

PACS numbers: 42.81.Dp; 42.79.Dj
DOI: 10.1070/QE2009v039n01ABEH013897

Calculation of the frequency band of a Bragg waveguide

D.V. Prokopovich

***Abstract*. Transmission regions of the silica-core Bragg optical waveguides are analysed. It is shown that a relatively small decrease in the refractive index of the core allows one to narrow down the waveguide transmission region so that to suppress in the spectrum the undesirable emission line propagating in the waveguide, thus ensuring a minimal loss of the fundamental mode at the working wavelength. The example of calculations of a frequency filter based on a Bragg optical fibre is considered, in which the fundamental mode has minimal losses at 0.925 μm but completely suppressed at 1.06 μm. The loss spectrum of a Bragg waveguide and the field distribution of the fundamental mode are presented.**

Keywords: *fibre optics, multilayer structures, Bragg waveguide.*

1. Introduction

It is well known that a multilayer quarter-wave structure is the most suitable cladding of a Bragg waveguide, allowing effective confinement of the mode within the core and the lowest radiation loss [1, 2]. Being strictly proved for the case of planar waveguides and quite applicable to cylindrical multilayer structures [3 – 5], this fact is widely used to select optimal parameters for real Bragg waveguides [6]. In practice, apart from minimising the loss at the working wavelength, it is sometimes necessary to satisfy specific requirements at other wavelengths (spectral bandwidth, dispersion, etc.) [7 – 10]. Although Bragg waveguides with a step refractive index profile have been studied in many papers [3 – 15], the general approach to the optimisation of their parameters has not been developed so far. The main difficulty is the multiparametric character of the problem even in the case of simple models. This paper deals with a particular example of calculating a frequency filter ensuring a minimum radiation loss A at the working wavelength λ_0 and the mode suppression at the neighboring wavelength $\lambda_1 > \lambda_0$. This problem arises when designing fibre lasers based on Bragg waveguides [16 – 18].

D.V. Prokopovich Fiber Optics Research Center, Russian Academy of Sciences, ul. Vavilova 38, 119333 Moscow, Russia;
e-mail: dvp@4ka.mipt.ru

Received 25 April 2008; revision received 21 September 2008
Kvantovaya Elektronika **39** (1) 105 – 109 (2009)
Translated by M.V. Politov

It was shown in papers [19, 20] that if refractive indices n_1 and n_2 of the structure layers, refractive index n_0 of the core, and radius a of the core are specified, then one can unambiguously determine thicknesses l_1 and l_2 of the layers satisfying the condition of a local minimum of loss at the wavelength λ_0 for a particular mode. In this case, it is impossible to satisfy any other conditions at the wavelength λ_1 because all the parameters of the waveguide are already determined. The paper offers a method for finding Bragg waveguide parameters satisfying simultaneously the requirements to minimisation of the loss at the working wavelength λ_0 and to suppression of the Bragg mode at the cutoff wavelength λ_1

$$A(\lambda_0) = A_{\min}, \quad A(\lambda_1) \to \infty \tag{1}$$

for any λ_0 и λ_1. It is shown that a relatively small change in the refractive index of the core, which in turn allows a significant change in the loss spectrum of the Bragg waveguide, is sufficient to meet conditions (1). A real Bragg waveguide whose parameters are optimised for the use in a neodymium fibre laser is calculated to illustrate the realisation of this method.

2. The model of a Bragg waveguide

Consider a planar Bragg waveguide with a homogeneous core of size $2a$ and the refractive index n_0, which has a Bragg cladding, namely, an infinite step structure with a periodic refractive index $n(x) = n(x + \Lambda)$. The l_1- and l_2-thick layers have the refractive indices n_1 and n_2, respectively and the structure period is $\Lambda = l_1 + l_2$.

Let an electromagnetic wave propagate along the z axis of the waveguide. We consider a TE wave for which the electric field strength has only one component $E_y = E(x, z)$, perpendicular to the xz plane (Fig. 1). We will seek for the electric field strength in the form of a running wave $E(x, z) = u(x)\exp(\mathrm{i}\beta z)$. Then, from Maxwell's equations we can easily derive a wave equation for $u(x)$:

$$\frac{\mathrm{d}^2u}{\mathrm{d}x^2} + (k^2n^2 - \beta^2)u = 0, \tag{2}$$

where $k = 2\pi/\lambda$ is the wave number in vacuum; β is the propagation constant (longitudinal wave number). Let us denote by

$$q_0 = \sqrt{k^2n_0^2 - \beta^2}, \; q_1 = \sqrt{k^2n_1^2 - \beta^2}, \; q_2 = \sqrt{k^2n_2^2 - \beta^2} \tag{3}$$

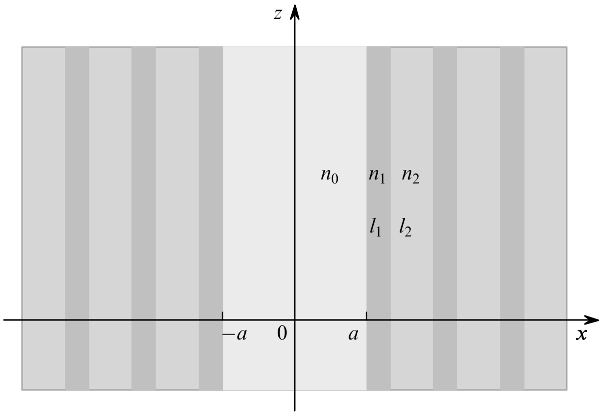

Figure 1. Schematic of a Bragg waveguide.

the transverse propagation constants in the media with refractive index n_0, n_1, and n_2, respectively. The solution of wave equation (2) in the Bragg cladding is a sum of harmonic functions with unknown coefficients, which should be found. When $a \leqslant x \leqslant a+\Lambda$, the solution of expression (2) has the from

$$\begin{aligned} u_1(x) &= A_1\cos q_1(x-a) + B_1 \sin q_1(x-a) \\ &\quad \text{for } a \leqslant x \leqslant a+l_1, \\ u_1(x) &= C_1 \cos q_2(x-a-l_1) + D_1 \sin q_2(x-a-l_1) \\ &\quad \text{for } a+l_1 \leqslant x \leqslant a+\Lambda. \end{aligned} \tag{4}$$

When $a+\Lambda \leqslant x \leqslant a+2$, we have

$$\begin{aligned} u_2(x) &= A_2\cos q_1(x-a-\Lambda) + B_2 \sin q_1(x-a-\Lambda) \\ &\quad \text{for } a+\Lambda \leqslant x \leqslant a+\Lambda+l_1, \\ u_2(x) &= C_2 \cos q_2(x-a-\Lambda-l_1) \\ &\quad + D_2 \sin q_2(x-a-\Lambda-l_1) \\ &\quad \text{for } a+\Lambda+l_1 \leqslant x \leqslant a+2\Lambda, \end{aligned} \tag{5}$$

etc.

It is easy to find the transition matrix

$$\hat{T} = \left\| \begin{matrix} c_1c_2 - \dfrac{q_1}{q_2}s_1s_2 & s_1c_2 + \dfrac{q_1}{q_2}c_1s_2 \\ -s_1c_2 - \dfrac{q_2}{q_1}c_1s_2 & c_1c_2 - \dfrac{q_2}{q_1}s_1s_2 \end{matrix} \right\|$$

from the condition of continuity of solutions u_1, u_2 and their derivatives at interfaces $x = a+l_1$ and $x = a+\Lambda$ so that

$$\bar{w}_{m+1} = \hat{T}\bar{w}_m. \tag{6}$$

Here we introduce the vector-column of coefficients

$$\bar{w}_m = \left\| \begin{matrix} A_m \\ B_m \end{matrix} \right\|, \; m = 1, 2, \ldots$$

and notations $c_1 = \cos q_1 l_1$, $c_2 = \cos q_2 l_2$, $s_1 = \sin q_1 l_1$, $s_2 = \sin q_2 l_2$. In a periodic structure, the matrix $\hat{T}$ is independent of the period number m. Therefore, the general solution of recurrence equation (6) can be written in the form

$$\bar{w}_m = M_1 v_1^{m-1}\bar{w}_1 + M_2 v_2^{m-1}\bar{w}_2. \tag{7}$$

Constants $M_{1,2}$ are determined by the boundary conditions at the core–cladding interface. Coefficients $v_{1,2}$ are the eigenvalues of the matrix $\hat{T}$, which are found from the characteristic equation

$$v^2 - 2v(c_1c_2 - \tau s_1 s_2) + 1 = 0. \tag{8}$$

The expression for them has the from

$$v_{1,2} = c_1c_2 - \tau s_1 s_2 \pm \sqrt{(c_1c_2 - \tau s_1s_2)^2 - 1}, \tag{9}$$

where $\tau = \frac{1}{2}(q_1/q_2 + q_2/q_1)$. We are only interested in the real eigenvalue v whose modulus is less than unity, i.e this solution corresponds to a decaying solution $u(x)$. This v always exists, if the discriminant of equation (8) is positive

$$(c_1c_2 - \tau s_1s_2)^2 - 1 > 0. \tag{10}$$

In view of the obvious relationship $v_1v_2 = 1$, the second solution corresponds to the wave with an exponentially growing amplitude, which was discarded for physical reasons. It is more convenient to rewrite inequality (10) in terms of phase variables $X = q_1 l_1$ and $Y = q_2 l_2$ [21] which correspond to the phase incursion of the quasi-periodic solution $u(x)$ in each layer of the Bragg cladding:

$$\left[\cos X \cos Y - \frac{1}{2}\left(\frac{l_2}{l_1}\frac{X}{Y} + \frac{l_1}{l_2}\frac{Y}{X}\right)\sin X \sin Y\right]^2 > 1. \tag{11}$$

It is necessary to fix layer thicknesses l_1 и l_2 to represent inequality (11) graphically. It can be done if the fundamental TE mode symmetric with respect to the z axis has the local minimum losses at the wavelength λ_0. In this case, l_1 and l_2 should meet the quarter-wave condition [22]

$$\begin{aligned} l_1 &= \frac{\pi(2m_1+1)}{2}\left(k_0^2 n_1^2 - k_0^2 n_0^2 + \frac{\pi^2}{a^2}\right)^{-1/2}, \\ l_2 &= \frac{\pi(2m_2+1)}{2}\left(k_0^2 n_2 2 - k_0^2 n_0^2 + \frac{\pi^2}{a^2}\right)^{-1/2}, \end{aligned} \tag{12}$$

where $k_0 = 2\pi/\lambda_0$; $m_1, m_2 = 0, 1, 2, \ldots$.

Figure 2 illustrates inequality (11) for the thicknesses derived from condition (12). Inequality (10) holds true in the painted regions, i.e. there exist decreasing and increasing real solutions $u(x)$. Similar to the solid-state physics, these regions are called forbidden bands in photonics [2, 13]. Following the character of the problem, we will call them transmission regions of the waveguide. Only complex solutions $u(x)$ exist in the unpainted regions. These regions correspond to the so-called allowed bands; we will call them non-transmission regions. The region boundaries correspond to purely periodic solutions $u(x)$ with respect to x. Therefore, Fig. 2 shows all types of solutions $u(x)$ in the Bragg cladding. It is necessary to satisfy additional requirements to select the solutions representing the Bragg waveguide modes from the whole set of possible solutions in a periodic medium.

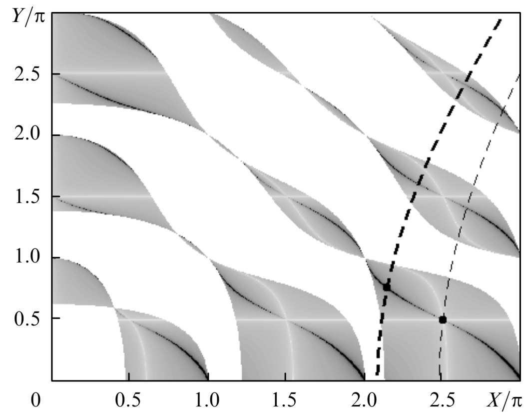

Figure 2. Transmission (painted areas) and non-transmission (unpainted areas) regions of a Bragg waveguide with $n_0 = n_2$ optimised for the wavelength λ_0. The thin and thick dashed curves correspond to calculations by using (13) for λ_0 and λ_1, respectively. The black solid curves is the calculations with the help of (14).

The first condition can be easily obtained by eliminating the propagation constant β from (3):

$$\frac{X^2}{l_1^2} - \frac{Y^2}{l_2^2} = k^2(n_1^2 - n_2^2). \tag{13}$$

Expression (13), which is another form of Snell's law, gives a relationship between phase variables X and Y. This relationship is independent of the propagation constant β and is shown by dashed curves in Fig. 2.

The second requirement is derived by conjugating smoothly the solution in the core $u_0(x) = \cos q_0 x$ and solution (4) in the first layer of the Bragg cladding. It has the from

$$-\frac{Z}{X}\frac{l_1}{a}\tan Z = \left(v - \cos X \cos Y + \frac{l_2}{l_1}\frac{X}{Y}\sin X \sin Y\right)$$
$$\times\left(\sin X \cos Y + \frac{l_2}{l_1}\frac{X}{Y}\cos X \sin Y\right)^{-1}, \tag{14}$$
$$Z = q_0 a = \left[\frac{a^2}{n_1^2 - n_2^2}\left(\frac{n_0^2 - n_2^2}{l_1^2}X^2 + \frac{n_1^2 - n_0^2}{l_2^2}Y^2\right)\right]^{1/2},$$

where v is the eigenvalues (9) whose modulus is less than unity. The relationship between X and Y defined by equation (14) is shown by black solid curves in Fig. 2. The intersection of the solid and dashed curves corresponds to a certain mode of the Bragg waveguide at a certain wavelength.

3. Analysis of the model

The above graphic representation of the mode structure of a planar Bragg waveguide is convenient for solving applied problems related to the synthesis of such structures. As an example, we will use it to construct a Bragg waveguide in which the fundamental mode has the lowest losses at the working wavelength λ_0 and is completely suppressed at a given wavelength $\lambda_1 > \lambda_0$.

Let the size of the core be $2a$ and the refractive indices n_0, n_1, and n_2, or rather, the range of their variation ($n_1, n_2 \leqslant |n_0 + \Delta n|$), be specified. We need to find the layer thicknesses l_1 and l_2 at which condition (1) is fulfilled.

The transmission spectrum of the Bragg waveguide is well studied. Papers [19, 20] show that when $a \gg \lambda$, the transmission minimum is mainly determined by the thickness of the first layer and is observed at the wavelength

$$\lambda_l \approx \frac{2n_2 l_1}{m}\left[\left(\frac{n_1}{n_2}\right)^2 - 1\right]^{1/2}, \quad l = 1, 2, \ldots, \tag{15}$$

while the maximum is observed at the wavelength

$$\lambda_k \approx \frac{4n_2 l_1}{(2k+1)}\left[\left(\frac{n_1}{n_2}\right)^2 - 1\right]^{1/2}, \quad k = 0, 1, 2, \ldots. \tag{16}$$

In other words, it is necessary to take such a structure that conditions $\lambda_0 \approx \lambda_k$ and $\lambda_1 \approx \lambda_l$ were fulfilled simultaneously. However, this is possible not for any λ_0 and λ_1, because satisfy one of conditions (15), (16) does not guarantee the fulfillment of the second condition. It is possible to satisfying (16) at first (i.e. $\lambda_0 \approx \lambda_k$) and then to try to broaden the non-transmission band (in wavelength units) so that the cutoff wavelength λ_1 fell into this band. This operation is similar to narrowing the transmission band. According to [2], the transmission bandwidth (in frequency units) is

$$\Delta\omega \sim \omega_k \frac{4}{\pi}\frac{|n_1 - n_2|}{n_1 + n_2}, \tag{17}$$

where $\omega_k = 2\pi c/\lambda_k$. Because variations in the frequency $\delta\omega$ and wavelength $\delta\lambda$ are related as $\delta\omega = -\delta\lambda 2\pi c/\lambda^2$, the transmission band (in wavelength units) is decreased according to (17) by increasing the contrast $|n_1 - n_2|$, which is not feasible because of technological constraints.

We use a somewhat different algorithm for broadening the transmission bandwidth. Let us analyse Fig. 2 in more detail. As was shown above, the coordinates X, Y of the intersection point of (13) and (14) give the phase incursion in each of the layers with refractive indices n_1 and n_2, respectively. Due to the efficiency of the quarter-wave plates [1, 2], if the waveguide is optimised for the wavelength λ_0, dependence (13) for $k_0 = 2\pi/\lambda_0$ intersects dependence (14)

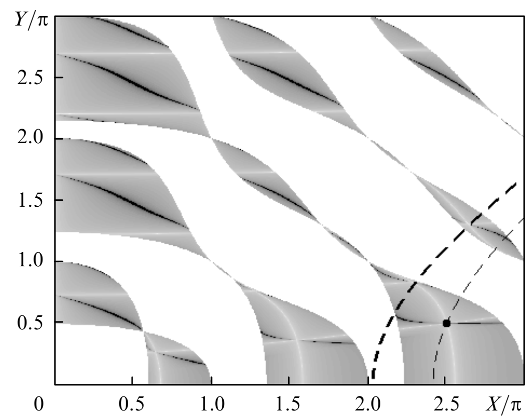

Figure 3. Transmission (painted areas) and non-transmission (unpainted areas) regions of a Bragg waveguide with $n_0 = n_2 - 5 \times 10^{-4}$ optimised for the wavelength λ_0. The thin and thick dashed curves correspond to calculations by using (13) for λ_0 and λ_1, respectively. The black solid curves refer to calculations with the help of (14). The fundamental mode is suppressed at λ_1.

at a point with half-integer coordinates in fractions of π. Thus, for example, in Fig. 2 these dependences intersect at point (2.5,0.5). For the mode to be suppressed precisely at the wavelength λ_1, it is necessary that dependence (13) for λ_1 goes through a point of integer coordinates, which corresponds to approximate fulfillment of condition (15) and, as was mentioned above, is not always feasible. It is also possible to change dependence (14), so that dependences (13) and (14) intersect outside the painted area, i.e. the wavelength λ_1 lying within the waveguide non-transmission region. This can be readily realised by slightly decreasing the refractive index n_0 of the core. One can see that by comparing Fig. 2 plotted for $n_0 = n_2 = 1.45$, $n_1 = n_2 + 0.015$ with Fig. 3, where $n_0 = n_2 - 5 \times 10^{-4}$, $n_1 = n_2 + 0.015$. What is important is that the Bragg waveguide remains optimal for the wavelength λ_0 in this case.

4. Application of the model

The model constructed in section 2 can be easily generalised for the case of cylindrical symmetry. For this purpose it is necessary to replace cosines and sines in expressions (4) and (5) for the field in the Bragg cladding by Bessel and Neumann functions and to take $u_0(r) = J_0(q_0 r)$ for the solution in the core, where J_0 is the zero-order Bessel function. Moreover, it follows from the theoretic results [11, 12] that the field in the cladding is well described with asymptotic expressions similar to those used in the planar case. Further analysis is similar to that given in section 2 for the planar structure. For Bragg waveguides (optical fibres), the spectral region in which the developed method can be applied is limited by the transmission band of the core material ($\sim 0.25 - 2$ μm).

Let us present a numerical example that finds a practical application; it is necessary to design a Bragg fibre whose LP_{01} mode would have minimal losses at the working wavelength $\lambda_0 = 0.925$ μm and be suppressed at $\lambda_1 = 1.06$ μm. The wavelength $\lambda_0 = 0.925$ μm corresponds to one of the allowed transitions ($^4F_{3/2} \rightarrow {}^4I_{9/2}$) of the neodymium ion, and $\lambda_1 = 1.06$ μm to the basic laser transition ($^4F_{3/2} \rightarrow {}^4I_{11/2}$). In this case, loss at λ_0 should not exceed 10 dB km^{-1}. The calculated radius of the core is $a = 10$ μm, refractive indices are $n_2 = 1.449$ and $n_1 = n_2 + 0.015$. If the refractive index of the waveguide core is $n_0 = n_2$, the optimal thicknesses l_1 and l_2 are 5.45 and 6.53 μm, respectively. For a four-layer cladding, the losses are $A(\lambda_0) \approx 10$ dB km^{-1} and $A(\lambda_1) \approx 5.5\times\ 10^3$ dB km^{-1}.

Based on the method considered in sections 2 and 3, the problem under study can be solved with the help of a fibre with a slightly decreased refractive index of the core $n_0 = n_2 - 0.001$. In this case, the optimal thicknesses l_1 and l_2 are 5.29 and 3.59 μm, respectively. For a six-layer cladding, $A(\lambda_0) \approx 5$ dB km^{-1}, and the wavelength $\lambda_1 = 1.06$ μm corresponds to the boundary of the waveguide transmission window with a loss $A(\lambda_1) > 10^5$ dB km^{-1}. Figure 4 shows the change in the loss spectrum with decreasing the refractive index of the core. The calculations were performed with the help of a numerical solution of the wave equation [22]. One can see that a small change in the refractive index allows us to expand the non-transmission region, so that the unwanted wavelength $\lambda_1 = 1.06$m falls into it. Figure 5 presents the field distribution of the fundamental LP_{01} mode at $\lambda_0 = 0.925$ μm and the refractive index profile.

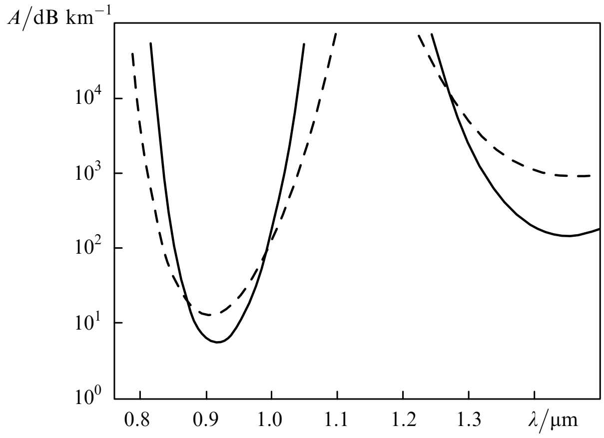

Figure 4. Wavelength dependence of the losses in the Bragg fibre optimised for $\lambda_0 = 0.925$ μm. The dashed curve corresponds to $n_0 = n_2$ and the solid line to $n_0 = n_2 - 0.001$.

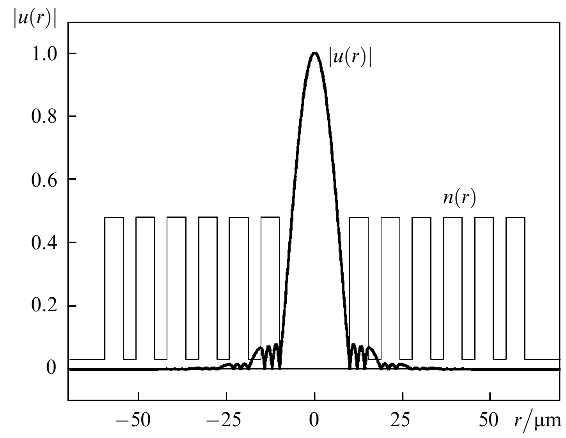

Figure 5. Modulus $|u(r)|$ of the field amplitude of the fundamental mode for a Bragg fibre ($n_0 = n_2 - 0.001$, $\lambda_0 = 0.925$ μm) and a schemeatic of the refractive index profile $n(r)$.

Note that inaccuracies in designing the refractive index profile affect the fibre parameters, deviations in the refractive index of the core being particularly essential. For qualitative estimations, we present optimal step refractive index profiles $n_2 = 1.449$ and $n_1 = n_2 + 0.015$ at the wavelength $\lambda_0 = 0.925$ μm corresponding to a local minimum of the waveguide loss (Table 1).

If we initially form profile 1 but decrease the refractive index n_0 of the core by 0.0005 (all other parameters remaining the same: $l_1 = 5.45$ μm, $l_2 = 6.53$ μm), it causes an increase in fibre loss by a factor of ~ 100. If we form, for example, profile 3, a decrease in n_0 by 0.0005 leads to an increase in the loss by factor of ~ 10. This difference is due to the fact that in the first case the conditions for resonance

Table 1.

Profile number	Core refractive index	l_1/μm	l_2/μm
1	$n_0 = n_2$	5.45	6.53
2	$n_0 = n_2 - 0.0005$	5.37	4.45
3	$n_0 = n_2 - 0.0010$	5.29	3.59
4	$n_0 = n_2 - 0.0015$	5.21	3.09

reflection from the Bragg cladding are violated more strongly.

The technology does not allow for step-index profiles with sharp boundaries, whereas research works devoted to optimisation of structures with smoothed boundaries are almost absent. Here we should mention recent paper [23] which showed that in a smooth profile close to a step quarter-wave one, the attenuation decrement of the transverse wave in the Bragg cladding is $\sim 3/4$ of the absolute maximum equal to $\ln(n_1/n_2)$.

5. Conclusions

We have considered in this paper the transmission and non-transmission regions of the silica-core Bragg waveguide. The performed analysis allows us to determine whether a certain mode solution can exist for a particular wavelength without solving the dispersion equation. It has been shown that a relatively small decrease in the reflective index of the core allows one to change significantly the loss spectrum of the Bragg waveguide. Inthis caswe it is possible to have the best localisation of the fundamental mode at the working frequency and simultaneously to narrow the transmission region so that to suppress this mode in the immediate vicinity of the working wavelength.

An example of calculating a Bragg-fibre frequency filter is presented, which can be used as a waveguide medium for a neodymium fibre laser. In this fibre the fundamental LP_{01} mode has a minimum loss at 0.925 μm and is fully suppressed at 1.06 μm. The spectral transmission bandwidth and the mode field distribution have been calculated for the wavelength corresponding to the smallest losses in the fibre.

Acknowledgements. The author thanks A.V. Vinogradov, A.V. Popov and M.E. Likhachev for fruitful discussions. This work was supported by the Russian Foundation for Basic Research (Grant Nos 07-02-01244-a and 07-02-01177-a).

References

1. Born M., Wolf E. *Principles of Optics* (Oxford: Pergamon Press, 1986; Moscow: Nauka, 1970).
2. Yariv A., Yukh P. *Optical Waves in Crystals* (New York: Wiley, 1984; Moscow: Mir, 1987).
3. Johnson S.G., Ibanescu M., Skorobogatiy M. *Opt. Express*, **9**, 748 (2001).
4. Hadley G.R., Fleming J.G., Lin S.-Y. *Opt. Lett.*, **29**, 309 (2004).
5. Bogdanovich D.V. *Pis'ma Zh. Eksp. Teor. Fiz.*, **86**, 265 (2007).
6. Fevrier S., Jamier R., Blondy J.-M., et al. *Opt. Express*, **14**, 562 (2006).
7. Birks T.A., Luan F., Pearce G.J., et al. *Opt. Express*, **14**, 5688 (2006).
8. Lin C., Zhang W., Huang Y., et al. *Appl. Phys. Lett.*, **90**, 031109 (2007).
9. Argyros A. *Opt. Express*, **10**, 1411 (2002).
10. Marcou J., Brechet F., Roy P. *J. Opt. A: Pure Appl. Opt.*, **3**, 144 (2001).
11. Xu Y., Lee R.K., Yariv A. *Opt. Lett.*, **25**, 1756 (2000).
12. Xu Y., Ouyang G.X., Lee R.K., et al. *J. Lightwave Technol.*, **20**, 428 (2002).
13. Joannopouls J.D., Johnson S.G., Winn J.N., et al. *Photonic Crystals* (New Jersey: Princeton University Press, 2008).
14. Sakai J.-I., Nouchi P. *Opt. Commun.*, **249**, 153 (2005).
15. Guo S., Albin S., Rogowski R.S. *Opt. Express*, **12**, 198 (2004).
16. Wang A., George A.K., Knight J.C. *Opt. Lett.*, **31**, 1388 (2006).
17. Isomaki A., Okhotnikov O.G. *Opt. Express*, **14**, 4368 (2006).
18. Bigot L., Pureur V., Jaouen Y., et al. *Proc. ECOC 2006* (Berlin, Germany, 2007) th.1.4.5.
19. Abeeluck A.K., Litchinitser N.M., Headley C., et al. *Opt. Express*, **10**, 1320 (2002).
20. Litchinitser N.M., Dunn S.C., Usner B., et al. *Opt. Express*, **11**, 1243 (2003).
21. Alam I., Sakai J.-I. *Opt. Commun.*, **250**, 84 (2005).
22. Prokopovich D.V., Popov A.V., Vinogradov A.V. *Kvantovaya Elektron.*, **37**, 873 (2007) [*Quantum Electron.*, **37**, 873 (2007)].
23. Popov A., Vinogradov A., Prokopovich D. *Proc. ICTON 2008* (Athens, Greece, 2008) Vol. 1, pp 242 – 245.