times (2u_1u_2)^{-1}$ is the detuning of inverse group velocities of modes; $d_j = \partial^2\beta_j/\partial\omega^2|_{\omega=\omega_j}$ are group velocity dispersions of mode pulses; σ is the intermode coupling coefficient determined by the overlap integral of mode functions of the fibre profile; γ_{sj} and γ_{cj} are the parameters of the self-phase and cross-phase modulation determined by the overlap integrals of the profile functions of the corresponding wave packets taking into account the distributions of the optical inhomogeneity across the cross section and the modulation depth by the fibre length [9]; $\delta_\beta = \beta_1 - \beta_2 - 2\pi/\Lambda$ and $\delta_\omega = \omega_1 - \omega_2 - \Omega$ are intermode detunings; $\Lambda = 2\pi u_{\rm ph}/\Omega$; $u_{\rm ph}$ is the rate of the refractive-index-change grating or the rate of the refractive-index wave. Note that Eqns (3) are of the maximum general type virtually for all systems with a strong linear coupling of copropagating waves.

The solution of system (3) in the general case can be presented in the form of superposition of partial pulses:

$$A_1(\tau,z) = [a_1(\tau,z)\exp(\mathrm{i}qz) + a_2(\tau,z)\exp(-\mathrm{i}qz)] \times \exp[\mathrm{i}(\delta z - \delta_\omega\tau)/2],$$
$$A_2(\tau,z) = [\chi a_1(\tau,z)\exp(\mathrm{i}qz) + \chi^{-1}a_2(\tau,z)\exp(-\mathrm{i}qz)] \times \exp[-\mathrm{i}(\delta z - \delta_\omega\tau)/2], \tag{4}$$

where

$$q = \left(\sigma^2 + \frac{\delta^2}{4}\right)^{1/2}; \quad \delta = \delta_\beta - \delta_\omega/v; \quad \chi = \frac{(2q+\delta)\psi + 2\sigma}{2q - \delta + 2\sigma\psi};$$

$\psi = A_{20}/A_{10}$; A_{j0} are the peak initial amplitudes of mode pulses producing a single wave packet.

The amplitude dynamics of the corresponding partial pulses in the second approximation of the dispersion theory, when the inequality $\delta_\omega \leqslant 1/\tau_0$ is fulfilled (τ_0 is the initial duration of radiation coupled into the fibre) limiting the phase detuning of mode pulses by the width of their spectrum, is described according to (3) and (4) by the equations:

$$\frac{\partial a_f}{\partial z} - \frac{\xi_f\delta}{2qv}\frac{\partial a_f}{\partial\tau} - \mathrm{i}\frac{D_f}{2}\frac{\partial^2 a_f}{\partial\tau^2} + \mathrm{i}(G_{sf}|a_f|^2 + G_{cf}|a_{3-f}|^2)a_f = 0, \tag{5}$$

where $f = 1, 2$ is the partial pulse number and $\xi_f = (-1)^f$. We introduced into (5) the effective group-velocity dispersion D_f and effective parameters of self- and cross-modulation of the corresponding partial pulse:

$$D_f = \frac{d_1+d_2}{2} + \frac{\xi_f}{q}\left[\frac{\sigma^2}{q^2v^2} + \frac{\delta(d_1-d_2)}{4}\right], \tag{6a}$$

$$G_{sf} = [(q+\xi_f\delta/2)(\gamma_{s1} + \chi^{-2\xi_f}\gamma_{c1}) + (q-\xi_f\delta/2)\times (\chi^{-2\xi_f}\gamma_{s2} + \gamma_{c2})]/(2q), \tag{6b}$$

$$G_{cf} = \{(2q+\xi_f\delta)\gamma_{s1} + (q+\xi_f\delta/)(\chi^{2\xi_f}-1)\gamma_{c1} + [q(\chi^{2\xi_f}+1) - \xi_f(\delta/2)(\chi^{2\xi_f}-1)]\gamma_{s2} - \xi_f\delta\gamma_{c2}\}/(2q). \tag{6c}$$

In this case, the initial conditions for the partial pulse taking (4) into account take the form

$$a_f(\tau,0) = \frac{1}{2}\left[A_{10} + \xi_f\left(\frac{\delta}{2q}A_{10} - \frac{\sigma}{q}A_{20}\right)\right]\theta(\tau) \equiv a_{f0}\theta(\tau), \tag{7}$$

where $\theta(\tau)$ is the function determining the shape of the time envelope of a pulse.

3. Degeneracy case

Of most interest from the point of view of obtaining the analytic solutions of Eqns (5) is the degenerate situation in which the behaviour of the entire wave packet can be described only by using only one partial pulse. It is for this situation that it is possible to obtain optimal dispersion and nonlinear parameters of a pulse propagating in the fibre [10]. The coupling between the copropagating modes is most efficiently realised under full phase-matching conditions when $\delta = 0$. In this case, the degeneracy takes place upon symmetric ($\psi = 1$) or antisymmetric ($\psi = -1$) excitation of the fibre, when $a_{20} = 0$ and $a_{10} \neq 0$ or $a_{10} = 0$ and $a_{20} \neq 0$, respectively. The amplitude of one of the partial pulses is equal to zero not only at the initial moment but also during the pulse propagation for all the mentioned types of the fibre excitation [11].

In the general case of detuning from phase matching, the degenerate situation according to (7) is also possible for the non-symmetric excitation of the fibre. Thus, if the condition $\psi = (-\xi_f q + \delta/2)/\sigma$ is fulfilled, $a_f \neq 0$, $a_{3-f} = 0$, and the system of equations (5) also degenerates into one nonlinear Schrödinger equation [12, 13]:

$$\frac{\partial a_f}{\partial z} - \frac{\mathrm{i}D_f}{2}\frac{\partial^2 a_f}{\partial\tau_f^2} + \mathrm{i}G_{sf}|a_f|^2 a_f = 0, \tag{8}$$

where $\tau_f = t - z/u_f$ is the running time related to the corresponding partial pulse; $u_f^{-1} = u^{-1} - \xi_f\delta/(2qv)$ is the group velocity of the partial pulse. The derived equation describes the pulse dynamics in the cubically nonlinear medium with the effective dispersion D_f and nonlinearity G_{sf}. The frequency Ω of the travelling refractive-index wave affecting in this way the wave-packet dynamics in the fibre enters this equation via the detuning δ. The specific feature of the wave-packet propagation in the fibre under study described by expression (8) is the self-action leading to a temporal broadening of the wave packet or its compression as well as production of stable wave packets of Schrödinger solitons whose appearance is caused by the balance between the action of the effective nonlinearity and the influence of the effective dispersion of the waveguide medium [12–15]. In the case of strong intermode coupling, dispersion properties are determined by the effective partial pulse dispersion D_f caused by the material dispersion, intermode coupling and detuning from phase matching. In the case of anomalous effective dispersion ($D_f < 0$) and the presence of focusing properties of a waveguide medium with respect to the corresponding partial pulse ($G_{sf} > 0$), expression (8) has the solution determining the so-called bright solitons of a secant-hyperbolic shape. In this case, the solution of Eqn (8) for the partial pulse amplitude has the form

$$a_f(\tau,z) = a_{f0}\,\mathrm{sech}(\tau/\tau_f)\exp(-\mathrm{i}\Gamma z), \tag{9}$$

where the pulse phase, duration and the initial amplitude are related by $2\Gamma = G_{sf}a_{f0}^2 = |D_f|/\tau_f^2$. One can see that its duration is determined by the effective dispersion and nonlinearity:

$$\tau_f = \left(\frac{|D_f|}{G_{sf} a_{f0}^2}\right)^{1/2} = \left(\frac{|D_f|\tau_0}{G_{sf} W_f}\right)^{1/2} = \left(\frac{W_{sf}}{W_f}\right)^{1/2} \tau_0, \quad (10)$$

where $W_{sf} = |D_f|/(G_{sf}\tau_0)$ is the energy of soliton production. According to (7), the partial pulse energy $W_f = a_f^2(\tau, 0)\tau_0$ is proportional to the energy $W_0 = (A_{10}^2 + A_{20}^2)\tau_0$ coupled into the fibre:

$$W_f = \left(1 + \xi_f \frac{\delta - 2\sigma\psi}{2q}\right)^2 \frac{W_0}{4(1+\psi^2)}. \quad (11)$$

It follows from (11) that $W_f = W_0/2$ at $\delta = 0$ and $\psi = \pm 1$. If the partial pulse energy W_f is rather close to W_{sf}, the soliton propagation regime of the partial pulse, and, hence, of the entire wave packet is realised. At $W_f < W_{sf}$, the pulse spreads and at $W_f > W_{sf}$, it is compressed and the approximating relation

$$\frac{\tau_0}{\tau_{\min}} = \left(\frac{\tau_0 G_{sf} W_f}{|D_f|}\right)^{1/2} \quad (12)$$

is valid for the degree of its compression [12–14]. Here, $\tau_{\min}$ is the minimal pulse duration. In the case of mode phase matching ($\delta = 0$), the effective nonlinearity parameters for both partial pulses turn equal and according to (6b) the expression for them assumes the form $G_{sf} = (\gamma_{c1} + \gamma_{c2} + \gamma_{s1} + \gamma_{s2})/2$. At $\gamma_{sj} = \gamma_s$ and $\gamma_{cj} = \gamma_c$ this expression takes the form $G_{sf} = \gamma_c + \gamma_s$. The dependence of the effective dispersion and nonlinearity on the detuning and intermode coupling and the fibre excitation type makes it possible to control efficiently the degree of compression $\tau_0/\tau_{\min}$, which mainly determines the pulse dynamics in the fibre.

Figure 1 presents the dependence $\tau_0/\tau_{\min}$ on the intermode coupling coefficient σ upon phase matching of modes ($\delta = 0$) and symmetrical ($\psi = 1$) excitation of the fibre at which $a_1 \neq 0$ and $a_2 = 0$. The plotted dependences are obtained for different input pulse powers $P_0 = A_{10}^2 + A_{20}^2 = 2A_{10}^2$. The phase matching condition $\delta = 0$ is fulfilled at the frequency of the refractive-index wave

$$\Omega = \frac{u_{\rm ph}}{\upsilon - u_{\rm ph}}(\upsilon\Delta\beta - \Delta\omega), \quad (13)$$

where $\Delta\beta = \beta_1 - \beta_2$; $\Delta\omega = \omega_1 - \omega_2$. One can see from the presented dependences that in the case under study the degree of compression the higher the larger the input pulse power and the intermode coupling quantity. This is explained by the decrease in the effective dispersion for the corresponding partial pulse, which takes place in the case of phase matching when the intermode coupling coefficient σ increases.

We will analyse numerically relations (6) and (12) for the general case of degeneracy taking place $\psi = (q + \delta/2)/\sigma$, when $a_1 \neq 0$ and $a_2 = 0$. In this case, the energy of the first pulse is $W_1 = W_0\sigma^2/[q(2q+\delta)]$. The effective dispersion for it according to (6a) in the case of strong linear coupling between wave packets (having significantly different group velocities) is anomalous in a broad range of values Ω. The anomality of the effective dispersion is possible even in the case, when both parameters (d_1 and d_2), characterising the group-velocity dispersion of each of the interacting wave packets, are positive quantities. In this case, $\delta = \Delta - 2\pi(1-\eta)/\Lambda$, where $\Delta = \beta_1 - \beta_2 - (\omega_1 - \omega_2)\upsilon^{-1}$ and $\eta = u_{\rm ph}/\upsilon$. If the pulse copropagates with the refractive-index wave, then $u_{\rm ph} > 0$ and $\eta > 0$, while if the pulse propagates in the direction opposite to the refractive-index wave, $u_{\rm ph} < 0$ and $\eta < 0$. It is obvious that the phase detuning δ takes different values at the forward and backward pulse propagations which results in the appearance of nonreciprocal effects, i.e. the dependence of the pulse dynamics on the direction of the pulse propagation.

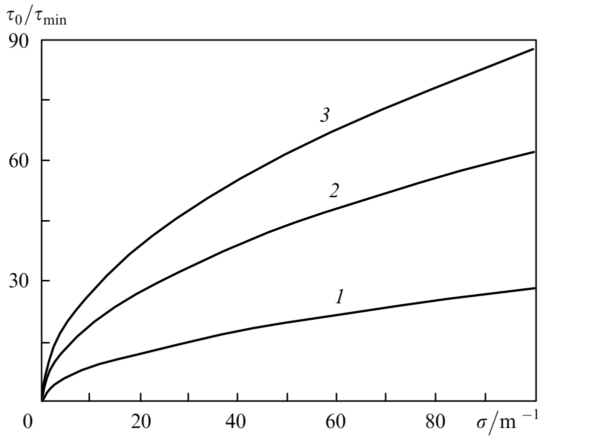

Figure 1. Dependences of the degree of the pulse compression $\tau_0/\tau_{\min}$ on the intermode coupling coefficient σ for the input pulse power $P_0 = 10$ (*1*), 50 (*2*) and 100 W (*3*).

Note that the realisation of the degenerate case is of special interest. It follows from relation (6a) that the situation, at which $D_f < 0$, can be realised due to interaction of pulses with substantially different group velocities for almost any values of the material dispersion d_f of each wave packet, i.e. in any frequency range. Thus, in the presence of a strong intermode interaction and the appropariate fit of the parameters, the regime of nonlinear compression for one of the partial pulses can be realised almost at any frequencies. In this case, the effective dispersion of partial pulses can achieve $\sim 10^{-23}$ s^2 m^{-1} (for $\upsilon \sim 10^{11}$ m s^{-1}), which is more than two orders of magnitude larger than the usual values of the material dispersion for optical fibres.

Figures 2 and 3 present the dependences of the effective dispersion of the partial pulse D_f and the inverse length of its self-phase modulation $L_f^{-1} = G_{s1}W_1/\tau_0$ on the parameter $\eta = u_{\rm ph}/\upsilon$ for a fibre with different Λ. These dependences are plotted for $\Delta = 10$ m^{-1}, $\upsilon = 10^{11}$ m s^{-1} and different intermode coupling coefficients σ. The group-velocity dispersion of interacting wave packets $d_1 = 2d_2$ are selected normal and equal to 10^{-26} s^2 m^{-1}, the nonlinear mode parameters are equal to $\gamma_{s1} = 0.9$ W^{-1} m^{-1}, $\gamma_{s2} = 1.1$ W^{-1} m^{-1}, $\gamma_{c1} = 1.8$ W^{-1} m^{-1}, $\gamma_{c2} = 2.2$ W^{-1} m^{-1}, and the parameters of radiation being coupled are equal to $A_{10}^2 = 100$ W, $A_{20} = \psi A_{10}$. Of interest are large effective anomalous dispersions of the partial pulse (10^{-23} s^2 m^{-1} and above), which appear due to strong intermode coupling upon excitation of a travelling refractive-index wave in the fibre as well as the appearance of strong nonreciprocity. In the case under study, the nonreciprocity is manifested

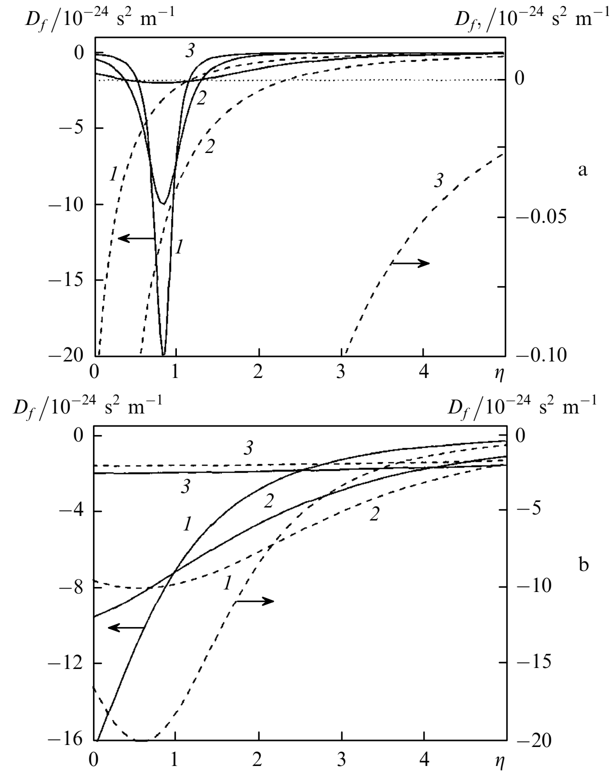

Figure 2. Effective dispersion D_f of a partial pulse in the degeneracy case at $\sigma = 5$ (*1*), 10 (*2*) and 50 m^{-1} (*3*), $\Lambda = 0.1$ (a) and 1 m (b). Solid curves (left axes) correspond to the codirectional propagation of a pulse and refractive-index wave and dashed curves (right axes) – to their counterpropagation.

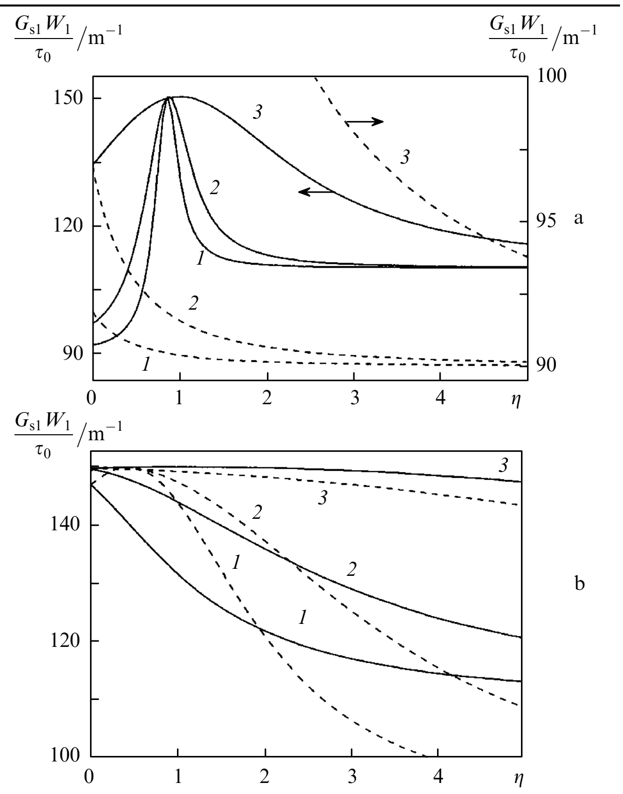

Figure 3. Effective nonlinearity of a partial pulse in the case of degeneracy at $\sigma = 5$ (*1*), 10 (*2*) and 50 m^{-1} (*3*), $\Lambda = 1$ (a) and 0.1 m (b). Solid curves (left axes) correspond to the codirectional propagation of a pulse and refractive-index wave and dashed curves (right axes) – to their counterpropagation.

both in the behaviour of the dependences $D_f(\eta)$ and $L_f^{-1}(\eta)$ for a copropagating and counterpropagating pulse and refractive-index wave (the presence of extrema of the mentioned functions for the 'forward' pulse and a monotonic decrease for the 'backward' pulse) and in the values of these parameters, which for counterpropagating pulses can differ by several orders of magnitude.

4. Pulse dynamics in the non-degeneracy case

If the problem under study is not reduced to degenerate cases, its exact analytic solution is impossible. In this case, we will use the variational method to find the approximate solution, which was proposed in paper [16] and successfully used to solve a wide range of problems in nonlinear optics [17–20]. Without going into detail of the used variational method, note that in the non-degenerate case, the self-phase modulation (caused by cubic nonlinearity) can lead to the pulse compression if the system of inequalities

$$G_{\mathrm{eff}} D_{\mathrm{eff}} < 0, \quad W_0 > W_{\mathrm{s}} = \frac{|D_{\mathrm{eff}}|}{G_{\mathrm{eff}}\tau_0} \tag{14}$$

is fulfilled, where W_{s} is the energy of production of a coupled soliton-like wave packet and the corresponding effective dispersions and nonlinearity of the entire wave packet have the form

$$\begin{aligned} D_{\mathrm{eff}} &= \frac{W_1 D_1 + W_2 D_2}{W_1 + W_2} \approx \frac{d_1 + d_2}{2} \\ &+ \left[\frac{\sigma^2}{q^2 v^2} + \frac{\delta(d_1 - d_2)}{4}\right] \frac{\delta - 2\sigma\psi}{q^2 + (\delta/2 - \sigma\psi)^2}, \\ G_{\mathrm{eff}} &= \frac{G_{\mathrm{s}1} W_1^2 + G_{\mathrm{s}2} W_2^2 + (G_{\mathrm{c}1} + G_{\mathrm{c}2}) W_1 W_2}{(W_1 + W_2)^2} \\ &\approx \frac{G_{\mathrm{s}1}\mu_1^4 + G_{\mathrm{s}2}\mu_2^4 + (G_{\mathrm{c}1} + G_{\mathrm{c}2})(\mu_1\mu_2)^2}{4(2 - \mu_1\mu_2)^2}, \end{aligned} \tag{15}$$

where $\mu_1 = 1 - \varkappa$; $\mu_2 = 1 + \varkappa$; $\varkappa = (\delta - 2\sigma\psi)/(2q)$. Under phase matching conditions (when the proposed variational model works best) expressions for the effective group-velocity dispersions and nonlinearity take the form:

$$\begin{aligned} D_{\mathrm{eff}} &= \frac{d_1 + d_2}{2} - \frac{2\psi}{1 + \psi^2} \frac{1}{\sigma v^2}, \\ G_{\mathrm{eff}} &= \frac{\gamma_{\mathrm{s}1} + \gamma_{\mathrm{s}2} + \gamma_{\mathrm{c}1} + \gamma_{\mathrm{c}2}}{2} + \frac{\gamma_{\mathrm{s}1} + \gamma_{\mathrm{s}2} - \gamma_{\mathrm{c}1} - \gamma_{\mathrm{c}2}}{4} \left(\frac{1 - \psi^2}{1 + \psi^2}\right)^2. \end{aligned} \tag{16}$$

In this case, if inequality (13) is valid for a pulse of a secant-hyperbolic shape, the minimum duration of a soliton-like pulse is achieved at the compression length

$$z_{\mathrm{c}} \approx \tau_0 \left[\frac{\tau_0}{|D_{\mathrm{eff}}| G_{\mathrm{eff}}(W_1 + W_2)}\right]^{1/2}, \tag{17}$$

and the expression for it has the form

$$\tau_{\min} \approx \tau_0 \left[\frac{|D_{\mathrm{eff}}|}{G_{\mathrm{eff}}(W_1 + W_2)\tau_0}\right]^{1/2}. \tag{18}$$

Thus, the dynamics of a single two-wave packet is fully described by the effective nonlinear and dispersion parameters $G_{\rm eff}$ and $D_{\rm eff}$. It follows from the obtained relations that realisation of a soliton-like wave packet is possible even under conditions $d_1 > 0$ and $d_2 > 0$. Therefore, for the coupled wave packet under study, the regime of self-compression or soliton-like propagation can be realised for visible and even UV range.

Note that in the case under study the condition for the efficient intermode phase matching, i.e. the condition, for which the single wave packet is not decomposed into separate autonomous partial pulses, can be written in the form

$$\delta = \beta_1 - \beta_2 - \frac{\omega_1 - \omega_2}{v} - \frac{2\pi}{\Lambda} + \frac{u_1 - u_2}{2u_1u_2}\frac{2\pi u_{\rm ph}}{\Lambda} \approx 0. \qquad (19)$$

It follows from (19) that in the case $u_1 \approx u_2$, i.e. when the equality $\partial\beta_1/\partial\omega|_{\omega=\omega_1} = \partial\beta_2/\partial\omega|_{\omega=\omega_2}$ is fulfilled, the wave-packet dynamics is independent of the modulation frequency Ω of the waveguide medium. In this case, it is possible to realise effective coupling between copropagating waves having substantially different carrier frequencies. The effective coupling between these waves can take place in a fibre without the refractive-index modulation ($2\pi/\Lambda = 0$ and $\Omega = 0$), if

$$\beta_1 - \beta_2 - \frac{\omega_1 - \omega_2}{2}\frac{u_1 - u_2}{u_1u_2} \approx 0. \qquad (20)$$

This condition can be fulfilled if there exist zones with the anomalous group-velocity dispersion in the spectral region located between the carrier frequencies of interacting wave packets ($\omega_1 < \omega < \omega_2$). Therefore, the coupled states of soliton-like wave packets can exist in the spectral regions corresponding to both the anomalous and normal group-velocity dispersion of the fibre under study.

5. Conclusions

Our analysis has shown that the dynamics of a two-wave packet in the medium with a cubic nonlinearity and a travelling refractive-index wave repeats the behaviour of a pulse in a single-mode nonlinear fibre with the effective dispersion and cubic nonlinearity. In this case, this medium has a significant nonreciprocity in effective nonlinear and, especially, dispersion parameters, which disappears in the case of a stationary refractive-index grating (i.e. $\Omega = 0$ and, as a result, at $u_{\rm ph} = 0$). It is shown, in particular, that the conditions for radiation coupling significantly affect the corresponding effective parameters and strongly depend on the phase matching detuning, which, in turn, depends of the frequency of radiation being coupled and the parameters of the refractive-index wave (first of all, its phase velocity). We have established the possibility of obtaining large effective dispersions propagating in a refractive-index fibre. All this makes the use of such systems promising for fabricating all-optical logical elements, compact control systems of laser radiation, compressors and etc.

We give particular attention to the possibility of fabricating elements based on these structures to compensate for dispersion in fibreoptic communication lines. Today, for these purposes Bragg fibres with a stationary modulation of the refractive index [13] are used, but in this case obtaining giant values of the group velocity dispersion is almost always associated with an undesirable distortion of the pulse shape due to partial reflection from an inhomogeneous structure of the Bragg fibre. In this connection, of interest is the use of Bragg gratings with a nonstationary change in the refractive index as resonator elements of laser systems, which we will consider in our next paper.

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