Quantum Electronics **39** (2) 197 – 200 (2009)

OPTICAL WAVEGUIDES

PACS numbers: 42.81.Dp
DOI: 10.1070/QE2009v039n02ABEH013919

Dispersion and energy parameters of the HE_{1m} modes of two-layer optical fibres

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***Abstract*. Dispersion and energy parameters of the HE_{1m} modes of multimode two-layer step-index optical fibres are studied. The results of studies on chromatic dispersion for wavelengths $\lambda < 1.3$ μm and $\lambda > 1.3$ μm, phase and group delays, waveguide and material dispersions of the modes, as well as results of calculations of fibre core diameters, normalised powers and corresponding effective areas of HE_{1m} modes are presented under conditions of the zero chromatic dispersion at $\lambda_0 = 1.55$ and 1.06 μm.**

Keywords: *multimode two-layer optical fibre, higher modes, chromatic dispersion coefficient, effective mode area.*

1. Introduction

Today such urgent applied and theoretical problems as coherent coupling, optical arbitrary waveform generation, precise optical metrology, etc. necessitate the development of optical fibres, which have a zero-chromatic-dispersion wavelength $\lambda_0 < 1.3$ μm and are capable of transmitting high-power picosecond and femtosecond laser pulses. This has aroused interest in the use of multimode two-layer step-index optical fibres [1], which can operate in a single-mode regime at fibre lengths of some tens of meters [2 – 4]. The working modes, which are excited most effectively in such fibres by linearly polarised radiation, are HE_{1m} modes with the radial index $m \geqslant 2$. To simplify the analysis, model linearly polarised LP_{nm} modes are often used instead of actual HE_{nm} modes and their superposition. In this approximation LP_{0m} modes correspond to HE_{1m} modes.

The waveguide characteristics of HE_{1m} modes allow one to compensate the material dispersion for λ being both less and greater than 1.3 μm. Multimode fibres have a larger cross-section area of the core, which makes it possible to transmit high-power radiation pulses at HE_{1m} modes ($m \geqslant 2$) in the single-mode regime. These and other advantageous dispersion and energy parameters of HE_{1m} modes require thorough research.

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Received 17 June 2008; revision received 20 November 2008
Kvantovaya Elektronika **39** (2) 197 – 200 (2009)
Translated by M.V. Politov

All calculations in the paper were performed for SiO_2 fused silica fibres 1 and 2, which operate at $\lambda_0 = 1.55$ and 1.06 μm, respectively, and have GeO_2-doped cores with 5 mol. % and 20 mol. % of the dopant concentration (see Table 1). The refractive indices of the core (n_1) and claddings (n_2) corresponding to these λ_0 were calculated using the Sellmeier dispersion formula [5].

2. Phase and group delays of HE_{1m} modes

Figures 1 and 2 show the qualitative characteristics of the HE_{1m}-mode phase ($n_{\rm eff}$) and group ($n_{\rm gr}$) delay coefficients as functions of the characteristic fibre parameter $V = (2\pi a/\lambda)(n_1^2 - n_2^2)^{1/2}$, where a is the fibre core radius.

The phase delay coefficient $n_{\rm eff} = c/v_{\rm ph}$ of each HE_{1m} mode varies from n_2 (at the mode cut-off) to n_1 (away from the mode cut-off). The group delay coefficient $n_{\rm gr} = c/v_{\rm gr} = n_{\rm eff} - \lambda {\rm d}n_{\rm eff}/{\rm d}\lambda = n_{\rm eff} + V{\rm d}n_{\rm eff}/{\rm d}V$ of each HE_{1m} mode varies from n_2 (at the mode cut-off) to a maximum value which grows with the mode radial index m and tends to n_1 (away from the mode cut-off). The peculiarities of the behaviour of $n_{\rm eff}$ and $n_{\rm gr}$ upon variation in V are important for finding the fibre parameters in which the chromatic dispersion is zero [6].

In calculations of the waveguide dispersion (see below), we also used the value of the normalised phase delay $B_{\rm eff} = (n_{\rm eff}^2 - n_2^2)/(n_1^2 - n_2^2)$, which varies for each mode from 0 to 1 upon varying V from its value at the mode

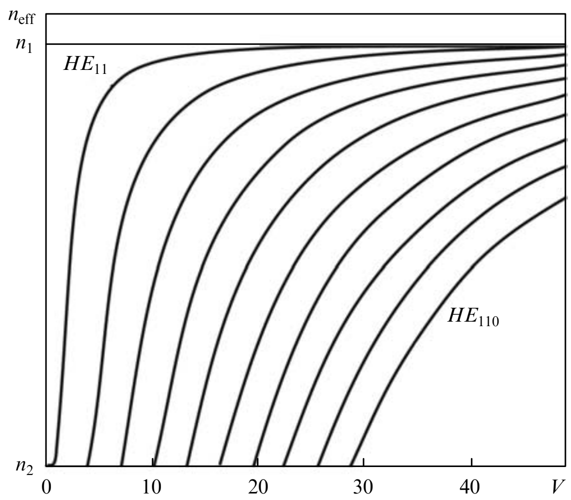

Figure 1. Phase delay of HE_{1m} modes ($m = 1 - 10$).

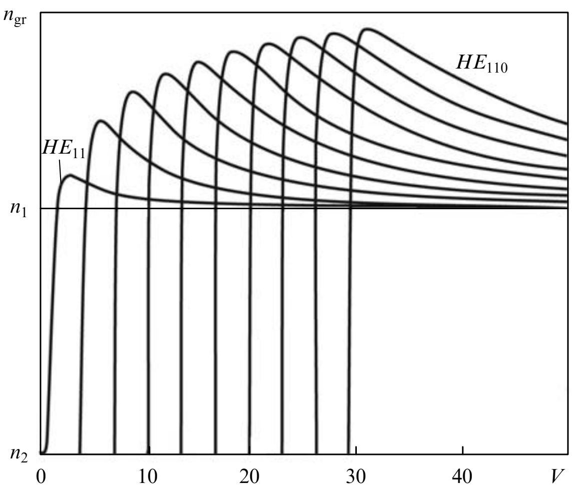

Figure 2. Group delay of HE_{1m} modes ($m = 1 - 10$).

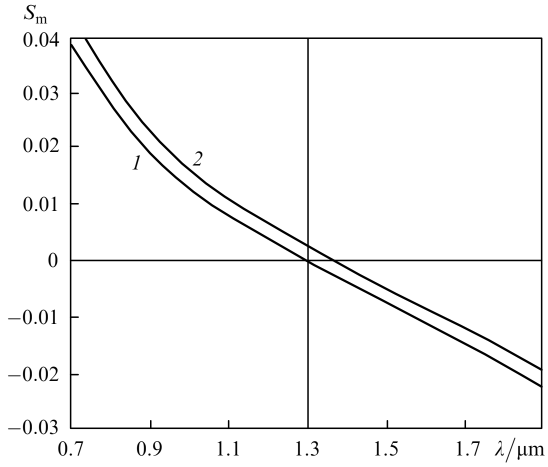

Figure 3. Wavelength dependences of the material dispersion for fibres 1 and 2.

cut-off to infinity. The values of B are virtually independent of the difference $\Delta n = n_1 - n_2$ under the condition that $\Delta n < 0.1$.

3. Chromatic dispersion of HE_{1m} modes

The chromatic dispersion coefficient calculated with the help of a rigorous expression [7]

$$S = \frac{1}{c}\frac{\mathrm{d}n_{\mathrm{gr}}}{\mathrm{d}\lambda} = -\frac{\lambda}{c}\frac{\mathrm{d}n_{\mathrm{eff}}^2}{\mathrm{d}\lambda^2} \tag{1}$$

depends on the wavelength and fibre parameters, i.e. the core diameter $2a$ and refractive indices of the core and cladding. Thus, $S = S[2a, \lambda, n_1(\lambda), n_2(\lambda)]$ is a function of many variables.

It is rather difficult and impossible sometimes to find the fibre parameters and wavelength λ_0 corresponding to the zero chromatic dispersion ($S = 0$) with the help of expression (1). Thus, it is convenient to make use of an approximate expression to calculate S at first, and then, after determining the ranges of V and λ where the zero chromatic dispersion is possible, to find exact fibre parameters and λ_0 using expression (1).

It is known [8] that for optical fibres with a relatively small difference in the refractive indices ($\Delta n < 0.1$) the chromatic dispersion coefficient can be approximated as a sum of the material (S_m) and waveguide (S_w) components:

$$S \approx -\frac{1}{\lambda c}(S_m + S_w) = -\frac{1}{\lambda c}\left[\lambda^2\frac{\mathrm{d}^2 n}{\mathrm{d}\lambda^2} + \Delta n V\frac{\mathrm{d}^2(VB)}{\mathrm{d}V^2}\right]. \tag{2}$$

The material dispersion coefficient S_m (material dispersion below) depends on fibre material properties and wavelength, and the waveguide dispersion coefficient S_w (waveguide dispersion below) depends only on Δn and $2a$. For $\lambda < 1.3$ μm, the material dispersion is positive, and for $\lambda > 1.3$ μm it is negative (Fig. 3). The waveguide

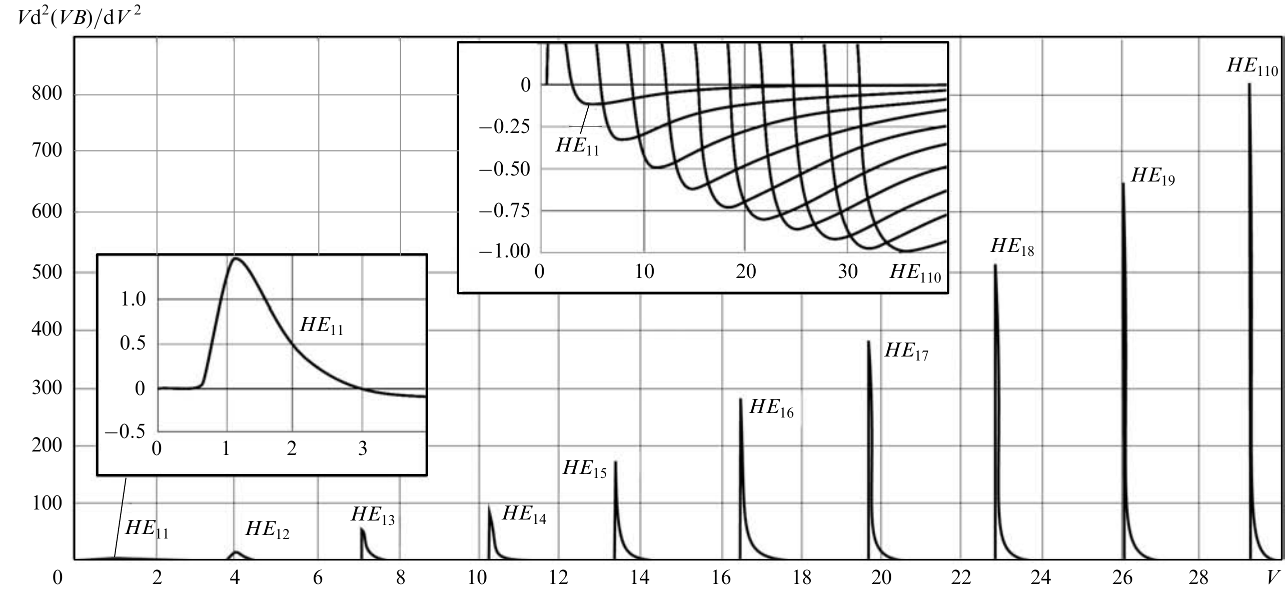

Figure 4. Normalised coefficient of the waveguide dispersion $Vd^2(VB)/dV^2$ as functions of V for HE_{1m} modes.

dispersion also has positive and negative regions (Fig. 4). If the fibre parameters are chosen well, the waveguide dispersion can compensate for the material dispersion.

Figure 4 presents the dependences of $V\mathrm{d}^2(VB)/\mathrm{d}V^2$ on V for the first ten HE_{1m} modes. For given Δn, these dependences allow one to find the waveguide dispersion S_w. One can see from the plots that for each HE_{1m} mode, $V\mathrm{d}^2(VB)\times\mathrm{d}V^{-2}$ grows with V from zero to a maximum value (the more, the higher the order m of the HE_{1m} mode), then decreases to achieve a minimum negative value (see the inset in Fig. 4) and then tends to zero at $V\to\infty$.

Note that for all HE_{1m} modes, there are such values of V (or $2a$) that permit material and waveguide dispersions to compensate for each other at the working wavelength λ_0, giving $S\approx 0$. With increasing m, the possibility to realise such compensation grows in both positive and negative regions of the material dispersion. Based on this approach, rough calculations showed that with a multimode two-layer optical fibre operating in a single-mode regime at relatively large differences in the refractive indices of the core and cladding ($\Delta n\approx 0.05$) and large orders ($m\geqslant 6$) of HE_{1m} modes, it is possible to have $S\approx 0$ for wavelengths as short as 0.7–0.8 μm. When $\lambda > 1.3$ μm, the condition $S\approx 0$ can hold true even for small Δn ($\Delta n < 0.01$).

It follows that to find the optimal fibre parameters ensuring $S = 0$, it is best to use rough formula (2) at first and then to refine these parameters using rigorous expression (1).

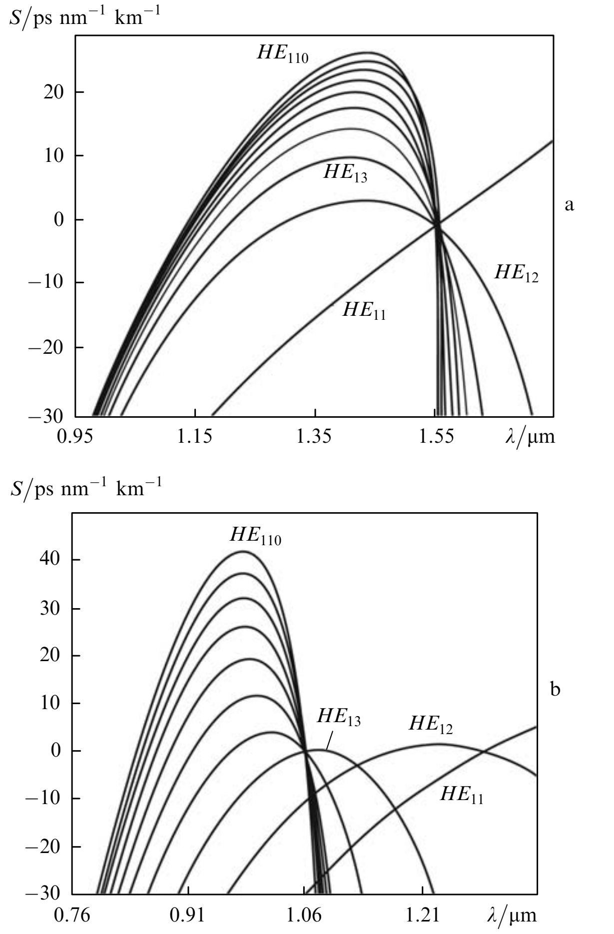

Figure 5. Wavelength dependences of the chromatic dispersion coefficient of HE_{1m} modes for fibres 1 (a) and 2 (b).

One can see from Fig. 5 that the coefficient S has two zero values for higher-order HE_{1m} modes ($m\geqslant 2$), the second value corresponding to a wavelength shorter than λ_0. Besides, for higher-order HE_{1m} modes it is possible for fibre 1 to obtain a small chromatic dispersion in the wavelength range lying between zero values of S and extending both to ranges below 1.3 μm and above 1.3 μm. It is especially the case with HE_{12}– HE_{14} modes for which $|S| < 10$ ps nm^{-1} km^{-1}.

Greater differences in Δn are required to make multimode two-layer optical fibres operate in a zero-chromatic-dispersion regime at shorter wavelengths. In particular, at $\lambda_0 = 1.06$ μm, Δn is equal to ~ 0.03 for fibre 2 (Fig. 5b); even in this case, the condition $S = 0$ could be fulfilled only for HE_{1m} modes with ($m\geqslant 3$).

4. Calculation results of the core diameters of multimode two-layer optical fibres to provide zero chromatic dispersion for the HE_{1m} modes

Table 1 presents the calculation results of the core diameter $2a$ of multimode two-layer optical fibres for first ten HE_{1m} modes for fibres 1 and 2 (the dash means that it is impossible to obtain zero chromatic dispersion for this particular mode). It follows from the table that the use of higher modes allows one to increase significantly the fibre core area and the effective mode area, which in turn permits increasing almost distortion-free transmission of higher laser-radiation powers. The calculations showed that the core diameter $2a$, for which $S = 0$ at the given wavelength, decreases with Δn.

Table 1 also presents the calculated values for the fraction of the power P_1 propagating in the fibre core [9]. The fraction of the power that travels in the cladding is $P_2 = 1 - P_1$. In the region of small V (close to the mode cut-off regime) the main fraction of the power propagates in the fibre cladding ($P_1\approx 0$). In the region away from the mode cut-off, almost all the power propagates in the core ($P_1\approx 1$). It follows from Table 1 that a major part of the power propagates in the fibre core only at higher modes ($m\geqslant 2$). Moreover, this part is larger at $\lambda_0 = 1.06$ μm than at 1.55 μm and grows with the radial index of the mode, reaching 90 %.

Table 1also presents the effective areas A_{eff} for HE_{1m} modes calculated with the help of the expression [10]

$$A_{\mathrm{eff}} = \frac{\left[\int_0^{2\pi}\int_0^{\infty}|E(r,\varphi)|^2 r\mathrm{d}r\mathrm{d}\varphi\right]^2}{\int_0^{2\pi}\int_0^{\infty}|E(r,\varphi)|^4 r\mathrm{d}r\mathrm{d}\varphi}, \tag{3}$$

where $E(r,\varphi)$ is the exact electric-field distribution across the fibre for the HE_{1m} mode. Calculations of A_{eff} for fibre 1 showed that it makes 111 μm^2 for the HE_{12} mode, which is ten times higher than the effective area of the fundamental HE_{11} mode and for the HE_{110} mode A_{eff} is 3683 μm^2. In the case of fibre 2, A_{eff} for HE_{1m} modes proved 6–8 times smaller.

5. Conclusions

Dispersion and energy parameters of HE_{1m} modes of a multimode two-layer step-index optical fibre have been studied. The fibre has been made to operate in a single-

Table 1. Values of $2a$, P_1 and A_{eff} at $\lambda_0 = 1.55$ μm for fibre 1 and at $\lambda_0 = 1.06$ μm for fibre 2 when $S = 0$.

Мода	Fibre 1			Fibre 2		
	$2a/\mu m$	P_1	$A_{\mathrm{eff}}/\mu m^2$	$2a/\mu m$	P_1	$A_{\mathrm{eff}}/\mu m^2$
HE_{11}	4.248	0.386	11.983	–	–	–
HE_{12}	15.936	0.708	110.713	–	–	–
HE_{13}	26.756	0.774	287.503	11.336	0.913	46.796
HE_{14}	37.496	0.811	543.411	14.578	0.906	76.757
HE_{15}	48.198	0.835	877.571	18.080	0.908	117.155
HE_{16}	58.878	0.852	1289.373	21.656	0.912	167.004
HE_{17}	69.542	0.865	1778.341	25.262	0.917	226.082
HE_{18}	80.192	0.876	2343.842	28.882	0.921	294.257
HE_{19}	90.834	0.884	2983.411	32.510	0.925	371.485
HE_{110}	101.468	0.892	3683.228	36.142	0.928	457.716

mode regime at one of HE_{1m} modes ($m \geqslant 2$). It has been shown that the use of multimode optical fibres operating at this working mode allow one to obtain zero chromatic dispersion in the wavelength range below and above 1.3 μm and nearly-zero chromatic dispersion (about 4 ps nm^{-1} km^{-1}) in wide enough wavelength range ($\Delta\lambda = 0.3 - 0.4$ μm). Large core diameters of these kinds of optical fibres allow almost distortion-free transmission of high-power optical signals.

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