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Study of a thermal lens in thin laser-ceramics discs

I.L. Snetkov, A.A. Soloviev, E.A. Khazanov

***Abstract*. Thermal distortions of the radiation phase are studied theoretically in laser ceramics. Special attention is paid to the small-scale phase modulation inherent in ceramics, which is caused by the arbitrary orientation of its single-crystal grains. Expressions are derived which describe the average phase distortion and its dispersion in disc elements in approximations of a thin disc cooled through optical surfaces and of weak heat exchange. The numerical calculation has confirmed the high accuracy of these expressions. The proposed approximate solutions of the heat conduction and elasticity equations are of their own importance. In particular, the obtained solutions can be used to describe phase and polarisation distortions of radiation in an arbitrarily oriented single-crystal disc.**

Keywords: *laser ceramics, thermal distortions of radiation, random small-scale phase modulation, disc optical elements.*

1. Introduction

The existing production technologies of laser ceramics allow manufacturing optical elements, which are not inferior to single-crystals in the purity of their chemical composition, thermal conductivity, linear expansion, etc. At the same time, ceramic elements can be fabricated with a large aperture [1], faster, have a better quality and, what is not the least of the factors, be much cheaper than single-crystal elements. Moreover, the viscosity of ceramics damage [2] and the parameter of the thermal damage in ceramics are much higher than those in a single-crystal [3]. In addition, it is possible to produce ceramics from materials, whose single-crystals cannot be grown at the present technological level (Y_2O_3, TAG, TSAG, etc.). All this generally explains the constant expansion in the field of applications of laser ceramics and the associated dynamic growth of the technological and production base for its manufacturing.

As was found earlier in papers [4 – 7], ceramics exhibits during heating a number of specific properties manifested in the small-scale inhomogeneity in the distribution of the phase and polarisation distortion across the beam cross section, which are especially noticeable in thin (disc) optical elements.

When considering thermal distortions of the radiation characteristics in disc optical elements made from ceramics, we encounter a number of peculiarities. First, the heat exchange with the environment mainly occurs through the end surfaces. As a result, the temperature distribution in the disc considerably differs from that in the rod. Second, the element surface deformation makes a noticeable contribution to the total phase distortions of radiation. Third, due to the small thickness of the disc ceramics element, there are few ceramic grains on the beam path, which leads to a significant inhomogeneity of the phase distortion [7].

The small-scale distortion modulation of the radiation phase and polarisation in ceramics is caused by the arbitrary polarisation of single-crystal grains. Hence, in the presence of the mechanical stress associated with the application of extraneous forces or with the temperature gradient, a photoelastic effect appears, which is determined for the specific point of the aperture by the orientations of all the grains located on the beam path. Note that during the transverse shift at a distance of the order of the grain size, a set of grains pierced through by the beam is replaced by another random set, which determines the transverse scale of the phase and polarisation modulation.

The authors of paper [7] considered ceramics in the form of a long cylinder (rod) and obtained analytic expressions for the phase distortion and its statistical parameters, i.e. the mathematical expectation and dispersion. In this case, they took into account the heat flux directed to the generatrix and neglected the flux through the endfaces. In addition, the contribution of deformation of the end surfaces was assumed low, which is quite reasonable in the case of the rod geometry.

In this paper, we study thermal effects in ceramic elements in the form of discs. We used two simplified analytic solutions for the heat conduction equation – in the case of a thin disc and during a weak heat exchange with the environment. Both these solutions together with the solution of the deformation equation in a disc allow one to derive, similarly to [7], expressions for the phase distortion and its statistical parameters. In addition, we took into account the deformation of the optical element surface, which makes it possible to determine the distortion parameters of radiation reflected, for example, from one of the faces.

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2. Formulation of the problem

Consider thermal phase distortions in a laser-ceramics cylinder of dimensons R_0 and $2l$ (Fig. 1). In solving the deformation and heat conduction equations we will assume that the thermal conductivity κ, the Poisson ratio υ and the linear expansion coefficient α_T of the ceramic medium and single-crystal coincide. This assumption was made based on papers [8, 9].

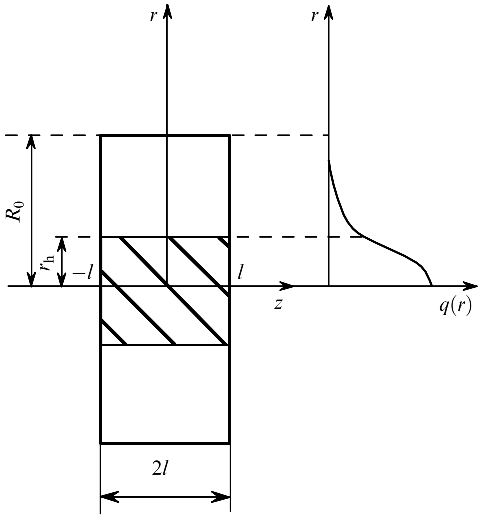

Figure 1. Disc optical element heated by a volume heat source $q(r)$.

Let us restrict ourselves to the case, when the density of the heat release power q depends not on the coordinate z and the polar angle φ but on the coordinate r:

$$q(r) = \frac{P_{\mathrm{h}}}{2l\pi r_{\mathrm{h}}^2 \int_0^{\rho} F(u)\mathrm{d}u} F(u), \tag{1}$$

where P_{h} is the power of the heat release in the entire sample; r_{h} and F are the radius and shape of the heat source; $u = (r/r_{\mathrm{h}})^2$; $\rho = (R_0/r_{\mathrm{h}})^2$; The problem symmetry with respect to the rotations around the cylinder axis allows one in the stationary heat conduction equation to neglect the derivative with respect to the temperature T along the coordinate φ:

$$\frac{\mathrm{d}^2 T}{\mathrm{d}z^2} + \frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}T}{\mathrm{d}r}\right) = -\frac{q(r)}{\kappa}. \tag{2}$$

Solution (2) in the form of series is known in the general case (it can be found, for example, in [10]), but it is not always convenient for calculating the specific temperature and, all the more, the corresponding temperature deformations. Therefore, of special interest are approximate solutions, which take into account the problem symmetry and the form of boundary conditions.

The inhomogeneous temperature distribution [the solution of equation (2)] leads to deformations, which in the isotropic case are described by the equation for the displacement vector $\boldsymbol{u}$ [11]:

$$\frac{3(1-\upsilon)}{1+\upsilon}\,\mathrm{grad\,div}\,\boldsymbol{u} - \frac{3(1-2\upsilon)}{2(1+\upsilon)}\,\mathrm{rot\,rot}\,\boldsymbol{u} = 3\alpha_T \nabla T. \tag{3}$$

The solution of Eqn (3) for the disc in the plane stressed state in the absence of the temperature dependence on z was considered in detail in [12], where the expressions for the components of the deformation tensor were obtained in cylindrical coordinates:

$$\varepsilon_{rr} = \alpha_T(1+\upsilon)\left[-\tilde{T}(r) + T(r) + \frac{1-\upsilon}{1+\upsilon}\tilde{T}(R_0)\right],$$

$$\varepsilon_{\varphi\varphi} = \alpha_T(1+\upsilon)\left[\tilde{T}(r) + \frac{1-\upsilon}{1+\upsilon}\tilde{T}(R_0)\right], \tag{4}$$

$$\varepsilon_{zz} = \alpha_T(1+\upsilon)\left[T(r) - \frac{2\upsilon}{1+\upsilon}\tilde{T}(R_0)\right],$$

where

$$\tilde{T}(r) = \frac{1}{r^2}\int_0^r T(r)r\mathrm{d}r. \tag{5}$$

Information on the temperature and stress tensor distributions is sufficient to determine the optical path distortions Ψ_{ph} and the polarisation change in an arbitrarily oriented ceramics grain [4, 5, 13] and, hence, in the entire ceramic element.

We will consider each ceramics grain as a phase plate in which both the direction of the intrinsic polarisations and the phase incursion for the 'fast' and 'slow' waves depend on the temperature T, deformations ε and Euler angles α, β, and φ specifying the direction of the crystallographic axes.

The directions of the crystallographic axes in each grain are assumed random with the following probability density functions for the Euler angles:

$$P_{\alpha} = \frac{1}{2\pi}, \quad P_{\beta} = \frac{1}{\pi}\cos\alpha, \quad P_{\varphi} = \frac{1}{2\pi}. \tag{6}$$

We assume that ε and T weakly change on the grain length and can be replaced by their average values. Then, in the absence of absorption the Jones matrix M_j of the jth grain has the form [14]:

$$M_j = \mathrm{e}^{\mathrm{i}\Psi_j}\times$$

$$\begin{pmatrix} \cos(\delta_j/2)+\mathrm{i}\cos(2\Theta_j)\sin(\delta_j/2) & \mathrm{i}\sin(\delta_j/2)\sin(2\Theta_j) \\ \mathrm{i}\sin(\delta_j/2)\sin(2\Theta_j) & \cos(\delta_j/2)-\mathrm{i}\cos(2\Theta_j)\sin(\delta_j/2) \end{pmatrix}, \tag{7}$$

where $\delta_j = (\Psi_{\mathrm{o}} - \Psi_{\mathrm{e}})$ is the phase difference of the 'fast' and 'slow' polarisation waves; Θ_j is the angle between the 'fast' polarisation wave and the axis x of the laboratory coordinate system. The quantities δ_j and Ψ_j are expressed via the tensor components of the dielectric impermeability ΔB_j in the form [4, 13]:

$$\delta_j = \pm lkn_0^3\left[(\Delta B_{11} - \Delta B_{22})^2 + 4\Delta B_{12}^2\right]^{1/2} = \frac{-lkn_0^3}{2}\frac{2\Delta B_{12}}{\sin 2\Theta_j},$$

$$\tan(2\Theta_j) = \frac{2\Delta B_{12}}{\Delta B_{11} - \Delta B_{22}},$$

$$\Psi_j = \frac{kn_0^3 l}{4}(\Delta B_{11} + \Delta B_{22}) + \beta Tkl + \varepsilon_{zz}(n_0 - 1)kl, \tag{8}$$

where k is the wave number; n_0 is the refractive index; the subscript j in the tensor components ΔB_{mn} is omitted.

Expressions for $\Delta B_{11} - \Delta B_{22}$, $\Delta B_{11} + \Delta B_{22}$ and ΔB_{12} via the deformation tensor components are obtained in [4].

The Jones matrix $M_{\rm tot}$ for the entire ceramic element is obtained by N Jones matrices of individual grains located in the beam path:

$$M_{\rm tot} = M_N M_{N-1} ... M_2 M_1 = M \exp\Big(\mathrm{i}\sum_j \Psi_j\Big). \tag{9}$$

There are no preferential directions in ceramics; therefore, without the loss of generality, we will assume that the field E_0 at the input is linearly polarised along the x axis in the laboratory coordinate system. Then,

$$\begin{pmatrix} E_x \\ E_y \end{pmatrix} = M_{\rm tot}\begin{pmatrix} E_0 \\ 0 \end{pmatrix}, \tag{10}$$

where E_x and E_y are the components of the Jones vector at the output from ceramics. The phase incursion $\Psi_{\rm tot}$ for radiation polarised along the x axis can be found via the matrix $M_{\rm tot}$:

$$\Psi_{\rm tot} = \arg(M_{\rm tot\,11}) = \sum_j \Psi_j + \arg(M_{11}) = \Psi_{\rm ph} + \Psi_{\rm pol}. \tag{11}$$

Thus, the phase incursion $\Psi_{\rm tot}$ is divided into two terms, i.e. the phase ($\Psi_{\rm ph}$) and polarisation ($\Psi_{\rm pol}$) distortions. The term $\Psi_{\rm ph} = \sum_j \Psi_j$ means the average phase distortion for any two mutually perpendicular polarisations. For ceramics with an arbitrary grain orientation the average value of the term is independent of the angle φ. If we want to obtain the phase distortion of radiation with the linear polarisation coinciding with the polarisation at the input to the ceramic element, it is necessary to take into account the second term, which depends on the angle φ even after averaging over the ceramics grains. In analytic consideration of statistical parameters of the phase distortions, the account for this term is rather problematic and hence, it is easier to consider this term numerically (see section 5). The dispersion and the average value for the term $\Psi_{\rm ph}$ can be found analytically in two approximations (see sections 3 and 4).

Local depolarisation $\Gamma = (|E_y|/|E_0|)$ (the fraction of the power in the polarisation perpendicular to the initial one) is defined by the matrix $M_{\rm tot}$:

$$\Gamma = |M_{\rm tot\,21}|^2. \tag{12}$$

For the quantitative estimate of polarisation distortions, the depolarisation

$$\gamma_{\rm d} = \frac{\int_S \Gamma(r,\varphi) E_0^2(r)\mathrm{d}s}{\int_S E_0^2(r)\mathrm{d}s} \tag{13}$$

is usually used, which is integral across the transverse cross section, where the integration is performed over the aperture S of the optical element.

Thus, expressions (7)–(13) make it possible to describe completely thermal distortions of the radiation parameters in ceramics, if the temperature and deformation in the optical element are known. For the disc geometry, these quantities can be found analytically within the framework of two approximations considered below.

3. A thin disc cooled through optical planes

In considering a very thin disc, the term $\mathrm{d}^2T/\mathrm{d}z^2$ in heat conduction equation (2) can be neglected. Together with the boundary conditions, for example, the convective heat exchange,

$$\mp\kappa\frac{\mathrm{d}T}{\mathrm{d}z}\bigg|_{z=\pm l} = \alpha_{\rm conv} T\big|_{z=\pm l} \tag{14}$$

($\alpha_{\rm conv}$ is the convective heat exchange coefficient), which leads to the solution:

$$T(r,z) = q(r)\left[\frac{l}{\alpha_{\rm conv}} + \frac{1}{2\kappa}\left(l^2 - z^2\right)\right]. \tag{15}$$

In practice, the temperature distribution of type (15) is realised in rather thin discs cooled intensely through the optical surfaces [15].

Let us estimate the domain of applicability of this solution. Consider the width of the temperature drop corresponding to the heat source of the form $q(r) = \Theta(r)$, where the function

$$\Theta(r) = \begin{Bmatrix} \text{const} & r \leqslant r_{\rm h} \\ 0 & r > r_{\rm h} \end{Bmatrix}.$$

Then, at $r \gg r_{\rm h}$, the temperature will be zero and at $r \ll r_{\rm h}$, the temperature will be equal to ΔT. The dimension of the transition region a is given by the expression

$$\kappa l\frac{\Delta T}{a} = \frac{a}{2}\Delta T \alpha_{\rm conv}, \tag{16}$$

which means the equality of the average heat flux released from the end surface of the transition region due to the heat exchange with the environment and the heat flux released long the r coordinate due to the thermal conductivity. To fulfil the thin disc approximation, it is necessary to have $r_{\rm h} \gg a$:

$$\left(\frac{2\kappa l}{\alpha_{\rm conv}}\right)^{1/2} \ll r_{\rm h}. \tag{17}$$

The comparison of the numerical calculations, which will be considered in detail below (see section 5) showed that the approximation under study well describes (the error does not exceed 10 %) thermal distortions in the case, when $r_{\rm h}$ exceeds by more than four times the left-hand side of expression (17).

In solving equation (3) for the temperature distribution of type (15), we failed to obtain simple expression for the deformation components. Hence, similarly to [15], we consider the case, when the dependence on the coordinate z is assumed weak. This allows us to perform averaging over this coordinate in expression (15):

$$T(r,z) = T(r) = q(r)\left(\frac{l}{\alpha_{\rm conv}} + \frac{l^2}{3\kappa}\right). \tag{18}$$

The procedure for determining the average value and dispersion of the phase incursion is described in [7]. Similarly, we obtain from (11) the expressions for the mathematical expectations of the phase distortion $\langle\Psi_{\rm ph}\rangle$ and its dispersion $D_{\Psi 1}$:

$$\frac{\langle\Psi_{\rm ph}\rangle}{2kl} = {\rm const}_{\rm disk} + P_{\rm disk}T(r) - Q_{\rm disk}(1-\xi)$$

$$\times\frac{11}{64}\left[T(r) - 2\tilde{T}(R_0)\right] + (n_0-1)\alpha_T(1+\upsilon)T(r), \quad (19)$$

$$D_{\Psi1} = 4Q_{\rm disk}^2(1-\xi)^2 l^2\left\{\left[\frac{265}{2^{15}}\left[T(r) - 2\tilde{T}(r)\right]^2\right.\right.$$

$$\left.+\frac{330}{2^{15}}\left[-T(r) + 2\tilde{T}(R_0)\right]^2\right]N_{\rm g}\langle l_{\rm g}\rangle^2\left(1 + \frac{D_{l_{\rm g}}}{\langle l_{\rm g}\rangle^2}\right)$$

$$\left.+\left(\frac{11}{64}\right)^2 N_{\rm g}D_{l_{\rm g}}\left[-T(r) + 2\tilde{T}(R_0)\right]^2\right\}, \quad (20)$$

where

$$Q_{\rm disk} = \alpha_T\frac{n_0^3}{4}(1+\upsilon)(p_{11} - p_{12});$$

$$P_{\rm disk} = \frac{{\rm d}n_0}{{\rm d}T} - \alpha_T\frac{n_0^3}{4}(1+\upsilon)(p_{11} + 3p_{12});$$

$\langle l_{\rm g}\rangle$ is the average grain length; $D_{l_{\rm g}}$ is the dispersion of the grain length; $Q_{\rm disk}$ and $P_{\rm disk}$ are thermooptic parameters of the medium (Q characterises thermally induced anisotropy and P – isotropic distortions); ξ is the optical anisotropy parameter introduced according to [4]. In (19) the temperature distribution is given by expression (18).

4. A disc in the case of a weak heat exchange

Under the conditions of a weak (for example, convective with the air) heat exchange, we will seek for the approximate solution of equation (2) in the form

$$T(r,z) = T_r(r) + az^2. \quad (21)$$

The substitution of (21) into (2) leads to the equation for $T_r(r)$, which is easily integrated. To determine the integration constants, we will use an assumption according to which the heat is released identically from all the surface points of the cylindrical sample:

$$\left.\frac{{\rm d}T}{{\rm d}\boldsymbol{\eta}}\right|_{\rm surf} = {\rm const}, \quad (22)$$

where $\boldsymbol{\eta}$ is the normal to the surface. In this case, the derivatives with respect to the temperature in the sample are defined by the expressions:

$$\frac{{\rm d}T}{{\rm d}r} = \frac{1}{4\pi\kappa l}\left[\frac{P_{\rm h}r}{R_0(R_0+2l)} - \frac{P_{\rm in}(r)}{r}\right], \quad (23)$$

$$\frac{{\rm d}T}{{\rm d}z} = -\frac{P_{\rm h}z}{2\pi\kappa R_0 l(R_0+2l)}, \quad (24)$$

where $P_{\rm in}(r) = 4\pi l\int_0^r xq(x){\rm d}x$ is the heat release power in a cylinder of radius r. Boundary conditions of type (14) and assumptions (22) can be satisfied, when two inequalities

$$\left|\frac{\Delta T_r}{T_{\rm surf}}\right| \ll 1, \quad \left|\frac{\Delta T_z}{T_{\rm surf}}\right| \ll 1 \quad (25)$$

are simultaneously fulfilled. Here, ΔT_r and ΔT_z are the temperature drops on the endfaces and the generatrix, respectively; $T_{\rm surf}$ is the average surface temperature of the sample. The quantity $T_{\rm surf}$ determines the total heat release power from the surface, which is equal in the stationary case to the heat release temperature $P_{\rm h}$ inside the sample. This implies that

$$T_{\rm surf} = \frac{P_{\rm h}}{2\pi\alpha_{\rm conv}R_0(R_0+2l)}. \quad (26)$$

The quantity ΔT can be found from (23) and (24):

$$\Delta T_r = \frac{\alpha_{\rm conv}R_0}{2\kappa l}\left(\frac{R_0}{2} - \frac{R_0+2l}{P_{\rm h}}\int_0^{R_0}\frac{P_{\rm in}}{r}{\rm d}r\right),$$

$$\Delta T_z = \frac{P_{\rm h}l^2}{4\pi\kappa R_0 l(R_0+2l)}. \quad (27)$$

Conditions (25) taking into account (26) and (27) take the form:

$$\left|\frac{\alpha_{\rm conv}R_0}{4\kappa l}\left[R_0 - (R_0+2l)I\right]\right| \ll 1, \quad \frac{l\alpha_{\rm conv}}{2\kappa} = {\rm Bi} \ll 1, \quad (28)$$

where Bi is the Bio similarity criterion and I is determined by the form of the heat source:

$$I = \int_0^{R_0}\left(\frac{P_{\rm in}(r)}{r}\right){\rm d}r. \quad (29)$$

It is easy to show that if the heat source has a Gaussian shape of width $r_{\rm h}$, the integral I is close to two at $R_0 = 2r_{\rm h}$. Note that for the disc geometry ($R_0 > l$) at $I > 1$, it is enough to fulfil only the first inequality in (28) because the second inequality will be fulfilled automatically. Solution (23), (24) can be used for cylindrical samples with an arbitrary length-to-radius ratio if Eqn (28) is fulfilled.

Consider appearing deformations. As was noted above, in the case of a disc, the deformation caused by $T(r,z) = T_r(r)$ is given by expression (4). The substitution of $T(r,z) = az^2$ into expression (3) in the case of a thin disc leads to the solution:

$$\varepsilon_{rr} = \varepsilon_{\varphi\varphi} = \alpha_T\frac{al^2}{3},$$

$$\varepsilon_{zz} = \alpha_T\frac{1+\upsilon}{1-\upsilon}az^2 + \alpha_T\frac{al^2}{3}\frac{2\upsilon}{\upsilon-1}. \quad (30)$$

Solution (30) together with solution (4) yields the solution of equation (3) for the disc with the temperature of type (21). Note that in papers [14, 15] considering the discs with $R_0 < 1$, the temperature dependence on the coordinate z was neglected.

In the weak heat release approximation, expression (19) for $\langle\Psi_{\rm ph}\rangle$ remains valid because the addition to deformation (30) does not affect $\langle\Psi_{\rm ph}\rangle$. Expression (20) for $D_{\Psi1}$ taking into account the dependence on z is supplemented with two terms:

$$D_{\Psi2} = D_{\Psi1} + 4Q_{\rm disk}^2(1-\xi)^2 l^2\left\{\frac{4}{(1-\upsilon)^2}\frac{1298}{2^{15}}\times\right.$$

$$\times\left\langle\sum_{j=1}^{N}\left[\int_{l_j}\left(\frac{4l^2}{3}-z^2\right)\mathrm{d}z\right]^2\right\rangle_{l_g}+\frac{4}{1-\upsilon}\frac{1298}{2^{15}}\left[-T_r(r)\right.$$

$$\left.+2\tilde{T}_r(R_0)\right]\left\langle\sum_{j=1}^{N}l_n\int_{l_j}\left(\frac{4l^2}{3}-z^2\right)\mathrm{d}z\right\rangle_{l_g}\Bigg\}, \quad (31)$$

where the integration is performed over the jth grain. The comparison with the numerical simulations showed that the contribution of these terms in the approximation under study is insignificant.

5. An arbitrary cylinder (numerical simulation)

For intermediate cases, when the described approximations and the approximations of the rod element [7] do not work, we used the program code [16, 17], which makes it possible to calculate the temperature and deformation of a cylindrical optical element with an arbitrary ratio of R and l. We used the following algorithm to find the average phase distortion and its phase. A random set of single-crystal grains with random lengths l_j and Euler angles α, β, γ was found on the path of the beam with the polar coordinates r and φ. In this case, the number of the grains on the beam path was also a random quantity with the mathematical expectation equal to N. Then, the Jones matrix M_j was calculated for each grain by using expressions (7) and (8). The deformation ε_{ij} inside one grain was assumed constant. From Eqn (9) we found the resultant matrix $M_{\rm tot}$, which together with expression (11) was used to determine the phase incursion $\Psi_{\rm tot}$. The average value of the phase and the phase dispersion were found with the help of the Monte-Carlo method.

6. Analysis of the results

The presence of both numerical and analytic solutions allows us, on the one hand, to verify reciprocally the obtained results and, on the other hand, to find the fields of applicability of thin disc approximations used in the analytic solution (see section 3) and disc approximations in the case of a weak heat exchange with the environment (see section 4).

For the heat conduction equation the thin disc approximation is well fulfilled when inequality (17) is fulfilled. For example, in a YAG disc element of thickness $2l = 4$ mm at $\alpha_{\rm conv} = 1000$ W m^{-2} K^{-1} (this heat exchange corresponds to cooling by water [18, 19]), the difference from the numerical solution does not exceed 10 % if the radius $r_{\rm h}$ of the heating Gaussian beam is greater than 4 cm. When $\alpha_{\rm conv}$ is decreased down to 100 W m^{-2} K^{-1}, which is achieved, for example, by the air blow-off [18, 19], the thin-disc approximation works well at $r_{\rm h} > 13$ cm.

The applicability of the weak heat exchange approximation requires condition (32) to be fulfilled. For example, for a YAG element at $R_0 = 2$ cm and convective heat exchange with air ($\alpha_{\rm conv} = 20$ W m^{-2} K^{-1}), it is necessary to have the length l in the range from 0.5 mm to 10 cm. When the sample radius R_0 decreases, restrictions on the length l become weaker.

In the deformation equation, the plane stressed state corresponds to our case when $R > 5l$. The same can be said about the adequacy of the addition to deformation (30), which takes into account the dependence on the coordinate z. Note that according to [12], to realise the plane stressed state, the temperature dependence in the sample on the coordinate z should be weak.

Consider, for example, the YAG ceramics in two cases. In the first case, $R_0 = 10$ mm, $2l = 4$ mm, $r_{\rm h} = 4$ mm, $P_{\rm h} = 100$ W and $\alpha_{\rm conv} = 100$ W m^{-2} K^{-1}; in the second case, $R_0 = 50$ mm, $2l = 4$ mm, $r_{\rm h} = 20$ mm, $P_{\rm h} = 100$ W and $\alpha_{\rm conv} = 1000$ W m^{-2} K^{-1}. Figure 2 presents the temperature averaged over the coordinate z. Because for further calculations only temperature gradients are important, we plotted the dependences $T(r) - T(0)$.

One can see from this figure that in the first case (Fig. 2a), solution (27) obtained in the weak heat exchange approximation is very close to the numerical solution. The solution obtained for the strong heat release through the optical surfaces, on the contrary, yields a large error, i.e. it is inapplicable for these parameters. In the second case (Fig. 2b), quite the contrary situation is observed, i.e. the more suitable is the approximation of cooling through optical surfaces within which solution (18) was obtained.

Consider the dependences for the deformation tensor components (Fig. 3). For comparison, they are plotted by subtracting the maximum value. Initially, the curves presented in one figure were at different heights, which corresponds to different homogeneous thermal expansion without stresses. The account for this expansion does not affect the final result.

As the temperature, the deformations in the first case are described more accurately in the weak heat exchange approximation and in the second case – in the approx-

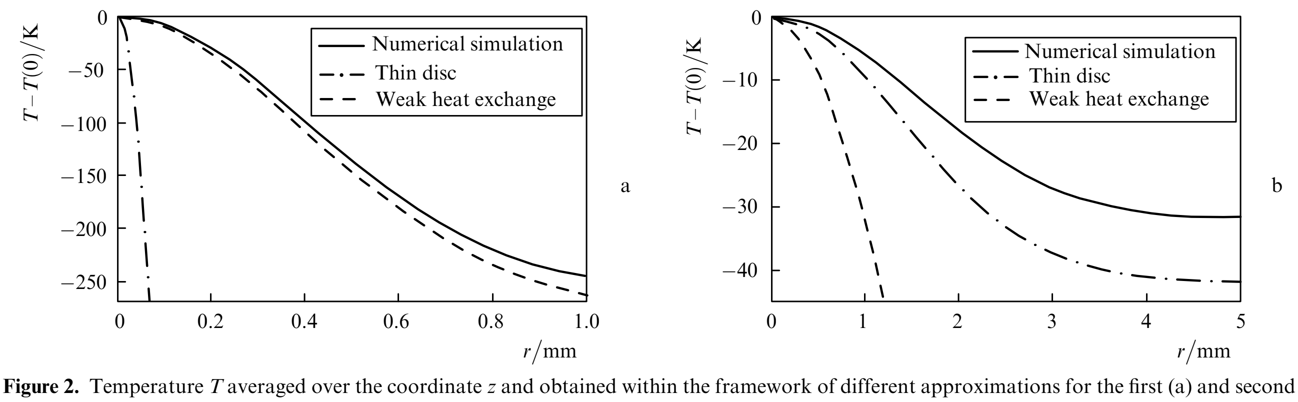

Figure 2. Temperature T averaged over the coordinate z and obtained within the framework of different approximations for the first (a) and second (b) cases of the heat exchange as a function of r.

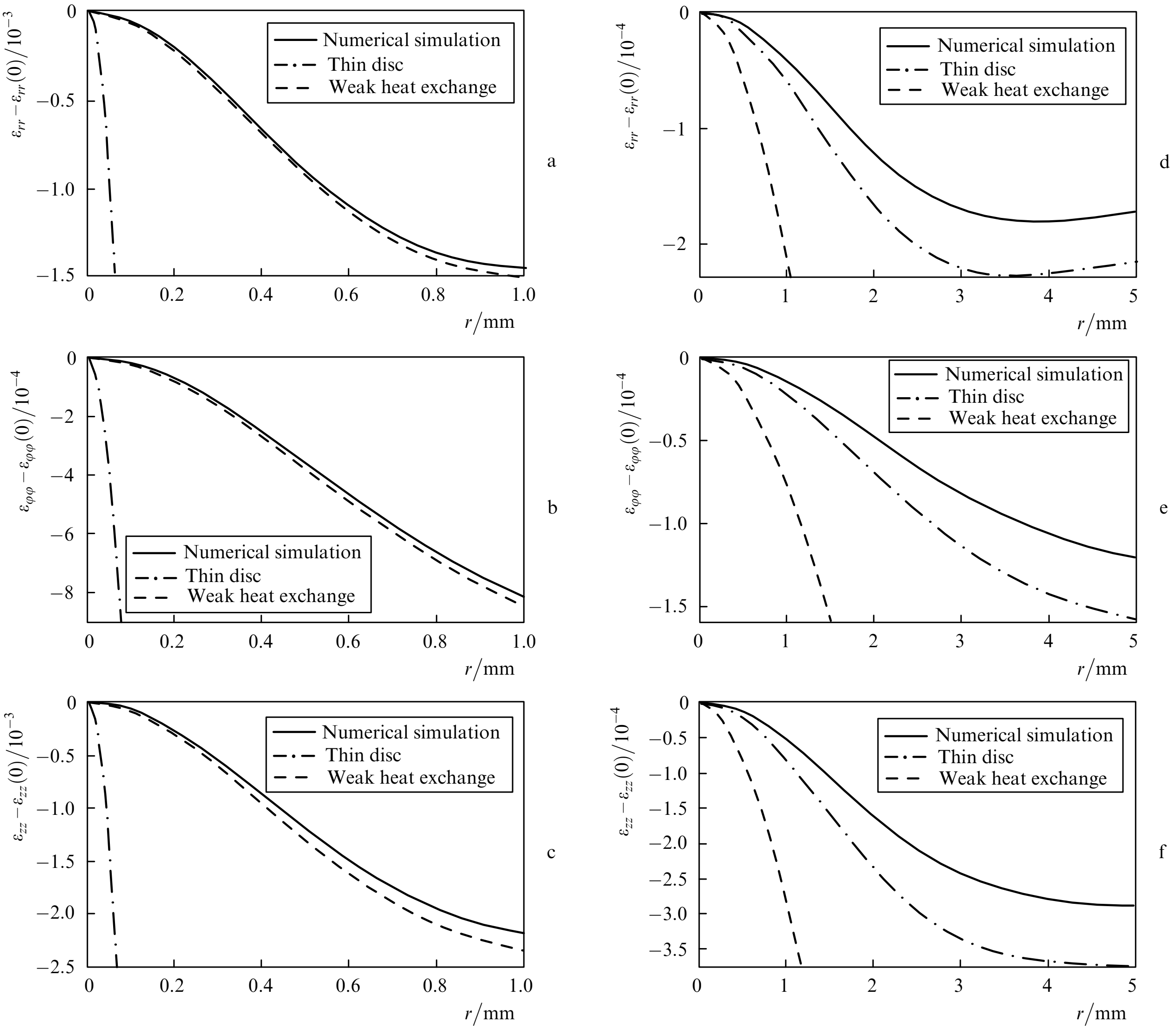

Figure 3. Components of the deformation tensor ε averaged over the coordinate z and obtained numerically (solid curves) and analytically in the weak heat exchange approximation (dashed curves) and in the planar-stressed state approximation (dot-and-dash curves) in the first (a – c) and second (d – f) cases of the heat exchange.

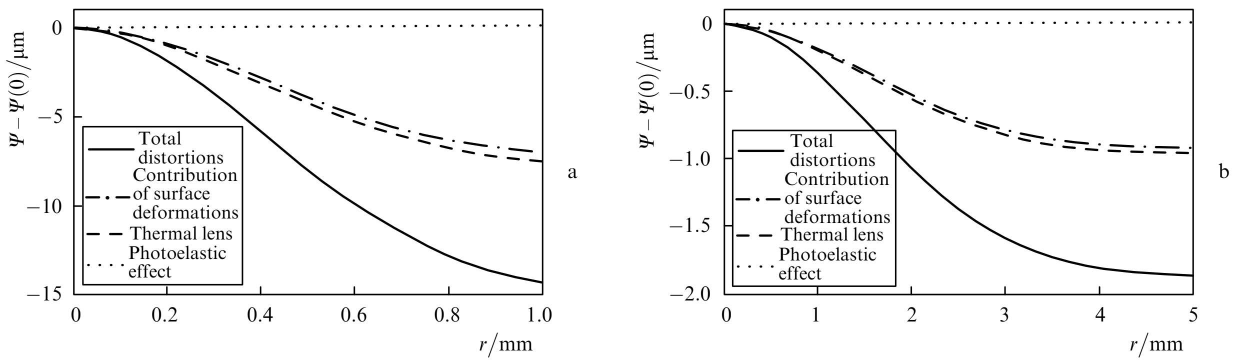

Figure 4. Phase distortion of propagated radiation averaged over the angle φ in the first (a) and second (b) cases. The solid curve shows total distortions, dash-and-dot curve – distortions caused by the surface deformation, dashed curve – the contribution dn/dT, dotted curve – the photoelastic effect.

imation of cooling through optical surfaces. Note that this assumption is valid also for linear combinations of the deformation tensor components.

Figures 4 and 5 present the dependences for the average value (Fig. 4) and root-mean-square deviation (Fig. 5) of the thermal phase distortion of radiation propagated through

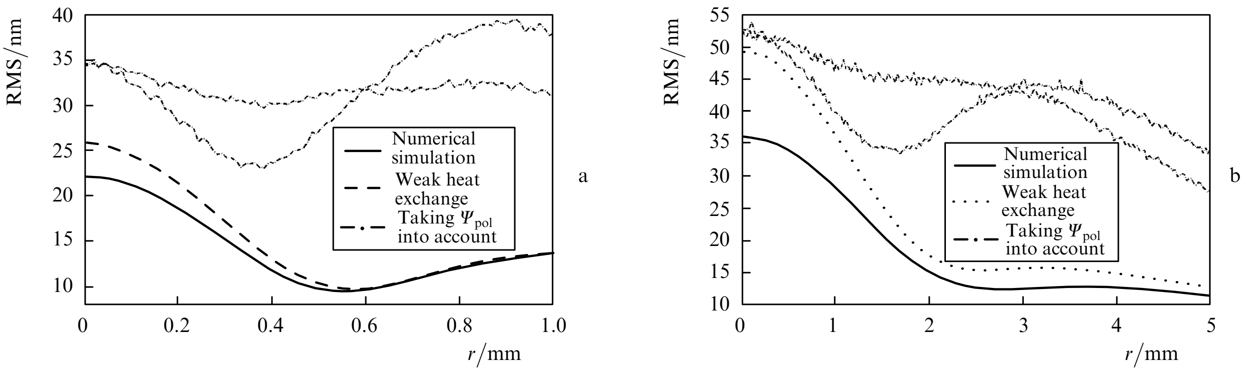

Figure 5. Root-mean-square phase distortions (RMS) from the average value as a function of the coordinate r, obtained numerically (solid curve) and analytically in the approximations of the weak heat exchange (dashed curve) (a) and thin disc cooled through optical surfaces (dotted curve) (b) for the first (a) and second (b) cases. The dot-and-dash curves show vertical and horizontal cross sections of the RMS surface obtained taking Ψ_{pol} into account.

the ceramic element. Apart from the dependence of the refractive index on the temperature and photoelastic effect, the geometric deformation of end surfaces also contributes to the φ-averaged mathematical expectation of the phase distortion. The contribution to the phase dispersion is made only by the photoelastic effect.

As was noted above, the contribution Ψ_{pol} in the left-hand side of expression (11) to the average value and dispersion of the phase distortion cannot be considered analytically, that is why we found it numerically. In this case, both the average value of the phase distortion and its dispersion acquired the dependence on the angular coordinate φ.

For comparison, Fig. 5 shows not only the curves for the root-mean-square deviation obtained numerically and analytically within the framework of approximations under study but also the curves corresponding to the vertical and horizontal cross sections when Ψ_{pol} is taken into account. As was above, the weak heat exchange approximation is more suitable in the first case, while the approximation of a thin disc cooled though the optical surface is more suitable in the second case.

One can see from Fig. 5 that the account for Ψ_{pol} in expression (11) for the YAG ceramics introduces additional dispersion depending both on the radius and the polar angle, and comparable with the dispersion Ψ_{ph}. Nevertheless, the characteristic quantity of the small-scale phase modulation can be obtained from analytic expression (19) and (31).

7. Conclusions

In this paper, we have obtained the analytic expressions for the phase distortion in disc ceramic elements taking into account the random small-scale inhomogeneity associated with the arbitrary orientation of single-crystal grains in ceramics. We have derived the expressions for the average value and dispersion of the phase distortion.

In particular, we have solved analytically the heat conduction equation in two different approximations and found the conditions for their applicability. These solutions allow one to calculate analytically the deformation and obtain expressions for the average phase distortion and its dispersion. These expressions and the domain of their applicability have been numerically verified. Together with the expressions obtained before for the rod elements they allow one to describe analytically the average value and dispersion of thermal distortions of the radiation phase in a large range of the parameters both for ceramic and single-crystal optical elements made of cylindrical cubic crystals.

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