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Calculation, optimisation, and measurements of optical resonator parameters of the Novosibirsk terahertz free-electron laser

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***Abstract*. A simple analytical method for calculating open stable laser cavities suggested earlier by the author is used for calculating losses and for performing efficient optimisation of an open optical resonator of the Novosibirsk terahertz free-electron laser. The gain and cavity losses are measured at various wavelengths. Good agreement between theory and experiment is observed. Optimisation of useful losses is considered and a correct expression is derived for their calculations in lasers with uniform saturation of the active medium. Possible potential modernisation of the laser resonator is considered, which may provide greater than twice the output power.**

Keywords: *open laser resonator, cavity losses, optimal output coupling, free-electron laser.*

1. Introduction

In many free-electron lasers (FELs), similarly to other types of lasers efficient generation necessitates an optical low-loss resonator. In high-power FELs, open cavities are usually preferable, in which it is easier to realise the required increase in the dimensions of mirrors, provide the input/output holes for the electron beam and pumpdown system, and allocate systems for mirror adjustment and control, intracavity calorimetry, and electron beam control. Despite the 'openness', a real FEL resonator always has a closed cavity of complicated configuration. The walls of the cavity except for mirrors are imperfect and have hardly predictable optical parameters [1]. This heavily hinders or even makes impossible to employ known numerical methods for electrodynamic calculations of optical resonators. Even at known optical parameters of the resonator walls, the numerical methods will be inefficient because they are intended for solving a direct problem with a prescribed geometry, whereas in designing an optimal resonator it is just the geometry that should be found. In the case of the complicated resonator geometry as it is in FELs, the enumerative technique is inefficient and does not guarantee an actually optimal solution because of the presence of possible local minima in losses. It is obvious that in designing such resonators approximate analytic calculation methods are mandatory. Then such methods can be complicated or even corrected by numerical methods in recalculating the found optimal geometry. However, for stable low-loss resonators no recalculation is necessary at all because in this case the lower the losses, the more precise the analytic approximation. In this case, it is these resonators that are of practical interest.

In [2], a universal approximate method for efficient analytic calculation of open stable laser resonators was suggested. The universal character of the method is explained by the fact that it is based on general principles. Comparison of calculations by [2] with other known numerical methods for particular resonators [3 – 5] demonstrates a good agreement of results. Comparison of theory with direct experiments is presented in this article. Note that such a comparison for the case of a diffraction loss is but rarely encountered in literature because the loss is only one component, which is not the most substantial part of the total losses in ordinary lasers. In addition, measurements of losses are a complicated task due to the inertia of the active medium. This is why the losses are often determined via indirect calculations. In this aspect, a free-electron laser presents unique possibilities. It has a virtually inertia-free active medium close in properties to a uniform vacuum, namely, an electron beam with a low density, which can be easily switched off. A sufficiently long resonator makes measurements of losses easier.

First results of the loss measurements in the resonator of the Novosibirsk free-electron laser (NFEL) possessing minimal useful losses in a sufficiently narrow wavelength range were published in paper [1] and demonstrated a good agreement between the experimental and theoretical data. The same paper thoroughly described the design of resonator. In this paper, we measure losses and other parameters of the resonator differing only by a part of useful losses, which are substantially greater and close to optimal. The measurements are performed in a spectral range maximally extended to long wavelengths. All kinds of diffraction losses are presented in the measurements. It was found that at short wavelengths the losses on central holes of mirrors prevail, whereas at long wavelengths these are losses on the external limiting diaphragms. Note for reference that the spectral range of the first harmonic of the NFEL is 110 – 235 μm, the maximum average power at 130 μm is 500 W, the maximum pulsed power is 0.4 MW, the light pulse duration is 100 ps, the repetition rate of light pulses is 5.6,

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11.2, or 22.4 MHz, and the minimum spectral width of emission line is 0.3 %.

2. Theoretical calculation and minimisation of internal losses

The diffraction losses c_i due to small perturbations at the centre of a Gaussian TEM_{00} mode (holes) and at its periphery (aperture of mirrors, diaphragms, adjustable mirrors for partial output of radiation from the resonator, which will be called scrapers) are equal, according to [2], to the doubled geometrical loss $c_i = 1 - (1 - c_g)^2 \approx 2c_g$. The geometrical loss c_g is equal to the part of the transverse cross section of the mode overlapped (cut) by the perturbation. The total losses are

$$c_\Sigma = 1 - \prod_i (1 - c_i) \approx \sum_i c_i. \quad (1)$$

This additivity property for the losses on holes and the external aperture of mirrors was obtained in [2]. Expression (1) holds true for one-type aperture losses at the beam periphery if the elements responsible for the losses are separated by a distance longer than $L_a = a\delta/\lambda$, where a and δ are the characteristic transverse dimensions of the mode and perturbation, respectively, and λ is the wavelength (the mode fills the part obscured by the diaphragm at the distance L_a). In the resonator under study this condition holds true for main components of losses. The losses in other parts of resonator can be neglected due to an exponential sensitivity of Gaussian beams to most narrow diaphragms.

The calculation model of the NFEL optical resonator (after certain preliminary optimisation has been performed) is shown in Fig. 1. The diameters of loss elements and their distances from the cavity centre (the axial position of the beam waist) along the axis are presented in Table 1. It is assumed that the diaphragms are completely absorbing because many of them have a special absorbing ceramic coating [1] and the diameters of vacuum pipes in the cavity are sufficiently large. Thus, the total resonator losses per round trip (with removed scrapers and symmetrical arrangement of the diaphragms relative to the cavity center) are

$$c_\Sigma = 1 - (1 - c_{mo})^2 (1 - c_{md})^4 (1 - c_{mh1})^2 (1 - c_{mh2})^2 \times (1 - c_{d1})^8 (1 - c_{d2})^8 (1 - c_{d3})^8, \quad (2)$$

where c_{mo} and c_{md} are the ohmic losses and losses on the external apertures of mirrors; c_{mh1} and c_{mh2} are the losses on mirror holes; c_{d1}, c_{d2} and c_{d3} are the losses on diaphragms (the sensor of the electron beam position, the aperture of the chamber of the turning magnet, and the undulator aperture, respectively).

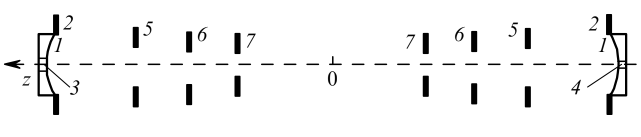

Figure 1. Calculation scheme of the NFEL optical resonator: (*1*) mirrors; (*2*) external diaphragms of mirrors; (*3*, *4*) holes in mirrors; (*5–7*) diaphragms.

Table 1. Diameters of elements responsible for losses and their positions on the axis.

Element number	Diameter/mm	Position on the axis/mm
2	190	±13294.5
3	9	+13294.5
4	3.5	−13294.5
5	105	±7500
6	101	±6300
7	78	±4500

Resonator mirrors have a gold coating, which is optimal from the viewpoint of minimal absorption and long-term stability in atmosphere conditions. Well-known experimental data on optical parameters for gold [6] entail the empirical expression for mirror ohmic losses $c_{mo} = 10^{-2}(0.71 - 1.2\lambda)$, where λ is taken in millimeters.

The losses of the TEM_{00} mode with the radial field distribution $E \sim \exp(-r^2/r_0^2)$, where $r_0 = \{(\lambda L_R/\pi)[1 + (z/L_R)^2]\}^{1/2}$, and L_R is the Rayleigh length, are in the form [2]:

$$c_{dk,md} = \exp\left[-\frac{\pi d_k^2 L_R}{2\lambda(L_R^2 + z_{dk,md})}\right], \quad (3)$$

$$c_{mh} = \frac{\pi d_{mh}^2}{2\lambda L_R[1 + (L_0/2L_R)^2]}, \quad (4)$$

where $k = 1, 2, 3$; $L_0 = 26.589$ m is the resonator length determined by the matching condition for the terahertz and electron beams at the frequency of 5.6 MHz; d is the diameter of a lossy element.

All the components of losses in expression (2) can be divided into two groups. The first group not related to radiation coupling (holes) we will call the internal losses. To the second group refer the losses directly connected with coupling out the radiation (the ways of their optimisation are considered in the next Section). As for the internal losses it is desirable to minimise them. The wavelength dependences of the internal losses are shown in Fig. 2 for various Rayleigh lengths. One can see that they are minimal at $L_R = 6$ m. Numerical simulation shows that to increase the interaction efficiency between the electron and terahertz beams it is desirable to decrease the Rayleigh length [7]. Principally, in the general form the optimisation problem for an optical resonator should also include enhancing the efficiency. However, our estimates and data from [7] show

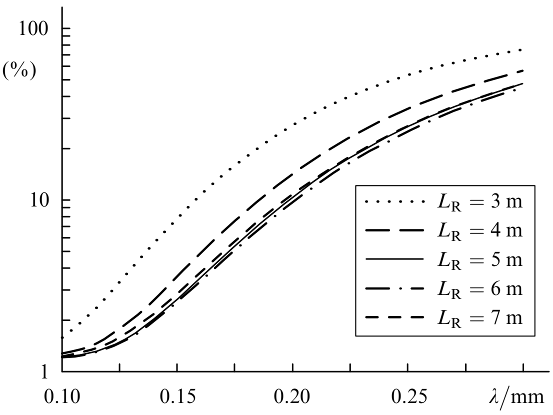

Figure 2. Internal losses per round trip in the NFEL optical resonator as a function of the wavelength for various L_R.

that in the used geometry and in the terahertz range the variation of internal losses is well above that of the gain in the considered range of Fresnel number changes (see Fig. 2). This fact allows us to optimise only losses by assuming the gain constant. One can see from Fig. 2 that the advance to small L_R values is impossible for NFELs because the internal losses rapidly rise in this case. In addition, serious problems arise with the stability and adjustment of the resonator. Hence, we chose for NFELs the following parameters at which the problems are not revealed noticeably: $L_R = 5$ m, the mirror radius is $R = L_0[1+(2L_R/L_0)^2]/2 = 15$ m, and the stability parameter is $g = 1 - L_0/R = -0.75$.

3. Optimisation of radiation coupling and useful cavity losses

In this section we discuss optimisation of the losses due to radiation coupling, namely, useful losses and additional diffraction losses related to useful losses. The additional diffraction losses arise during nonuniform radiation coupling, i.e., by using holes, scrapers, and other devices that perturb the mode structure. According to the theory given above these losses are exactly equal to the useful losses. Thus, the simple change of nonuniform coupling to uniform increases twice the laser output power. Uniform radiation coupling can be easily realised in the optical and IR ranges by means of semitransparent multilayer dielectric mirrors. Unfortunately, this variant cannot be applied to high-power submillimeter radiation because of the lack of suitable material pairs.

One more method of radiation coupling, which employs the Michelson interferometer with a splitter made from a thin film, has been used for a long time in comparatively low-power gas lasers [8, 9]. In our case of FELs such a splitter should be placed in vacuum and withstand a high radiation power inside the resonator. The only material satisfying these requirements is synthetic CVD-diamond, the maximum diameter of the film sample being now 100–120 mm. This diameter is determined by synthesis installations and is ever-increasing. For NFELs, the optimal splitter is 53-μm-thick diamond film inclined at an angle of 45° in such a way that the polarisation of laser radiation should be normal to the incidence plane. The required splitter dimension is approximately one and a half of that available presently. Figure 3 shows the wavelength dependence of the maximum coupling coefficient γ_{max} for this method of radiation coupling (useful losses). When the length of one arm of the interferometer changes, the useful losses vary from zero to this maximum value. One can see that the considered method of radiation coupling not only replaces the method presently realised (two holes) but it also makes it easy to establish an optimal coupling for a particular wavelength and, hence, to increase the output laser power by a factor of greater than two. In the calculations, the refractive index for CVD-diamond was $n = 2.378 \pm 0.002$ [10]. In the same work the absorption coefficient of CVD-diamond has been measured at 130 μm with a unique accuracy of $\alpha = 0.067 \pm 0.003$ cm^{-1}. These are the sufficiently low parameter α and very high heat conductivity of diamond (20 W cm^{-1} K^{-1}) that provide high radiation resistance of the material.

The total losses related to the radiation coupling should be optimised, similarly to the case of any other cw laser.

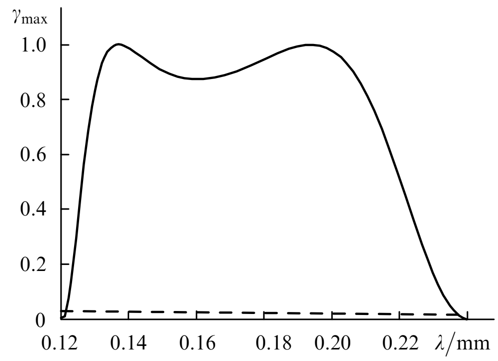

Figure 3. Wavelength dependences of the maximum uniform output coupling on the basis of the Michelson interferometer with the splitter from CVD-diamond (solid curve) and nonuniform coupling in the form of holes used presently (dashed curve).

With adjustable output coupling this can be easily performed experimentally. The optimal value of the fixed output coupling, which is presently employed in the NFELs, should be calculated or determined from special experiments. Because the optimal output coupling differs at different wavelengths further optimisation was performed at 150 μm, which is approximately in the middle of the generation range. The experiment was carried out on a special starting optical resonator possessing minimum possible useful losses (with minimum similar holes in the mirrors).

It is known that the optimal losses depend on the type of the active medium saturation, the gain K, and internal losses c. In the NFEL, the uniform broadening related to a finite duration of the terahertz pulse prevails in the gain spectral broadening. Saturation of the active medium results in the dependence of the intracavity radiation intensity on parameters K and c:

$$I \sim -\frac{\ln(1+K)}{\ln(1-c_\Sigma)} - 1, \qquad (5)$$

where $c_\Sigma = 1 - (1-c)(1-\gamma)^2$ are the total losses and $\gamma = 1 - (1 - c_{mh2})^2$ is the useful part of the losses due to nonuniform coupling of radiation from the cavity with similar holes. The value of K as an fitting parameter was

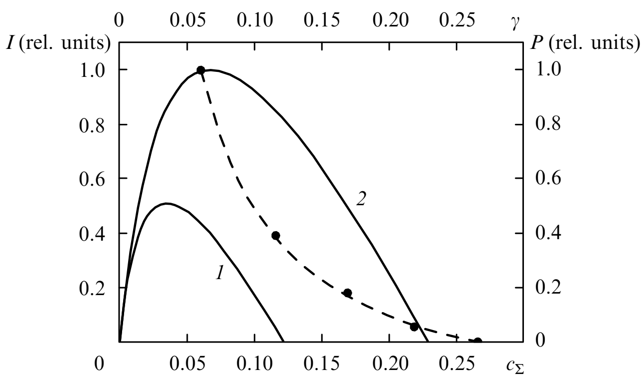

Figure 4. Experimental (points) and theoretical (5) (dashed curve) dependences of the intracavity radiation intensity I on the total cavity losses c_Σ and the dependences of the laser output power on useful losses for nonuniform (1) and uniform (2) radiation coupling.

determined from dependence (5) for the NFEL, which has been confirmed experimentally (see Fig. 4). At the wavelength of 150 μm we have $K = 0.36$. Although the design of the resonator allows one to control radiation coupling out to internal vacuum calorimeters of the optical resonator by means of movable scrapers [1], we employed a more precise method. In this approach, the output NFEL power through a fixed hole was measured as a function of the frequency factor for the electron and terahertz pulses inside the cavity. It is obvious that if the repetition rate of the electron pulses is lower than that of terahertz pulses by a factor of two, three, or more, then at the constant gain the cavity losses increase by approximately the same factor [the actual increase is somewhat less and is calculated by using expression (1)].

The output laser power depends on the useful losses as $P(\gamma) \sim \gamma I(\gamma)$. This function takes the maximum at $\gamma = \gamma_{opt}$, which for the case of nonunifrom radiation coupling is found from the expression

$$\ln(1+K) = -\frac{(1-c)(1-\gamma_{opt})^2 \ln^2\left[(1-c)(1-\gamma_{opt})^2\right]}{2\gamma_{opt}(1-\gamma_{opt}) + (1-c)(1-\gamma_{opt})^2 \ln\left[(1-c)(1-\gamma_{opt})^2\right]}, \quad (6)$$

and in the case of uniform radiation coupling it is found from

$$\ln(1+K) = -\frac{(1-c)(1-\gamma_{opt}) \ln^2[(1-c)(1-\gamma_{opt})]}{\gamma_{opt} + (1-c)(1-\gamma_{opt}) \ln[(1-c)(1-\gamma_{opt})]}. \quad (7)$$

Note that the expression for optimal useful losses in the case of homogeneously saturated active medium was first obtained in [11] and then widely used in literature [12]. However, paper [11] has some inconsistencies because all the expressions in it are actually written for the case of small useful and internal losses, whereas the plots presented there, show the useful losses of up to 0.3 (Fig. 2 in [11]) and even to 0.85 (Fig. 4 in [11]), which in no case are small. Expressions (6) and (7) were obtained by the method given in this paper; nevertheless, they are valid for all cases including those with substantial losses.

One can see that at small useful losses their optimal value in the case of nonuniform radiation coupling is twice less then that in the uniform case. Figure 4 presents the dependence of $P(\gamma)$. The optimal useful losses estimated from this figure, which can be also exactly calculated from (6) and (7) are 0.035 and 0.068, respectively. Note that due to the flatness of maxima these values are mainly formal. In view of this fact, slightly less nonuniform coupling equal to 0.025 at the wavelength of 150 μm was chosen for the NFEL resonator, which actually does not reduce the output power but extends the generation range. It is important because the NFEL is a radiation source for various physical, chemical, and biological experiments carried out at the Siberian Centre for Photochemical Investigations.

In the case of uniform radiation coupling a laser with the same parameters of the active medium and optimal coupling will have twice the output power.

4. Experimental measurement of losses

Similarly to the experiment with an optical resonator [1] in the present experiment we used the detector based on an array of Schottky diodes with a traveling-wave antenna in a corner reflector [13]. Not very fast detector and amplifier were intentionally used to provide a sufficient number of points per light pulse on the screen of a digital oscilloscope with the bandwidth of 0.5 GHz. The detector was specially calibrated for the pulse-periodic signal of our FEL though the detector linearity with respect to an incident cw radiation power has been verified many times. The linear dependence of the detector signal on the incident power was observed in the total range of signal amplitudes specific for the experiments described below.

Detector signals $S(t)$ after switching off the electron beam, which are proportional to the intracavity power of the NFEL are shown in Fig. 5 at various wavelengths. The instant of beam switching off is marked with an arrow. After the beam is switched off, the amplitudes of terahertz pulses exponentially damp at all the wavelengths no longer than 200 μm. The total losses per round trip (period) in this wavelength range were $c_\Sigma = 1 - T/\tau$, where $T = 177.4$ ns is the repetition rate for terahertz pulses (the operating frequency is 5.6 MHz) and τ is the characteristic decay time corresponding to the exponential interpolation $P(t) = P_0 \exp(-t/\tau)$ performed over the pulse vertices or over the differences between maximum and minimum signal values.

Damping distinct from exponential was observed when the losses in resonator were above 20 %. Figure 6 shows the losses as a function of the number of terahertz pulse round trips across the resonator averaged over four oscillograms, which are similar to those given in Fig. 5d. It is obvious that in this case it will be most reasonable to accept the losses corresponding to the first period, because it is just the case, which is realised in normal stationary operation of the NFEL.

The resonator losses obtained in this way at $\lambda = 129.8$, 152.5, 197.3, and 229.4 μm are 7.7 %, 9.4 %, 13.7 %, and 22.4%, respectively. Similar experiments with the resonator were carried out in 2004 in a narrower wavelength range. At the wavelengths $\lambda = 134$, 144, and 147 μm the losses were 6.4 %, 8.3 %, and 8.7 %, respectively.

5. Comparison of theory and experiment

Comparison of theoretical and experimental results is presented in Fig. 7. Different curves correspond to different components of the losses [according to the calculation model (see Fig. 1) and expressions (1)–(4)]. One can see that at short wavelengths the largest losses are due to holes, whereas at longer wavelengths these losses are due to external diaphragms. Two groups of experimental points represent the data of experiments performed in 2004 and 2007. One can see that the experiment agrees with the theory within the accuracy of calculations and experiments (~ 10 %). In [2] it was mentioned that the suggested method is well for calculating losses below 10 % for the resonators with $|g| = 0.5 - 0.9$. For the resonator with $|g| = 0.8$, the method yielded the best agreement for losses up to 20 % – 25 %, which is observed in our experiment ($|g| = 0.75$).

However, Fig. 7 shows an excess of the experimental losses over calculated losses. This fact can be preliminarily explained by possible underestimation of theoretical ohmic losses on mirrors especially in a short-wavelength range. The calculations suggested the parameters of bulk gold, whereas the mirrors were actually copper surfaces treated by diamond turning, which were then deposited by gold.

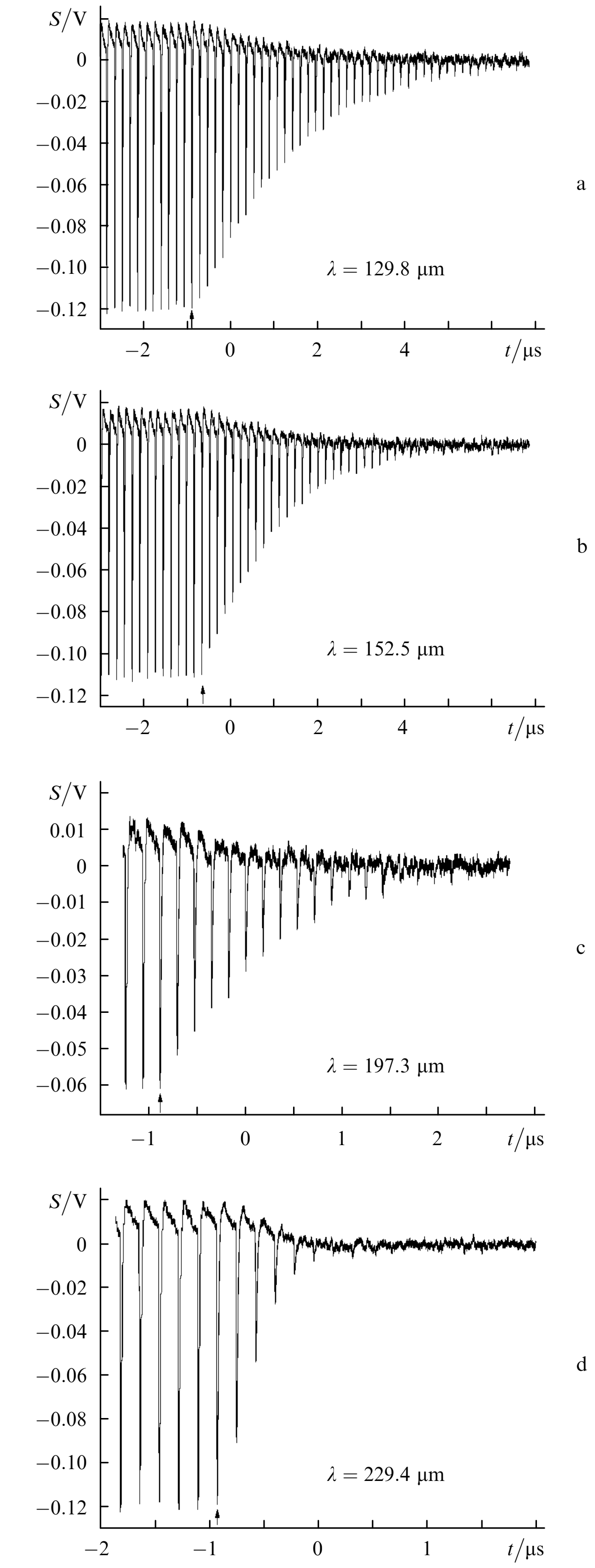

Figure 5. Oscillograms of detector signals from terahertz pulses in the NFEL resonator after the electron beam is switched off (marked by arrow) at various wavelengths.

The most deviated experimental point for losses at $\lambda = 152.5$ μm was measured under modulation instability of side bands, which might slightly increase the mode cross section. We plan to repeat this measurement in a stabilised regime.

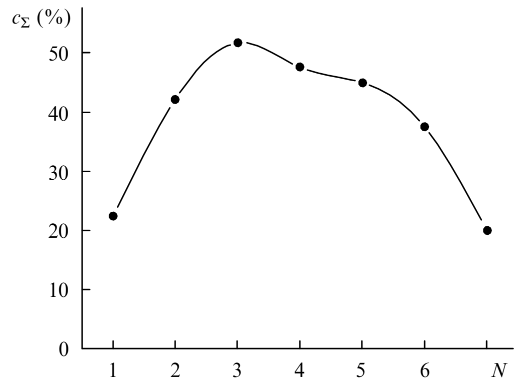

Figure 6. Total losses in the optical NFEL resonator at $\lambda = 229.4$ μm as a function of the number N of round trips for a terahertz pulse after the electron beam is switched off.

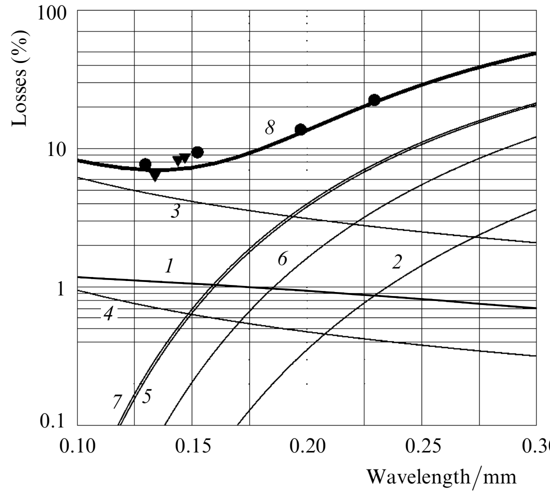

Figure 7. Calculated and experimental losses per round trip of the NFEL resonator versus the wavelength: (*1*) ohmic losses on mirrors; (*2*) losses on external diaphragms of mirrors; (*3*, *4*) losses on holes in mirrors; (5–7) losses on diaphragms; (*8*) total losses; points correspond to the experiment performed in 2007, triangles correspond to the experiment performed in 2004.

One more correction of calculations in a long-wavelength range is connected with the imperfect geodesic adjustment of the axis of absorbing diaphragms (see Fig. 1). Possible transverse displacements of the diaphragms correspond to the total losses at the wavelength $\lambda = 200$ μm increased by 1 %. These additional losses can be easily calculated by the method given above.

6. Conclusions

Direct comparison of the simple analytical method for calculating losses of open stable laser resonators, which was suggested earlier by the author of this paper, with the experiment shows a good agreement. Thus, the correctness of this method is confirmed now not only by comparison with other numerical methods but with the experiment also.

This method was applied to develop and optimise an optical resonator of the Novosibirsk terahertz free-electron laser. By using this method, we calculated an optimal uniform controlled method for radiation coupling with the help of the Michelson interferometer with a CVD-diamond splitter, which allows one to increase the output power of the NFEL by a factor greater than two. Correct expressions are presented for calculating optimal laser output coupling under uniform saturation of the active medium.

Specific features of the NFEL made it possible to carry out direct experiments on determining the parameters of the optical resonator and phenomenological parameters of the active medium, which usually can hardly be performed.

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