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Nonlinear stochastic effects during the action of noise on relaxation oscillations in a ring solid-state laser

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***Abstract*. The influence of the pump noise intensity on the characteristics of the fundamental relaxation frequency of the emission spectrum of a ring chip laser operating in the self-modulation regime of the first kind is studied theoretically and experimentally. It is shown that the noise effects give rise to such nonlinear stochastic phenomena as a change in the relaxation oscillation frequency, a noticeable broadening of the relaxation peak in the emission spectrum, and the resonance dependence of the amplitude of this peak on the noise intensity (the peak becomes maximal at a particular noise intensity).**

Keywords: *ring laser, nonlinear dynamics, stochastic effects, relaxation frequency, noise effect.*

1. Introduction

The general principles inherent in the nonlinear behaviour of various systems have been studied and demonstrated in many researches on solid-state laser dynamics. Note that most of these researches ignore stochastic effects related to intrinsic and extrinsic noise in a laser. Usually, when operating well above the laser threshold, one can neglect intrinsic fluctuations of laser radiation and almost eliminate technical noise. However, under certain conditions, the output radiation of a solid-state laser can experience considerable fluctuations caused by the quantum noise even if the pump power is much higher than the threshold value [1 – 4]. In addition, investigations of stochastic phenomena appearing in solid-state lasers, when their parameters are noise modulated, may also be of interest.

Much consideration is given to the study of stochastic phenomena in various dynamic systems. It was shown in some researches that noise can have not only a destructive, but, under certain conditions, constructive effect on the system performance, thus optimising the system parameters. The examples that prove the latter are stochastic [5 – 7] and coherent [8 – 10] resonances. In these cases, noise helps to select weak coherent signals against the noise background as well as to generate and amplify quasi-periodic signals brought about by noise, and to increase their coherence time [10, 11]. As applied to laser physics, stochastic and coherent resonances were studied in a semiconductor laser [10], a laser with a nonlinear absorber [12], and in a ring dye laser [13]. It was shown that bifurcations in dynamic systems can be accompanied by stochastic resonance phenomena, in particular stochastic and coherent resonances [10, 11, 14]. Noise precursors of period-doubling bifurcation were studied in papers [15 – 17]. Papers [11, 14] demonstrated that the precursor excitation efficiency depends on the noise intensity and becomes maximal at specific noise amplitudes. In the case of semiconductor lasers, the noise influence on the frequency and spectrum width of relaxation oscillations of the radiation intensity was studied in [18].

Similar effects in a bidirectional ring solid-state laser (SSL) are of obvious interest. This laser is an intricate enough nonlinear dynamic system whose properties depend on many parameters and their relationship. One of the most interesting nonstationary operation regimes of a ring SSL is self-modulation of the first kind, which is characterised by the out-of-phase sinusoidal intensity modulation of counter-propagating waves. The interaction of self-modulation and relaxation oscillations can lead to instability of the self-modulation regime of the first type and excitation of more complicated self-modulation regimes (including dynamic chaos) [19 – 22]. The period-doubling bifurcation of self-modulation oscillations is usually observed during a parametric resonance at the fundamental relaxation frequency.

The above analysis of relevant data allows one to conclude that in the case of solid-state lasers, a number of nonlinear stochastic effects have been poorly studied. In particular, the relaxation oscillation frequency and relaxation peak width have so far been assumed independent of the external noise intensity. Besides, earlier researches do not answer the question whether nonmonotonic noise-intensity dependence of the relaxation peak height (which is typical of stochastic and coherence resonances) can be observed when relaxation oscillations are triggered by noise?

The aim of this paper is to study experimentally and theoretically these phenomena when relaxation oscillations in a ring SSL operating in the self-modulation regime of the first kind are induced by the pump noise.

2. Experimental setup

In our experiments we used a monolithic Nd:YAG chip laser oscillating at 1.06 μm. The length of the ring resonator was 28 mm, the resonator nonplanarity angle was 85°. A

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0.81-μm diode laser with the power supply incorporating a noise generator was used for pumping (Fig. 1). The noise spectral width was 200 kHz, which exceeded the relaxation oscillation frequency by two – three times. For this reason, the noise can be considered white in the frequency range corresponding to the relaxation peak in the laser power-density spectrum. During the experiments the pump noise intensity varied with changing the noise generator output voltage from zero to maximum. The maximum intensity of the electric noise at the noise generator output achieved 10^{-6} W Hz^{-1}. When processing experimental data, the pump noise was measured in relative units (each unit of the pump noise corresponds to the 10^{-7}-W Hz^{-1} electric noise intensity at the generator output).

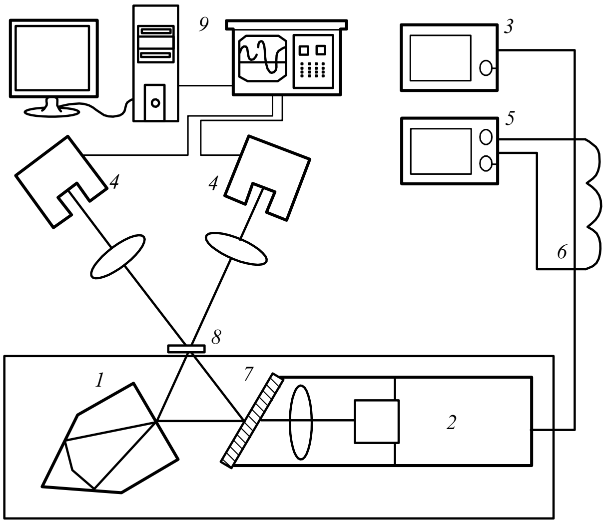

Figure 1. Scheme of the experimental setup: (*1*) ring Nd:YAG chip laser; (*2*) pump diode; (*3*) diode laser power supply; (*4*) LFD-2 photodetector; (*5*) GZ-124 random noise generator; (*6*) transformer coil; (*7*) selective mirror with the nearly 100 % reflectivity at the laser wavelength and with the 90 % reflectivity at the pump wavelength (the mirror allows simultaneous detection of two countepropagating waves); (*8*) IR filter preventing the pump radiation from falling on the photodetector; (*9*) ADC 20-12-PCI oscilloscope and spectral analyser.

The parameters of the bidirectional chip laser under study were chosen so that the ring laser operated in the self-modulation regime of the first kind in the absence of the noise modulation of the pump power. The frequency of self-modulation oscillations was 230 kHz, and the relaxation frequency was 103 kHz when the lasing threshold $\eta = 0.23$ was exceeded.

In the experiments, we measured time and spectral characteristics of the radiation intensity of counterpropagating waves as functions of the noise power and the excess of the pump power over the laser threshold. A 20-12-PCI analogue-to-digital converter (ADC) and Tektronix TDS 2014 digital broadband oscilloscope were used in the measurements. While processing experimental data, we used statistic averaging over a large number (about 100) of random measurements. In the paper, we pay special attention to the issue of how noise influences the spectral characteristics of the ring laser output in the vicinity of the fundamental relaxation frequency.

3. Theoretical model and numerical simulation

The phenomena under study were numerically simulated using the vector model of a ring SSL [23, 24] taking into account the effect of the pump noise. In this case, the basic equations of the vector model have the form

$$\frac{\mathrm{d}}{\mathrm{d}t}\tilde{E}_{1,2} = -\frac{\omega_c}{2Q_{1,2}}\tilde{E}_{1,2} \pm \mathrm{i}\frac{\Omega}{2}\tilde{E}_{1,2} + \frac{\mathrm{i}}{2}\tilde{m}_{1,2}\tilde{E}_{2,1} + \frac{\sigma l}{2T}(N_0\tilde{E}_{1,2} + N_{\mp}\tilde{E}_{2,1}),$$

$$\frac{\mathrm{d}N_0}{\mathrm{d}t} = \frac{1}{T_1}\Big[N_{\mathrm{th}}(1+\eta) - N_0 - N_0 a\big(|E_1|^2 + |E_2|^2\big) - N_+ a E_1 E_2^* - N_- a E_1^* E_2\Big] + g_{\mathrm{w}}, \qquad (1)$$

$$\frac{\mathrm{d}N_+}{\mathrm{d}t} = -\frac{1}{T_1}\Big[N_+ + N_+ a\big(|E_1|^2 + |E_2|^2\big) + \beta N_0 a E_1^* E_2\Big],$$

$$N_- = N_+^*.$$

The dynamic variables are complex amplitudes of counter-propagating waves $\tilde{E}_{1,2}(t) = E_{1,2}\exp(\mathrm{i}\varphi_{1,2})$ and spatial harmonics of the population inversion N_0, $N_{\pm}$ defined by expressions

$$N_0 = \frac{1}{L}\int_0^L N\mathrm{d}z, \quad N_{\pm} = \frac{1}{L}\int_0^L \boldsymbol{e}_1^*\boldsymbol{e}_2 N\exp(\pm\mathrm{i}2kz)\mathrm{d}z. \qquad (2)$$

Here, $\omega_c/Q_{1,2}$ are the resonator bandwidths; $Q_{1,2}$ are the cavity Q factors for the counterpropagating waves; $T = L/c$ is the round-trip time; T_1 is the longitudinal relaxation time; l is the active element length; $a = T_1 c\sigma/(8\pi\hbar\omega)$ is the saturation parameter; σ is the laser transition cross section; $\Omega = \omega_1 - \omega_2$ is the frequency nonreciprocity of the cavity; ω_1, ω_2 are the resonator eigenfrequencies for the counter-propagating waves. The pump rate has the form $N_{\mathrm{th}}(1 + \eta)/T_1$, where N_{th} is the threshold inversion population, and $\eta = P/P_{\mathrm{th}} - 1$ is the excess of the pump power over the threshold. The linear coupling between the counterpropagating waves is phenomenologically determined by using complex coupling coefficients

$$\tilde{m}_1 = m_1\exp(\mathrm{i}\vartheta_1), \quad \tilde{m}_2 = m_2\exp(-\mathrm{i}\vartheta_2), \qquad (3)$$

where $m_{1,2}$ are the moduli of the coupling coefficients, and $\vartheta_{1,2}$ are their phases. The polarisation of the counter-propagating waves is characterised by arbitrary unit vectors $\boldsymbol{e}_{1,2}$. The polarisation factor is $\beta = (\boldsymbol{e}_1\boldsymbol{e}_2)^2$. Note that equations (1) refer to lasing at the gain-line centre. The noise modulation of the pump is described with the help of a Gaussian white noise source g_{w} whose statistical characteristics are

$$\langle g_{\mathrm{w}}(t)\rangle = 0, \qquad (4)$$

$$\langle g_{\mathrm{w}}(t)g_{\mathrm{w}}(s)\rangle = D\delta(t-s), \qquad (5)$$

where D is the noise intensity, and $\delta(t)$ is the Dirac delta function.

During the numerical simulation, some parameters were assumed equal to the laser parameters measured in the experiments. The relaxation frequency $\omega_r = (\eta\omega_c/QT_1)^{1/2}$ was used to determine the resonator bandwidth $\omega_c/Q = 4.4 \times 10^8$ c^{-1}. When the pump was above the threshold $\eta = 0.218$, the main relaxation frequency in the laser under study was $\omega_r/2\pi = 101$ kHz. The amplitude nonreciprocity of the ring resonator was $\Delta = \omega_c/2Q_2 - \omega_c/2Q_1 = 5000$ s^{-1}, the polarisation parameter was $\beta = 0.75$. The latter value was found from the experimental dependence of the additional relaxation frequency ω_{r1} on the resonator frequency nonreciprocity Ω (see [24]).

However, we failed to measure experimentally such parameters as moduli and phases of complex coupling coefficients $\tilde{m}_{1,2}$. Below for simplicity the coupling coefficients are assumed to be complex conjugate ($\vartheta_1 - \vartheta_2 = 0$). The moduli $m_{1,2}$ of the coupling coefficients were chosen so that the experimental self-modulation frequencies and instability boundaries of the self-modulation regime of the first kind (i.e. the self-modulation period-doubling bifurcation points) agree with the numerical simulation results. Figure 2 presents the results of the numerical simulation, which were obtained for $m_1/2\pi = 130$ kHz, $m_2/2\pi = 318$ kHz, the self-modulation frequency being $\omega_m/2\pi = 220$ kHz.

4. Experimental results

In the paper we studied experimentally and simulated numerically the effect of the pump noise intensity on the relaxation peak parameters, i.e. its central frequency, width and amplitude. As a rule, studying noise effects on laser radiation parameters, only linear processes (the noise intensity is small) are taken into account, when the frequency and decay rate of noise-induced relaxation oscillations are independent of the noise intensity. In our research nonlinear stochastic effects are found, namely, a noise-induced shift of the relaxation frequency, broadening of the relaxation peak in the emission spectrum and others. We discovered that these phenomena can arise even at small enough fluctuations of the pump power.

In the absence of noise during the laser operation in the self-modulation regime of the first kind, the power-density spectrum of its output exhibits a peak at the self-modulation frequency ω_m. In the presence of noise two additional peaks appear: at the fundamental relaxation frequency ω_r and

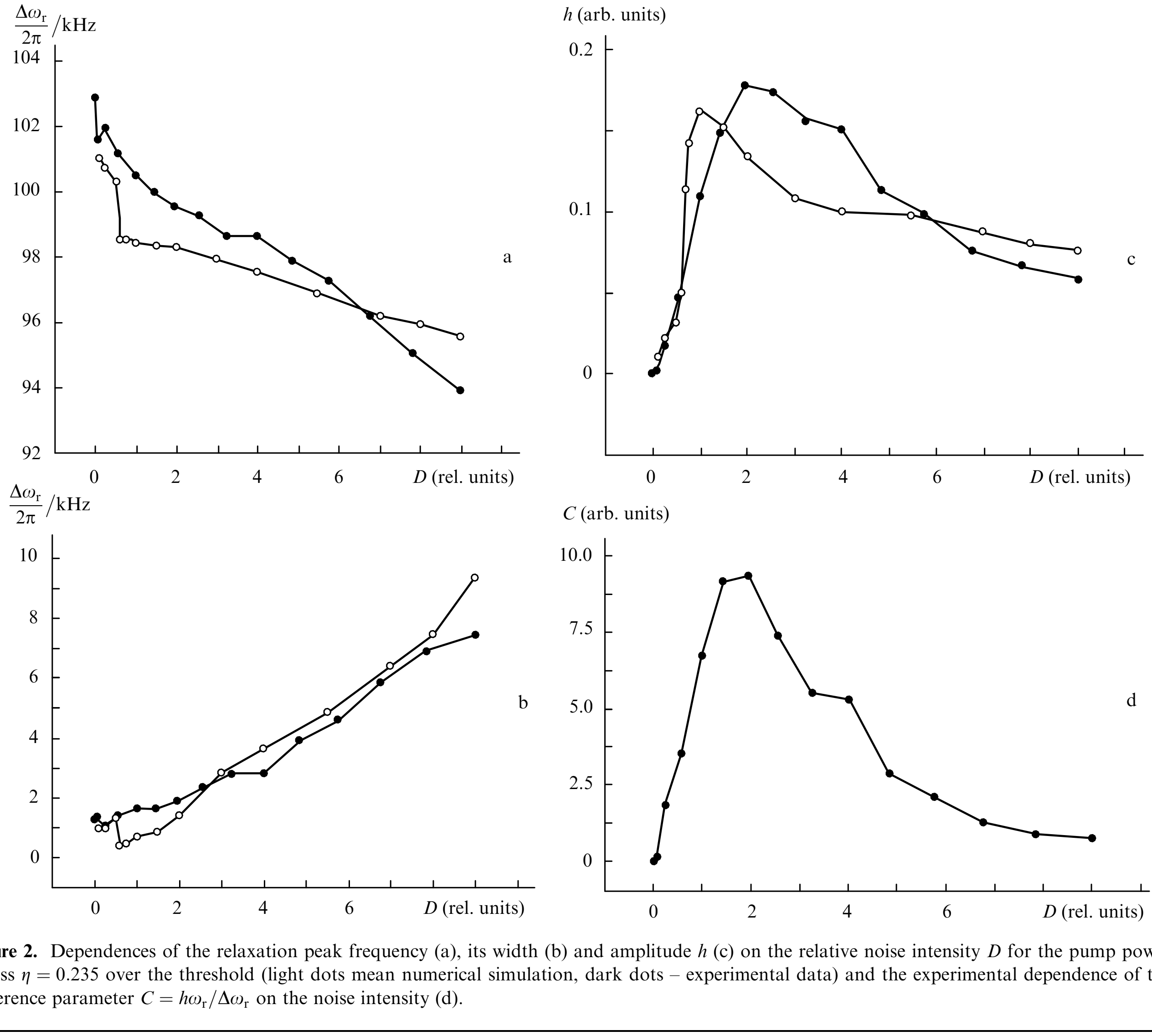

Figure 2. Dependences of the relaxation peak frequency (a), its width (b) and amplitude h (c) on the relative noise intensity D for the pump power excess $\eta = 0.235$ over the threshold (light dots mean numerical simulation, dark dots – experimental data) and the experimental dependence of the coherence parameter $C = h\omega_r/\Delta\omega_r$ on the noise intensity (d).

combination frequency $\omega_m - \omega_r$. The self-modulation regime of the first kind is stable within a specific range of η. Thus, when $\eta > \eta_{cr}$, period-doubling bifurcations of self-modulation oscillations appear. Here, we studied the stability range of the self-modulation regime of the first kind ($\eta < \eta_{cr}$, i.e, up to the period-doubling bifurcation point).

The noise effect on the relaxation oscillations was experimentally investigated for $\eta = 0.235$ and 0.18. Period-doubling bifurcation occurs in the laser under study at $\eta > \eta_{cr} = 0.25$. Figure 2 presents typical dependences of the relaxation peak frequency, its width and amplitude on the noise intensity for $\eta = 0.235$. The figure also shows the numerical stimulation results. Similar experimental dependences for $\eta = 0.18$ are shown in Fig. 3.

It follows from these dependences that the relaxation oscillation frequency decreases and the peak width increases with the noise intensity. The obtained dependences for different η qualitatively coincide; however, they exhibit quantitative differences. For example, the broadening of the spectral peak with the noise intensity is more pronounced for smaller η. The experimental dependences of the relaxation peak frequency and its width agree well with the results of numerical simulations (see Figs 2a, b).

One can see from Figs 2c and 3c that both the experimental and simulation results yield a nonmonotonic dependence of the relaxation peak amplitude on the noise intensity, which it is of the resonance type: the peak amplitude is maximum for some (optimal) noise intensity.

The coherence parameter $C = h\omega/\Delta\omega$ characterising the noise-induced efficiency of coherent signals ($\Delta\omega$, ω, h are the peak width, central frequency and amplitude) is used to analyse stochastic resonance phenomena in a number of papers. Figures 2d and 3d exhibit the dependence of this parameter on the noise intensity, which is also of the resonance type.

The shape $S(\omega)$ of the noise-induced relaxation peak changes with increasing the noise intensity D (Fig. 4). It is clearly seen from the results of the experiments (Figs 4c and d) and numerical simulation (Figs 4a and b). At low noise intensities, the spectral line is symmetric with respect to the frequency ω_r (Figs 4a,c). When the noise intensity increases, there appears a significant asymmetry, i.e. spectral components at the left wing of the line fall much slower than those at the right one with distance from the line centre (see Fig. 4b,d).

5. Discussion of the results

Consider qualitatively the physical mechanisms explaining some of stochastic phenomena observed in the research. The relaxation oscillations in the laser can be considered as oscillations of a nonlinear oscillator, e.g., oscillator Toda

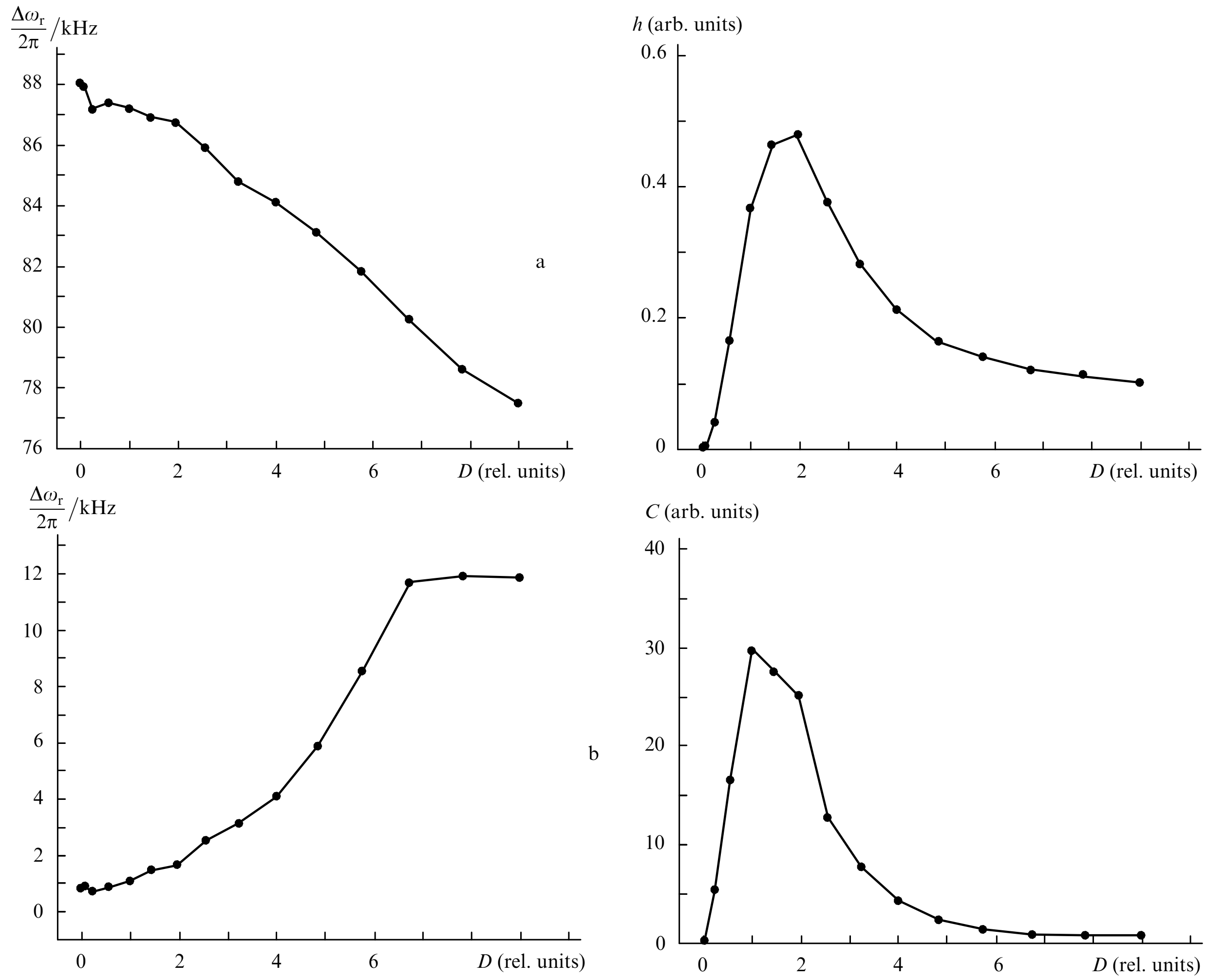

Figure 3. Same as in Figure 2, except that $\eta = 0.18$.

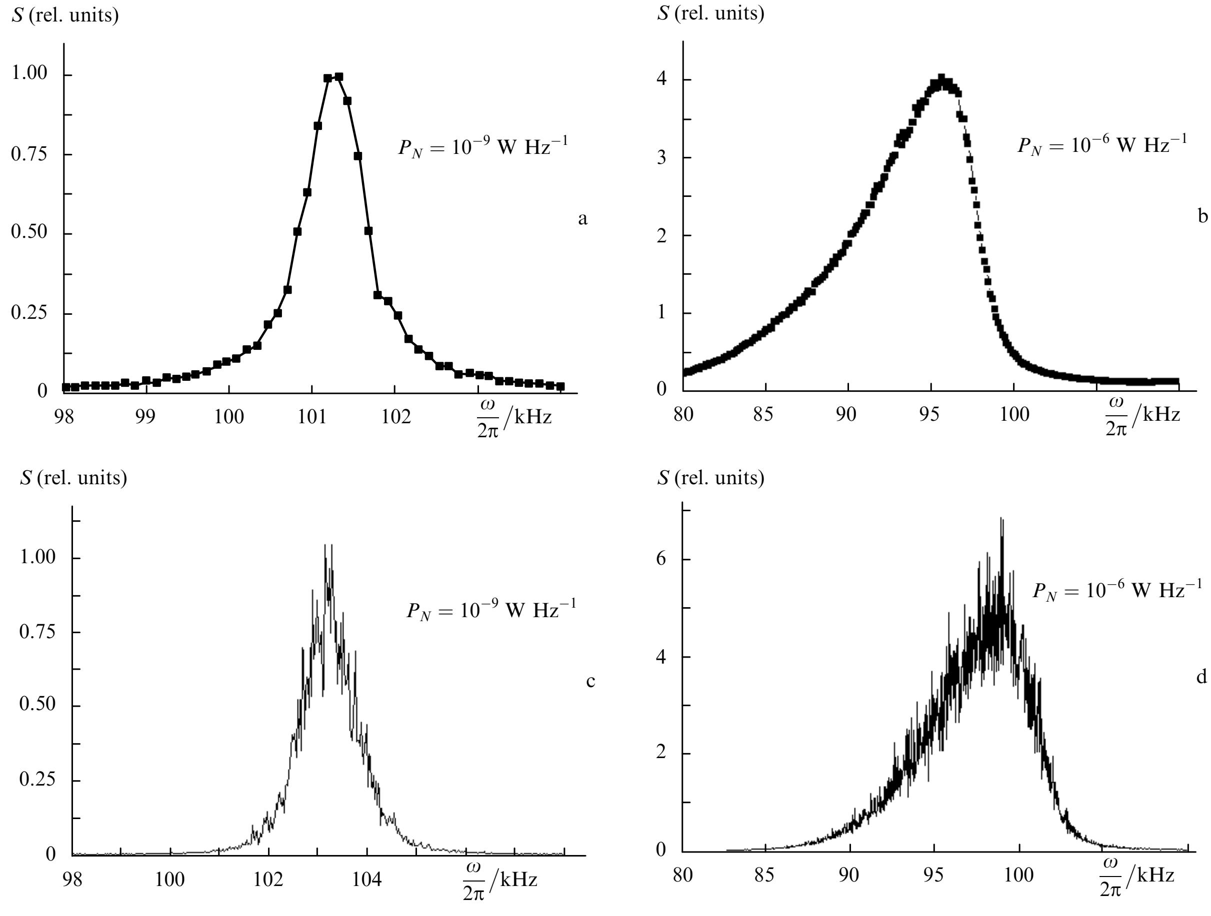

Figure 4. Effect of the pump noise intensity on the shape of the relaxation peak $S(\omega)$ obtained numerically (a, b) and in experiments (c, d) for the electric noise power $P_N = 10^{-9}$ W Hz^{-1} (weak noise pump modulation) and $P_N = 10^{-6}$ W Hz^{-1} (strong noise pump modulation).

(see, for example, [3, 25]). The anharmonicity exhibits itself in that the oscillation frequency of this oscillator depends on its amplitude (non-isochronism). In the oscillator Toda the oscillation frequency decreases with increasing the amplitude. The dependencies $\omega_r(D)$ presented in Figs 2 and 3 can be explained by the fact that when the noise increases, the root-mean-square amplitude of relaxation oscillations increases and their frequency decreases.

The peak width of relaxation oscillations (see Figs 2b and 3b) monotonically increases with increasing the noise intensity except for a range of small D. This growth is qualitatively explained as follows. Because the amplitude of noise-induced relaxation oscillations is a random function of time, the relaxation frequency is also a random function of time, i.e. the noise phase modulation appears. The phase modulation increases with the noise intensity, which results in the peak broadening of the relaxation oscillation. Explanation of the nonmonotonic dependence $\Delta\omega_r(D)$ at small noise intensities as well as the resonance dependence $h(D)$ requires further research.

The experiments showed that the noise-induced broadening of the relaxation peak decreases upon approaching the period-doubling bifurcation point, because parametric synchronisation of relaxation oscillations occurs near this point. Due to this, the main relaxation frequency becomes equal to half the self-modulation frequency, which is less dependent on the noise intensity than the relaxation frequency (see [17]). As a result of the parametric synchronisation, the noise dependence of the relaxation frequency becomes weaker, i.e. the width of the relaxation peak decreases. Besides, the parametric interaction of self-modulation and relaxation oscillations leads to a decrease in the damping decrement of relaxation oscillations, which becomes more pronounced upon approaching the bifurcation point. It, in turn, leads to a decrease in the relaxation peak width.

6. Conclusions

We have studied in this paper the effect of the pump noise on characteristics of relaxation oscillations in a ring SSL operating in the self-modulation regime of the first kind. The performed research has allowed us to find some nonlinear stochastic effects: the frequency of relaxation oscillations decreases and the relaxation peak in the output power-density spectrum broadens with increasing the noise intensity, and the amplitude of the peak depends resonantly on the noise intensity. Some of the obtained results may be qualitatively explained using the model of nonlinear oscillator Toda.

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