SUPERSTRONG FIELDS

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On the motion of a charged particle in a plane monochromatic electromagnetic wave

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***Abstract*. The motion of a charged particle in the external specified field of a plane electromagnetic wave of large amplitude, when the relativistic consideration is required, is analysed in detail. The cases of different initial conditions for the motion of the charged particle and different polarisations of the wave are studied. It is shown that the expression for the kinetic energy of an electron oscillating in the transverse field of the wave, proposed in [1], is valid only in the nonrelativistic limit.**

Keywords: *plane electromagnetic wave, acceleration of charged particles, ultrashort laser pulse.*

1. Introduction

The problem of acceleration of charged particles upon the interaction of ultrashort laser pulses with plasma has been recently extensively studied, both experimentally and theoretically (see, for example, reviews [2, 3]).

The process of energy accumulation by electrons on the frontal surface of a target in the electromagnetic field of an incident laser pulse plays a key role in the acceleration of target ions. The authors of paper [1] proposed to estimate the temperature of rapid electrons on the frontal surface of a target by using the expression for the kinetic energy of an electron oscillating in the transverse field of the incident light wave:

$$\bar{K}_{\mathrm{e}} = m_{\mathrm{e}}c^2\left[\sqrt{1+\left(\frac{eE_0}{m_{\mathrm{e}}c\omega}\right)^2}-1\right]$$

$$= m_{\mathrm{e}}c^2\left[\sqrt{1+\frac{I\lambda^2}{1.37\times10^{18}}}-1\right], \quad (1)$$

where m_{e} is the electron mass; c is the speed of light; E_0 is the amplitude of the electric field of the incident electromagnetic wave; ω is its circular frequency; I is the incident wave intensity (in W cm^{-2}); and λ is the wavelength (in μm).

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In fact the authors of [1] have not presented the derivation of expression (1). It seems that they substituted into the known formula for the electron energy

$$\varepsilon = m_{\mathrm{e}}c^2\sqrt{1+\frac{p^2}{(m_{\mathrm{e}}c)^2}}$$

the expression for the amplitude of the momentum $p = eE/\omega$ of an electron oscillating in the field of a plane monochromatic electromagnetic wave in the nonrelativistic case. The subsequent papers (see, for example, [4 – 8]), where expression (1) is used for theoretical estimates and analysis of experimental results, also do not contain the derivation of this expression.

The aim of our paper is to derive consistently the expression for the energy of a particle averaged over its oscillation period in the field of a plane monochromatic wave. We will show that expression (1) is valid only in the nonrelativistic limit.

2. Motion of a particle in a plane monochromatic electromagnetic wave

The equation of motion of a particle with mass m and charge q in an electromagnetic field has the form (see, for example, [9], paragraph 17)

$$\frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t} = q\left(\boldsymbol{E}+\frac{1}{c}\,\boldsymbol{V}\times\boldsymbol{H}\right), \quad (2)$$

where the particle momentum $\boldsymbol{p}$ and velocity $\boldsymbol{V}$ are related by the equality ([9], paragraph 9)

$$\boldsymbol{p} = \frac{m\boldsymbol{V}}{\sqrt{1-V^2/c^2}}. \quad (3)$$

The change in the particle energy

$$\varepsilon = \frac{mc^2}{\sqrt{1-V^2/c^2}} = \sqrt{m^2c^4+c^2p^2} \quad (4)$$

is determined by the equation

$$\frac{\mathrm{d}\varepsilon}{\mathrm{d}t} = q\boldsymbol{E}\boldsymbol{V}. \quad (5)$$

It follows from (3) and (4) that the energy, momentum, and velocity of the particle are related by the equalities

$$\boldsymbol{p} = \frac{\varepsilon \boldsymbol{V}}{c^2}, \qquad \boldsymbol{V} = \frac{c^2 \boldsymbol{p}}{\varepsilon}. \tag{6}$$

For a plane monochromatic wave, we have

$$E_x = H_y = b_x \cos \Phi, \quad E_y = -H_x = \pm b_y \sin \Phi,$$
$$E_z = H_z = 0, \quad \Phi = \omega t - kz + \alpha = \omega\tau + \alpha, \tag{7}$$

where $k = \omega/c$; α is a constant phase ([9], paragraph 48); the z axis is directed along the wave propagation direction, while the x and y axes coincide with the b_x and b_y axes of the polarisation ellipse of the wave and $b_x \geqslant b_y \geqslant 0$. The upper (lower) sign in the expression for E_y corresponds to the right (left) polarisation.

The solution of equations (2) and (5) with $\boldsymbol{E}$ and $\boldsymbol{H}$ from (7) has the form

$$p_x = \chi_x + \frac{q b_x}{\omega} \sin \Phi, \qquad p_y = \chi_y \mp \frac{q b_y}{\omega} \cos \Phi,$$
$$p_z = \gamma g; \qquad \varepsilon = c\gamma(1+g), \tag{8}$$

where χ_x, χ_y, and γ are constants ($\gamma \geqslant 0$ because $\varepsilon \geqslant mc^2$);

$$g = h + \frac{q}{\gamma^2 \omega}(\chi_x b_x \sin \Phi \mp \chi_y b_y \cos \Phi)$$
$$- \frac{q^2}{4\gamma^2\omega^2}\left(b_x^2 - b_y^2\right)\cos 2\Phi; \tag{9}$$

$$h = \frac{1}{2}\left[\left(\frac{m^2c^2 + \chi_x^2 + \chi_y^2}{\gamma^2} - 1\right) + \frac{q^2}{2\gamma^2\omega 2}\left(b_x^2 + b_y^2\right)\right]. \tag{10}$$

From (8) and (9) we obtain the parametric representation (the parameter Φ) of the particle velocity:

$$V_x = \frac{\mathrm{d}x}{\mathrm{d}t} = \frac{c}{\gamma}\left(1 - \frac{V_z}{c}\right)\left(\chi_x + \frac{q b_x}{\omega}\sin\Phi\right)$$
$$= \frac{c}{(1+g)\gamma}\left(\chi_x + \frac{q b_x}{\omega}\sin\Phi\right),$$
$$V_y = \frac{\mathrm{d}y}{\mathrm{d}t} = \frac{c}{\gamma}\left(1 - \frac{V_z}{c}\right)\left(\chi_y \mp \frac{q b_y}{\omega}\cos\Phi\right) \tag{11}$$
$$= \frac{c}{(1+g)\gamma}\left(\chi_y \mp \frac{q b_y}{\omega}\cos\Phi\right),$$
$$V_z = \frac{\mathrm{d}z}{\mathrm{d}t} = c\,\frac{g}{1+g}.$$

Constants χ_x, χ_y, and γ are determined by the initial phase $\Phi_0 = -kz_0 + \alpha$ of the wave and the initial velocity $\boldsymbol{V}_0$ of the particle. From (11), (4) and (8), we find

$$\chi_x = -\frac{q b_x}{\omega}\sin\Phi_0 + \frac{m V_{x0}}{\sqrt{1 - V_0^2/c^2}},$$
$$\chi_y = \pm\frac{q b_y}{\omega}\cos\Phi_0 + \frac{m V_{y0}}{\sqrt{1 - V_0^2/c^2}}, \tag{12}$$
$$\gamma = mc\,\frac{1 - V_{z0}/c}{\sqrt{1 - V_0^2/c^2}}.$$

From (11) we determine the coordinates of the particle as functions of the parameter Φ:

$$x = x_0 + \frac{\chi_x}{\gamma k}(\Phi - \Phi_0) - \frac{q b_x}{\gamma\omega k}(\cos\Phi - \cos\Phi_0),$$
$$y = y_0 + \frac{\chi_y}{\gamma k}(\Phi - \Phi_0) \mp \frac{q b_y}{\gamma\omega k}(\sin\Phi - \sin\Phi_0), \tag{13}$$
$$z = z_0 + \frac{h}{k}(\Phi - \Phi_0) - \frac{q}{\gamma^2\omega k}\left[\chi_x b_x(\cos\Phi - \cos\Phi_0)\right.$$
$$\left.\pm \chi_y b_y(\sin\Phi - \sin\Phi_0)\right] - \frac{q^2}{8\gamma^2\omega^2 k}\left(b_x^2 - b_y^2\right)(\sin 2\Phi - \sin 2\Phi_0).$$

Let us show that the motion of the particle is the superposition of motion with a constant velocity $\boldsymbol{V}$ and oscillatory motion with frequency $\tilde\omega = 2\pi/\tilde T$ (different from the field frequency ω):

$$x(t) = \tilde x + \tilde V_x t + \xi(t), \qquad y(t) = \tilde y + \tilde V_y t + \eta(t),$$
$$z(t) = \tilde z + \tilde V_z t + \zeta(t), \tag{14}$$

where x, $\tilde y$, $\tilde z$ are constants and

$$\xi\left(t + \tilde T\right) = \xi(t), \quad \eta\left(t + \tilde T\right) = \eta(t), \quad \zeta\left(t + \tilde T\right) = \zeta(t) \tag{15}$$

are periodic functions with the same period.

We seek the solution of the equation for the coordinate z in (13) in form (14). By substituting $z(t)$ from (14) into (13) and selecting constants $\tilde z$ and $\tilde V_z$ in the form

$$\tilde z = z_0 + \left[\frac{q}{\gamma^2\omega k}(\chi_x b_x \cos\Phi_0 \pm \chi_y b_y \sin\Phi_0)\right.$$
$$\left. + \frac{q^2\left(b_x^2 - b_y^2\right)}{8\gamma^2\omega^2 k}\sin 2\Phi_0\right]\frac{1}{1+h}, \qquad \tilde V_z = \frac{ch}{1+h}, \tag{16}$$

we obtain the equation for $\zeta(t)$:

$$(1+h)\zeta(t) = -\frac{q}{\gamma^2\omega k}$$
$$\times\left[\chi_x b_x \cos\Phi \pm \chi_y b_y \sin\Phi + \frac{q\left(b_x^2 - b_y^2\right)}{8\omega}\sin 2\Phi\right]. \tag{17}$$

Because the right-hand side of Eqn (17) is a periodic function, the function $\zeta(t)$ is also periodic. Let us find its period. It is obvious from (17) that the period $\tilde T$ is determined by the equality $\Phi(t + \tilde T) = \Phi(t) + 2\pi$, from which it follows, taking (7), (14), and (15) into account, that

$$\tilde T = \frac{2\pi}{\omega}\,\frac{1}{1 - \tilde V_z/c} = \frac{2\pi}{\omega}(1+h). \tag{18}$$

One can see that the oscillation period of the particle differs from that of the field.

We will seek now the solution of the first equation in (13) in the form $x(t)$ from (14). By representing constants $\tilde x$ and $\tilde V_x$ in the form

$$\tilde{x} = x_0 + \frac{\chi_x}{\gamma}(z_0 - \tilde{z}) + \frac{qb_x}{\gamma\omega k}\cos\Phi_0,$$
$$\tilde{V}_x = \frac{\chi_x}{\gamma}c\left(1 - \frac{\tilde{V}_z}{c}\right) = \frac{\chi_x}{\gamma}\frac{c}{1+h}, \tag{19}$$

we find that

$$\xi(t) = -\frac{\chi_x}{\gamma}\zeta(t) - \frac{qb_x}{\gamma\omega k}\cos\Phi. \tag{20}$$

Similarly, we obtain for $y(t)$ in (14):

$$\tilde{y} = y_0 + \frac{\chi_y}{\gamma}(z_0 - \tilde{z}) \pm \frac{qb_y}{\gamma\omega k}\sin\Phi_0,$$
$$\tilde{V}_y = \frac{\chi_y}{\gamma}\frac{c}{1+h}, \qquad \eta(t) = -\frac{\chi_y}{\gamma}\zeta(t) \mp \frac{qb_y}{\gamma\omega k}\sin\Phi. \tag{21}$$

3. Motion of a particle averaged over the oscillation period

In this section we will perform the averaging of the coordinate $\boldsymbol{r}(t)$, velocity $\boldsymbol{V}(t)$, momentum $\boldsymbol{p}(t)$, and energy $\varepsilon(t)$ of a particle over its oscillation period $\tilde{T} = 2\pi/\tilde{\omega}$ in the wave field.

For the coordinate x in (13), we have

$$\bar{x}(t) = \frac{1}{\tilde{T}}\int_t^{\tilde{t}} x(t')\mathrm{d}t' = \left(x_0 - \frac{\chi_x}{\gamma k}\Phi_0 + \frac{qb_x}{\gamma\omega k}\cos\Phi_0\right)$$
$$+ \frac{\chi_x}{\gamma k\tilde{T}}\int_t^{\tilde{t}}\Phi(t')\mathrm{d}t' - \frac{qb_x}{\gamma\omega k\tilde{T}}\int_t^{\tilde{t}}\cos\Phi(t')\mathrm{d}t', \tag{22}$$

where

$$\tilde{t} \equiv t + \tilde{T} \tag{23}$$

and [see (7), (14) and (24)]

$$\Phi(t) = \omega t - k\left[\tilde{z} + \tilde{V}_z t + \zeta(t)\right] + \alpha$$
$$= \tilde{\omega}t - k\tilde{z} + \alpha - k\zeta(t). \tag{24}$$

By using (23) and (24), we obtain the expression

$$\int_t^{\tilde{t}}\Phi(t')\mathrm{d}t' = (\alpha - k\tilde{z} + \tilde{\omega}t)\tilde{T} - k\int_t^{\tilde{t}}\zeta(t')\mathrm{d}t' \tag{25}$$

for the first integral in the right-hand side of (22).

The integral in the right-hand side of (25) is independent of t,

$$\frac{\partial}{\partial t}\int_t^{\tilde{t}}\zeta(t')\mathrm{d}t' = \frac{\partial}{\partial t}\int_t^{t+\tilde{T}}\zeta(t')\mathrm{d}t' = \zeta\left(t+\tilde{T}\right) - \zeta(t) = 0, \tag{26}$$

because $\zeta(t)$ is a periodic function with a period $\tilde{T}$. This integral is the zero Fourier component of the function $\zeta(t)$ multiplied by $\tilde{T}$, or

$$\int_t^{\tilde{t}}\zeta(t')\mathrm{d}t' = \tilde{T}\bar{\zeta}, \tag{27}$$

where $\bar{\zeta}$ is the average value of the function $\zeta(t)$ in the time interval equal to the period $\tilde{T}$. Taking (27) into account, expression (25) can be transformed to

$$\int_t^{\tilde{t}}\Phi(t')\mathrm{d}t' = \left[\alpha - k\left(\tilde{z} + \bar{\zeta}\right)\right]\tilde{T} + 2\pi t. \tag{28}$$

Other quantities are obtained by calculating the integral

$$I(t) = \int_t^{\tilde{t}} f(t')\mathrm{d}t' \tag{29}$$

with the corresponding function $f(t')$. Let us introduce a new integration variable

$$\Phi' \equiv \Phi(t'), \quad \mathrm{d}t' = \frac{\mathrm{d}\Phi'}{\omega}\frac{1}{1 - V_z(t')/c} = \frac{1+g}{\omega}\mathrm{d}\Phi' \tag{30}$$

[we used (11) in the latter equality]. Then,

$$I(t) = \frac{1}{\omega}\int_{\Phi(t)}^{\Phi(\tilde{t})} f(\Phi')[1 + g(\Phi')]\mathrm{d}\Phi', \tag{31}$$

where, taking (23), (24), and (26) into account,

$$\Phi(\tilde{t}) = \Phi\left(t + \tilde{T}\right) = \tilde{\omega}t - k\tilde{z} + \alpha - k\zeta\left(t + \tilde{T}\right) + 2\pi, \tag{32}$$

i.e. $\Phi(\tilde{t}) = \Phi(t) + 2\pi$. The calculation of integral (29) with the function $\zeta(t)$ gives $\bar{\zeta} = 0$.

By returning to the integral in (22) and using (29), (31), (9) and (32), we have

$$\int_t^{\tilde{t}}\cos\Phi(t')\mathrm{d}t' = \frac{1}{\omega}\int_{\Phi(t)}^{\Phi(\tilde{t})}\cos\Phi'[1 + g(\Phi')]\mathrm{d}\Phi'$$
$$= \mp\frac{\pi q\chi_y b_y}{\gamma^2\omega^2}. \tag{33}$$

By substituting into (22) the values of integrals from (28) with $\bar{\zeta} = 0$ and (33), we obtain finally

$$\bar{x}(t) = \tilde{x} + \tilde{V}_x(t + \tilde{T}/2) \pm \frac{cq^2\chi_y b_x b_y}{2\gamma^3\omega^3(1+h)}, \tag{34}$$

where $\tilde{x}$ and $\tilde{V}_x$ are defined by expressions (19).

In the same way, we find

$$\bar{y}(t) = \tilde{y} + \tilde{V}_y(t + \tilde{T}/2) + \bar{\eta}, \tag{35}$$

where $\tilde{y}$ and $\tilde{V}_y$ are defined by expressions in (21), and the average value of the periodic function $\eta(t)$ is described by the function

$$\bar{\eta} = \mp\frac{cq^2\chi_x b_x b_y}{2\gamma^3\omega^3(1+h)}. \tag{36}$$

Finally, taking into account that $\bar{\zeta} = 0$, the expression for $\bar{z}(t)$ takes the form

$$\bar{z}(t) = \tilde{z} + \tilde{V}_z(t + \tilde{T}/2)9, \tag{37}$$

where $\tilde{z}$ and $\tilde{V}_z$ are defined by expressions (16).

Consider now the particle velocity (11). By using (29), (31), and (9), we find that

$$\bar{V}_x = \tilde{V}_x, \qquad \bar{V}_y = \tilde{V}_y, \qquad \bar{V}_z = \tilde{V}_z, \tag{38}$$

i.e., as expected, the particle velocity averaged over the period coincides with $\tilde{\boldsymbol{V}}$, which can be found from expressions (19), (21) and (16).

For the particle momentum $\boldsymbol{p}$ (8), we obtain similarly the expressions

$$\bar{p}_x = \chi_x\left[1 + \frac{q^2 b_x^2}{2\gamma^2\omega^2(1+h)}\right], \quad \bar{p}_y = \chi_y\left[1 + \frac{q^2 b_y^2}{2\gamma^2\omega^2(1+h)}\right],$$

$$\bar{p}_z = \frac{\gamma}{1+h}\left\{h + h^2 + \frac{1}{2}\left(\frac{q}{\gamma^2\omega}\right)^2 \times\left[(\chi_x b_x)^2 + (\chi_y b_y)^2\right] + \frac{1}{32}\left(\frac{q}{\gamma\omega}\right)^4 (b_x^2 - b_y^2)^2\right\}. \tag{39}$$

This gives the energy ε [see (8)]

$$\bar{\varepsilon} = \frac{c\gamma}{1+h}\left\{(1+h)^2 + \frac{1}{2}\left(\frac{q}{\gamma^2\omega}\right)^2 \times\left[(\chi_x b_x)^2 + (\chi_y b_y)^2\right] + \frac{1}{32}\left(\frac{q}{\gamma\omega}\right)^4 (b_x^2 - b_y^2)^2\right\}, \tag{40}$$

where b_x and b_y are the field amplitudes [see (7)] and parameters γ, χ_x, and χ_y are defined in (12).

It is clear that $\bar{\varepsilon}$ depends on the wave intensity, its polarisation, initial phase, and the initial velocity of the particle.

4. Cases of the circular and linear polarisations of a wave for a particle initially at rest

Consider the case when a particle is initially at rest ($\boldsymbol{V}_0 = 0$). Then, according to (12),

$$\gamma = mc, \quad \chi_x = -\frac{qb_x}{\omega}\sin\Phi_0, \quad \chi_y = \pm\frac{qb_y}{\omega}\cos\Phi_0. \tag{41}$$

For a wave with the circular polarisation, we have $b_x = b_y = b/\sqrt{2}$. In this case,

$$\gamma = mc, \qquad \chi_x = -\frac{qb\sin\Phi_0}{\omega\sqrt{2}}, \qquad \chi_y = \pm\frac{qb\cos\Phi_0}{\omega\sqrt{2}},$$

$$(\chi_x b_x)^2 + (\chi_y b_y)^2 = \left(\frac{qb^2}{2\omega}\right)^2 \tag{42}$$

and, according to (10),

$$h = \frac{1}{2}\left(\frac{qb}{mc\omega}\right)^2 = \frac{1}{2}\left(\frac{2q^2}{\pi m^2 c^5} I\lambda^2\right) \equiv \frac{\mu}{2}, \tag{43}$$

where $I = cb^2/(8\pi)$ is the wave intensity and $\lambda = 2\pi c/\omega$ is the wavelength.

The oscillation period (18) of a particle in this case is

$$\tilde{T} = T(1+h) = T\left(1 + \frac{\mu}{2}\right). \tag{44}$$

By substituting (42) and (43) into (40), we obtain the average energy for a particle initially at rest in the circularly polarised wave:

$$\bar{\varepsilon} - mc^2 = \frac{1}{2}mc^2\mu\left(1 + \frac{\mu}{4+2\mu}\right). \tag{45}$$

One can see from (44) and (45) that the oscillation period of a particle and its average energy are independent of the initial phase of the wave.

In the case of linear polarisation, $b_x = b$, $b_y = 0$ [for a particle initially at rest, see (41)]; in this case,

$$\gamma = mc, \quad \chi_x = -\frac{qb\sin\Phi_0}{\omega}, \quad \chi_y = 0,$$

$$(\chi_x b_x)^2 + (\chi_y b_y)^2 = \left(\frac{qb^2}{\omega}\right)^2 \sin^2\Phi_0 \tag{46}$$

and, according to (10),

$$h = \frac{1}{4}\left(\frac{qb}{mc\omega}\right)^2 (1 + 2\sin^2\Phi_0) = \frac{\mu}{4}(1 + 2\sin^2\Phi_0). \tag{47}$$

The oscillation period of a particle is

$$\tilde{T} = T\left[1 + \frac{\mu}{4}(1 + 2\sin^2\Phi_0)\right]. \tag{48}$$

By substituting (46) and (47) into (40), we obtain the dependence of the average energy for a particle initially at rest in the linearly polarised wave:

$$\bar{\varepsilon} - mc^2 = \frac{1}{4}mc^2\mu\left[1 + 2\sin^2\Phi_0 + \frac{\mu(1/8 + 2\sin^2\Phi_0)}{1 + (1/4)\mu(1 + 2\sin^2\Phi_0)}\right]. \tag{49}$$

The maximum average energy is obtained for the phase $\Phi_0 = \pi/2$ or $3\pi/2$, when the field at the point where a particle is located initially is zero. In this case, we have

$$\bar{\varepsilon} - mc^2 = \frac{3}{4}mc^2\mu\left(1 + \frac{17\mu}{24 + 18\mu}\right). \tag{50}$$

The minimum average energy corresponds to the phase $\Phi_0 = 0$ or π and is determined by the expression

$$\bar{\varepsilon} - mc^2 = \frac{1}{4}mc^2\mu\left(1 + \frac{\mu}{8+2\mu}\right). \tag{51}$$

Finally, the energy of a charged particle averaged over the initial phase Φ_0 in the field of a plane monochromatic linearly polarised wave has the form

$$\langle\bar{\varepsilon}\rangle - mc^2 = \frac{1}{4}mc^2\mu\left(6 - \frac{32 + 7\mu}{2\sqrt{4+3\mu}\sqrt{4+\mu}}\right). \tag{52}$$

Figure 1 presents the dependences of the average kinetic electron energy on the intensity of a plane monochromatic linearly (52) and circularly (45) polarised electromagnetic wave and the energy calculated by expression (1). One can see that expression (1) gives considerably lower average electron energies in the electromagnetic field. For $I\lambda^2 > 4.5 \times 10^{18}$ W μm^2 cm^{-2}, these values are more than twice lower than the values calculated by expression (45) and more than 2.5 times lower than the values calculated from expression (52). The average energies prove to be com-

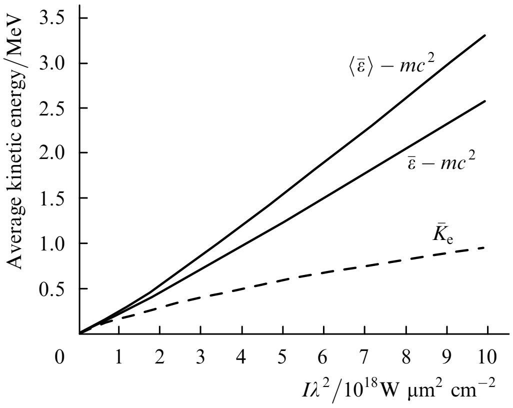

Figure 1. Dependences of the average kinetic electron energy on the intensity of linearly (52) and circularly (45) polarised plane monochromatic electromagnetic waves and energy calculated by expression (1).

parable only in the nonrelativistic limit, when their difference does not exceed 10 % for $I\lambda^2 < 2 \times 10^{17}$ W μm^2 cm^{-2}.

5. Conclusions

We have analysed in detail the motion of a charged particle in the external field of a plane electromagnetic wave. The motion of the particle has been studied for different initial conditions and different polarisations of the wave.

It has been shown that the motion of the particle is the superposition of motion at a constant velocity and oscillatory motion at a frequency different from the field frequency. As the field intensity is increased, the frequency of the oscillatory motion of the particle tends to zero according to (18). The velocity, momentum, and energy of the particle averaged over its oscillation period have been calculated. A comparison of the expressions obtained in the paper for the average energy of a charged particle in the field of linearly and circularly polarised plane monochromatic waves with expression (1) shows that the latter is valid only in the nonrelativistic limit.

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