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Classification analysis of CuBr laser parameters

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***Abstract*. A statistical classification analysis of parameters of a CuBr laser emitting at 510.6 and 578.2 nm is performed for the first time based on numerous experimental data. The ten basic parameters affecting the laser output power are investigated by using the multidimensional cluster analysis. Classification tables and dendrograms for these parameters are presented. The obtained results are consistent with our previous study of laser parameters based on the multidimensional factor and regression analysis and supplement it. Some experiment-planning tasks are solved.**

Keywords: *copper bromide laser, cluster analysis, output laser power*.

1. Introduction

The availability of numerous experimental data makes it possible to study independently phenomenologically basic dependences and relations for the description of complicated processes and systems. Of fundamental importance in this case is the classification analysis because it can be used for developing scientific theories and determining the specific properties of the individual components of systems playing an important role in the planning and control of experiments [1]. Note that multidimensional statistical methods of data classification and clustering are mainly applied in economics and social sphere; however, their applications in engineering sciences and physics are also quite promising [2].

In particular, the use of the classification analysis for studying lasers, including CuBr lasers allows one to solve the following problems: (i) to classify independent quantities (variables) of a laser system over macroscopic categories and reduce their number down to several significant groups; (ii) to determine the place of each of the independent variables in the general hierarchy; (iii) to find interrelations (near and far) between the quantities under study, their mutual influence and their influence on other quantities or entire groups; (iv) to determine the degree of influence (or distance) of a group of variables or an individual independent quantity on the dependent laser parameters such as the output power, efficiency, etc.; (v) to use the obtained classification groups and grouping variables for constructing models of different types for predicting the behaviour of the system under study, including the planning of screening and extremal experiments, the sequence of measurements, and the distribution of resources. The classification analysis also allows one to find the new properties of a laser system and characteristic dependences, which cannot be obtained by other theoretical and experimental methods.

In this paper, we consider a CuBr laser emitting at 510.6 and 578.2 nm. The study is performed based on a great amount of experimental data obtained during last decades at the Metal Vapour Lasers Department at Georgi Nadjakov Institute of Solid State Physics, Bulgarian Academy of Sciences [3 – 11]. The results obtained in [12 – 14] by the method of multidimensional factor analysis are briefly described. The multidimensional classification statistical analysis is performed by using the hierarchic cluster analysis of physical variables, the calculations are analysed, and some of the problems formulated above are solved. The results are compared with the classification data obtained by the factor analysis.

Statistical calculations are performed based on the 25 % sampling from all the data available for a CuBr laser by using the SPSS statistical software [15].

2. Basic results of the multidimensional factor analysis obtained earlier

Classification in the multidimensional factor analysis is based on the correlation of variables. The aim is to determine the beforehand unknown number of macroscopic variables (factors) grouping independent input variables according to the degrees of their mutual correlation. The obtained factors do not usually correlate with each other, which can be used for their subsequent analysis, for example, for constructing regression models and predicting the behaviour of a system. Variables of different types involved in physical processes proceeding in a laser system are first reduced to a standardized (dimensionless) form. The data should also satisfy certain statistical criteria. They should be randomised and the types of their distribution,

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the conditions of the validity of the model, etc. should be studied [2, 12–16].

In this paper, we perform classification by using the following basic variables (parameters): the internal diameter D of a laser tube, the internal diameter d_r of a diaphragm, the active zone length L (distance between electrodes), the input electric power P_{in}, the electric power per unit length P_L, the electric pulse repetition rate f, the buffer neon gas pressure p_{Ne}, the pressure p_{H_2} of the additional hydrogen, the equivalent capacity C of a capacitor bank, and the temperature T_r of a reservoir with CuBr. The output laser power P_{out} is considered as the main dependent variable.

It was found in [12, 13] that only variables D, d_r, L, P_{in}, P_L, and p_{H_2} correlate with the output power P_{out} and with each other. This correlation is weak and is absent at all for variables f, p_{Ne}, C, and T_r. The procedure of the factor analysis requires in this case the exclusion of these variables from further calculations. Below, we considered only remaining six (1–6) variables D, d_r, L, P_{in}, P_L and p_{H_2}. They were grouped into three mutually orthogonal factors. The first factor contained variables P_{in}, d_r, L, and D, the second one – P_L, and the third one – p_{H_2}. Their factor loadings are presented in Table 1.

Table 1. Rotated matrix of factor (component) loadings obtained by the varimax method with Kaiser normalisation [12–14].

Variable	Component (factor)		
	1	2	3
P_{in}	0.942		
d_r	0.905		
L	0.789		
D	0.744		
P_L		−0.913	
p_{H_2}			0.943

Note. Factor loadings smaller than 0.5 are not presented.

Note that the classification of independent variables obtained by using the factor analysis is only partial because it neglects the remaining four (7–10) quantities, f, p_{Ne}, C, and T_r, because of their weak correlations.

3. Classification by using the cluster analysis

Unlike factor analysis, the procedures of cluster analysis are based on the classification of objects according to the degrees of their homogeneity and closeness [1]. The formation of groups (clusters) according to the specified criteria is performed by combining homogeneous objects, while clusters themselves should remain inhomogeneous. The closeness of objects is quantitatively estimated by using a certain metrics, most often the usual Euclidean distance $d_{ij} = \left[\sum_{k=1}^{p} (x_{ik} - x_{jk})^2\right]^{1/2}$ between points X_i and X_j (objects) of the p-dimensional space [1].

There exists a great number of clustering methods. When the number of objects is small, as in our case, the most suitable are hierarchic agglomerative methods. The results are presented in the form of tables and dendrograms (dendrite diagrams), which express the hierarchic structure of the similarity matrix and the rules for obtaining clusters. There exist numerous methods for combining objects to cluster and then combining clusters themselves. In the case of 'chain' clustering, the between-groups-linkage and nearest-neighbour methods are used. However, a specific choice of metrics, of the appropriate number of clusters, and the method of their formation is important and sometimes complicated stage of the cluster analysis.

3.1 Results of the cluster analysis

First we perform a partial cluster analysis only for the first six independent variables (D, d_r, L, P_{in}, P_L, and p_{H_2}) used in the previous classification. Our aim is to compare the obtained results with the results of the factor analysis.

The first stage is to obtain a matrix containing the results of comparison of the objects (Table 2). We characterise the degree of similarity (difference) of objects by the square of the Euclidean distance. Note that Table 2 presents only the results of comparison at the first step, when each object is considered as a cluster. The independent variables are grouped into three clusters by the method of average between-groups linkage, as shown in Table 3. The first cluster contains variables D, d_r, L and P_{in}, the second one – P_L, and the third one – p_{H_2}. Thus, we obtain complete agreement with the results of the factor analysis (Table 1).

Table 2. Similarity matrix of six variables.

Variable	D	d_r	L	P_{in}	P_L	p_{H_2}
D	0	23.9	36.7	44.7	227.5	103.0
d_r	23.9	0	12.1	20.1	223.8	86.0
L	36.7	12.1	0	21.2	247.8	67.3
P_{in}	44.7	20.1	21.2	0	189.3	91.7
P_L	227.5	223.8	247.8	189.3	0	217.9
p_{H_2}	103.0	86.0	67.3	91.7	217.9	0

Table 3. Belonging of six variables to three clusters in a group of three clusters.

Variable	Cluster number
D	1
d_r	1
L	1
P_{in}	1
P_L	2
p_{H_2}	3

One can see from Table 2 that the minimal value of the coefficient characterising the degree of homogeneity of clusters being formed (in our case, the square of the Euclidean distance) is 12.1 and links variables d_r and L. Correspondingly, this is the first linkage characterising the degree of similarity observed in Fig. 1. The next coefficient equal to 20.1 links d_r and P_{in}. Therefore, the variable P_{in} is further grouped with already formed first cluster, etc. By using this procedure, we obtain a complete structure presented in Fig. 1.

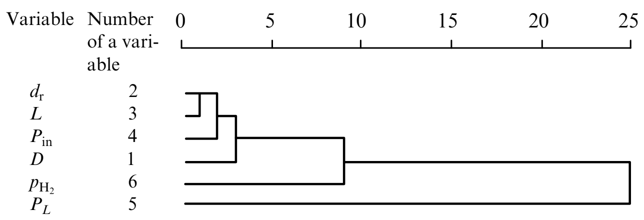

Figure 1. Dendrogram of six independent variables obtained by the method of between-groups linkage. The horizontal axis here and in Figs 2 and 3 shows the normalised squares of Euclidean distances; to the value 25, the maximum metrics 247.8 from Table 2 corresponds.

Table 4. Similarity matrix for all ten variables.

Variable	D	d_r	L	P_{in}	P_L	p_{H_2}	f	p_{Ne}	C	T_r
D	0	19.5	33.0	39.4	202.4	95.9	145.0	158.4	101.5	115.9
d_r	19.5	0	9.5	16.9	201.1	79.7	155.3	161.0	105.8	101.6
L	33.0	9.5	0	19.4	223.3	58.9	159.0	149.5	120.2	115.0
P_{in}	39.4	16.9	19.4	0	169.3	83.1	155.1	145.0	107.6	116.3
P_L	202.4	201.1	223.3	169.3	0	196.0	113.4	88.3	141.2	132.6
p_{H_2}	95.9	79.7	58.8	83.1	196.0	0	164.8	140.7	158.2	174.3
f	144.7	155.3	159.0	155.1	113.4	164.9	0	97.4	139.8	120.5
p_{Ne}	158.4	161.0	149.5	145.0	88.3	140.7	97.4	0	172.6	126.5
C	101.5	105.8	120.2	107.6	141.2	158.2	139.8	172.6	0	93.4
T_r	115.9	101.6	115.0	116.3	132.6	174.3	120.5	126.5	93.4	0

At the second stage, we will classify all the ten initial variables. Table 4 presents their similarity matrix. The main Table 5 presents the classification of variables over groups from two, three, four, and five clusters. It is necessary to determine the optimal parameters of the clusters. This problem can be solved using the dendrogram presented in Fig. 2, which was obtained similarly to that in Fig. 1.

Table 5. Belonging of all ten variables to five clusters in groups of N clusters.

Variable	Cluster number			
	$N = 5$	$N = 4$	$N = 3$	$N = 2$
D	1	1	1	1
d_r	1	1	1	1
L	1	1	1	1
P_{in}	1	1	1	1
P_L	2	2	2	2
p_{H_2}	1	1	1	1
f	3	3	2	2
p_{Ne}	2	2	2	2
C	4	4	3	1
T_r	5	4	3	1

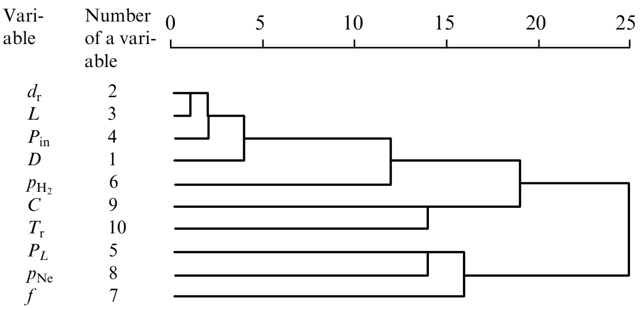

Figure 2. Dendrogram of ten indepenent variables obtained by the method of linkage between groups.

A deteiled analysis of the sequence of the clustering procedure shows that all ten independent variables form three clusters. The first cluster includes variables D, d_r, L, P_{in}, and p_{H_2}, the second one – P_L, p_{Ne}, and f, and the third one – C and T_r. This group corresponds to the column of three clusters in Table 5. Finally, we obtain three clusters classifying all the ten initial variables.

The third stage is the determination of the place of the dependent variable P_{out} (number 11 in Fig. 3) among independent variables. Figure 3 shows that they are close to each other. As expected, P_{out} is closer to variables D, d_r, L, P_{in}, and p_{H_2} and forms the first cluster with them. The latter confirms a considerable influence of these five variables on P_{out}.

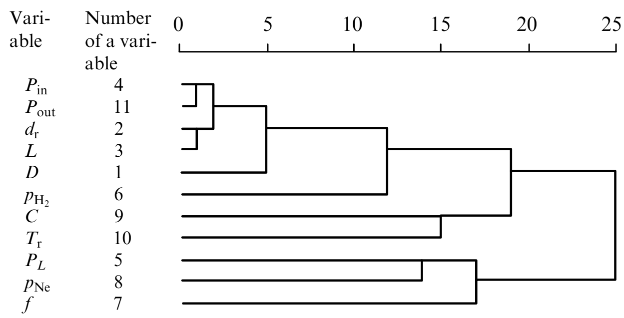

Figure 3. Dendrogram of ten indepenent variables and P_{out} obtained by the method of between-groups linkage.

3.2 Analysis of the results of cluster analysis

It follows from the results of analysis (Table 5 and Fig. 2) that five physical quantities (D, d_r, L, P_{in}, and p_{H_2}) have a strongly pronounced mutual homogeneity. Table 5 shows that they are grouped so that cannot be separated by increasing the number of clusters. The same conclusion follows from Fig. 2. The mutual arrangement of these quantities also remains invariable. Thus, we can conclude that the parameters of the laser radiation source are mainly determined by these quantities.

This classification can be used for planning the screening experiment, in which the basic group of variables should be separated from the entire set of quantities and further studied in detail [12, 17]. By performing the extremal experiment for optimisation of the object under study, the basic variables should be also first varied in the order of their homogeneity, i.e. in the sequence d_r, L, P_{in}, D, and p_{H_2}.

4. Conclusions

We have used for the first time the classification analysis of variables for metal vapour lasers. Ten independent variables were considered. Based on the previous factor analysis with the help of a representative sampling from the total set of all available experimental results and a correlation principle, two groups of variables, significant and insignificant, were conventionally selected. The significant variables were classified into three groups (factors). Then, the variables were classified by using the statistical technique of cluster analysis and homogeneity principle. The hierarchic dependence was obtained and the mutual relation between variables was established. Three clusters and the classification order were determined.

Some tasks related to the application of the obtained results for planning screening and extremal experiments were solved.

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