Quantum Electronics **39** (8) 759 – 764 (2009)

RESONATORS

PACS numbers: 42.60.Da; 42.55.Rz; 42.55.Xi
DOI: 10.1070/QE2009v039n08ABEH013980

Symmetry of the spatial structure of radiation upon transverse mode locking in an astigmatic resonator laser

V.V. Bezotosnyi, M.V. Gorbunkov, P.V. Kostryukov, V.G. Tunkin, E.A. Cheshev, D.V. Yakovlev

Abstract. **The influence of the astigmatic resonator parameters on the symmetry of the spatial structure of the radiation intensity is analysed upon transverse mode locking in a nonuniformly pumped laser. Conditions for the transition from the circular symmetry to its violation are found. At a fixed astigmatism of the resonator, the symmetry is determined, first of all, by the resonator length and losses. The theoretical conclusions are confirmed by the experiments with diode end-pumped Nd : YAG and Nd : YLF lasers.**

Keywords: *diode pumping, cavities, transverse mode locking.*

1. Introduction

The use of longitudinal diode pumping of solid-state lasers provides a high lasing efficiency due to the formation of an inhomogeneous gain profile [1]. The authors of papers [2 – 5] studied the spatial structure of radiation from cw diode-end-pumped Nd :YVO_4 and Nd : YAG lasers. It was found that when pumping is performed by a beam that is narrow compared to the transverse dimension of the Gaussian mode of an empty resonator, there exists a discrete set of the so-called critical resonator configurations, for which the spatial structure of the output radiation considerably differs from the Gaussian. The existence of critical configurations is related to the degeneracy of empty resonator modes with respect to the frequency [6], which is realised under the condition

$$\frac{\arccos \pm\sqrt{g_1 g_2}}{\pi} = \frac{r}{s}, \tag{1}$$

where r/s is an irreducible fraction; $g_{1,2} = 1 - L/R_{1,2}$ are the parameters of the resonator configuration; L is the resonator length; and $R_{1,2}$ are the radii of curvature of the mirrors. The spatial inhomogeneity of the pump, when condition (1) is fulfilled, leads to transverse mode locking [7, 8]. In this case, the fundamental mode represents a superposition of phase-locked degenerate modes of an empty resonator. The authors of the above papers used relatively short resonators formed by a spherical output mirror with the radius of curvature $R \leqslant 15$ cm, the transmission $T < 15\,\%$ and a plane mirror deposited on the laser crystal face. The fundamental mode produced in this case has the intensity distribution with a characteristic ring structure and represents a superposition of Laguerre – Gaussian modes of the empty resonator.

Transverse mode locking in a longitudinally nonuniformly pumped Nd : YAG laser having an output mirror with $R = 150$ cm and $T < 1\,\%$ in the case of astigmatism in the active medium was studied in [9]. It was found that the symmetry of the spatial radiation structure differs from the circular one. For each of 15 appearing degeneracies r/s, the presence of two resonator lengths – $L_x^{\rm deg}$ and $L_y^{\rm deg}$ was established for which the intensity distributions stretch in two mutually perpendicular directions, thus representing a superposition of Hermite – Gaussian modes. The violation of the circular symmetry found in [9] is explained by the presence of weak astigmatism in the active medium, i.e. the inequality of optical forces D_x and D_y of the laser crystal, which is obviously caused by the breakdown of the surface flatness during the crystal treatment and/or by the nonuniform distribution of the refractive index in it. It was established in [9] that the inequality of optical forces D_x, D_y of the crystal leads to the fact that the degeneracy conditions for Hermite – Gaussian modes HG_{m0} and HG_{0n} with nonzero first and second subscripts are realised separately at lengths $L_x^{\rm deg}$ and $L_x^{\rm deg}$, respectively. This splitting of degeneracy violating the circular symmetry made it possible to measure experimentally the astigmatism of laser crystals. The optical forces D_x, D_y characterising the astigmatism of the active medium of five different Nd : YAG crystals studied in [9] fall into the region $-0.03\ldots+0.2$ m^{-1}. It was noted that these values of astigmatism do not result in a noticeable ellipticity of the Gaussian intensity distributions in the case of sufficient detuning from the degeneracy: the calculated ratio of the beam radii at the output mirror is $w_x/w_y \sim 1.01$.

Thus, we can single out two characteristic cases realised upon transverse mode locking in an astigmatic resonator laser: the intensity distribution with a ring structure [2 – 5] and the degeneracy splitting leading to the violation of the circular symmetry [9]. In this paper, we determine the conditions for realisation of both cases.

V.V. Bezotosnyi, M.V. Gorbunkov, E.A. Cheshev P.N. Lebedev Physics Institute, Russian Academy of Sciences, Leninsky prosp. 53, 119991 Moscow, Russia;
e-mail: victorbe@sci.lebedev.ru, gorbunk@sci.lebedev.ru;
P.V. Kostryukov, V.G. Tunkin, D.V. Yakovlev Department of Physics, M.V. Lomonosov Moscow State University, Vorob'evy gory, 119991 Moscow, Russia; e-mail: kostryukov@gmail.com

Received 15 October 2008; revision received 13 March 2009
Kvantovaya Elektronika **39** (8) 759 – 764 (2009)
Translated I.A. Ulitkin

2. Conditions for realisation of the circular symmetry and its violation

Consider a laser whose thin active medium has the optical forces D_x and D_y. The highly reflecting mirror is deposited on the face, through which pumping is performed; the output spherical mirror has a radius of curvature R and the transmission coefficient T. The radii of mirrors will be assumed large to provide the existence of a sufficient number of modes in the resonator. As in the case of axial symmetry, we will introduce the g parameters of the resonator

$$g_1^x = 1 - D_x \frac{L}{2}, \quad g_1^y = 1 - D_y \frac{L}{2}, \quad g_2^x = g_2^y = 1 - \frac{L}{R}. \qquad (2)$$

Consider the vicinity of the point corresponding to $L_0 = R[1 - \cos^2(\pi r/s)]$ (Fig. 1). Here, L_0 is the resonator length responsible for the degeneracy r/s at $D_x = D_y = 0$. The inequality of optical forces ($D_x \neq D_y$) leads to slitting of L_0 into $L_y^{\rm deg}$ and $L_x^{\rm deg}$ for which degeneracy condition (1) is fulfilled for $g_1^x g_2^x$ and $g_1^y g_2^y$ separately:

$$\frac{\arccos \pm\sqrt{g_1^x g_2^x}}{\pi} = \frac{r}{s}, \quad \frac{\arccos \pm\sqrt{g_1^y g_2^y}}{\pi} \neq \frac{r}{s} \quad \text{for} \quad L = L_x^{\rm deg},$$

$$\frac{\arccos \pm\sqrt{g_1^x g_2^x}}{\pi} \neq \frac{r}{s}, \quad \frac{\arccos \pm\sqrt{g_1^y g_2^y}}{\pi} = \frac{r}{s} \quad \text{for} \quad L = L_y^{\rm deg}.$$

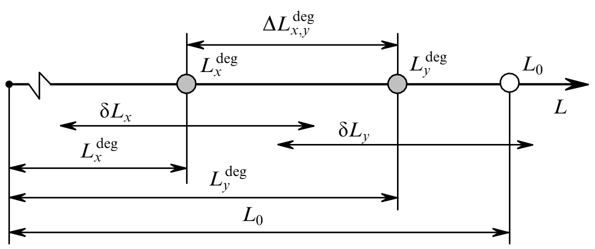

Figure 1. Characteristic lengths of the resonator near the degeneracy: L_0 is the length corresponding to $D_x = D_y = 0$; $L_x^{\rm deg}$ and $L_y^{\rm deg}$ are the lengths corresponding to the degeneracy of modes HG_{m0} and HG_{0n} at $D_x \neq D_y$; $\delta L_{x,y}$ are the effective widths of mode-locking regions.

Thus, at $L = L_x^{\rm deg}$ the Hermite–Gaussian modes HG_{m0} with $m = \Delta m j$ are degenerate and at $L = L_y^{\rm deg}$ – HG_{0n} modes with $n = \Delta n j$, where $j = 0, 1, 2 \ldots$; Δm and Δn are equal to each other and take the values s or $2s$ for even or odd s, respectively (see, for example, [10]). Equations (1) and (2) yield the expression relating the distance between the points $\Delta L_{x,y}^{\rm deg} = L_x^{\rm deg} - L_y^{\rm deg}$ with the optical forces $D_{x,y}$:

$$\Delta L_{x,y}^{\rm deg} \equiv \left|L_x^{\rm deg} - L_y^{\rm deg}\right| \approx \frac{1}{8} R^2 \sin^2\left(2\pi \frac{r}{s}\right)|D_x - D_y|$$

$$= \frac{1}{2} L_0^2 \cot^2\left(\pi \frac{r}{s}\right)|D_x - D_y|. \qquad (3)$$

This expression with an accuracy to notation coincides with the expression derived in [11]. Note that $\Delta L_{x,y}^{\rm deg}$ is proportional to the square of L_0 and, hence, the longer the resonator, the more pronounced the astigmatism.

We will assume that the effective radius of the pump beam in the active medium $w_{\rm p}$ is rather small to provide transverse mode locking [12]. The transverse modes of the HG_{m0} and HG_{0n} families are mode locked within some range of the resonator lengths with the centres at points corresponding to the lengths $L_x^{\rm deg}$ and $L_y^{\rm deg}$ with the widths δL_x and δL_y. Below, the mode-locking region width $\delta L_{x,y}$ is treated as the width of the detuning range from the strict degeneracy within which the power fraction per higher modes is less than half the value, which is realised in the case of strict degeneracy.

To estimate the widths of the mode-locking regions δL_x and δL_y, we will use the results obtained by the authors of [12]. The authors of paper [12] obtained, for the case of mode locking of transverse axially symmetric Laguerre–Gaussian modes (with the zero angular mode number), an estimate of the width of the mode-locking region in the coordinates g_1, g_2 {expression (23) in [12]}:

$$\Delta(g_1 g_2)^{\rm LG} = 2\sin\left(2\pi \frac{r}{s}\right)\left[2\Delta p\left(\frac{t_{\Delta p,\Delta p}\gamma_{\Delta p}}{t_{0,0}\gamma_0}\right)^{1/2}\right]^{-1}$$

$$\times\left(1 - \frac{\gamma_{\Delta p}}{\gamma_0}\frac{t_{\Delta p,\Delta p}}{t_{0,0}}\right), \qquad (4)$$

where Δp is the minimum difference in the values of the radial number of degenerate modes; the quantities $1 - \gamma_0^2$ and $1 - \gamma_{\Delta p}^2$ determine by the diffraction losses of the modes; $t_{0,0}$, $t_{\Delta p,\Delta p}$ are the matrix elements of the gain profile in the basis of Laguerre–Gaussian modes. Expression (4) yields the mode-locking region widths, which differ by more than 10 % from the widths obtained by the Fox–Li method. It was shown in [12] that the widths of the mode-locking region increase with increasing the gain and, hence, with increasing the resonator losses, and are virtually independent of the relative diameter of the pump beam (see Figs 1 and 2 in [12]). To apply expression (4) to the case of mode locking of Hermite–Gaussian modes of the HG_{m0} (HG_{0n}) family, it is necessary to replace the matrix elements $t_{0,0}$ and $t_{\Delta p,\Delta p}$ by analogues quantities $t_{00,00}$ and $t_{\Delta m0,\Delta m0}$ ($t_{00,00}$ and $t_{0\Delta n,0\Delta n}$) determined in the basis of Hermite–Gaussian functions:

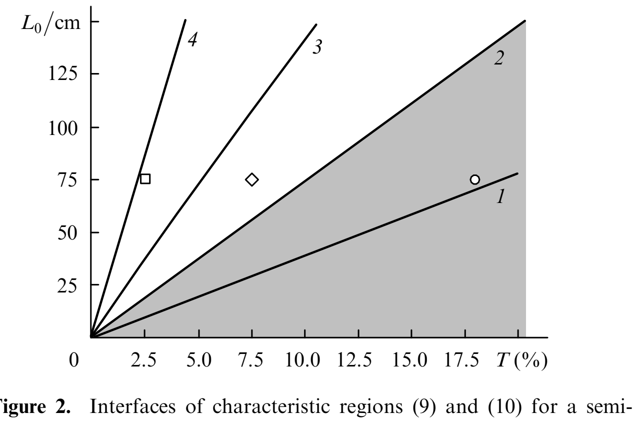

Figure 2. Interfaces of characteristic regions (9) and (10) for a semiconfocal resonator configuration ($r/s = 1/4$, $L_0 = R/2$) in the coordinates L_0, T, which correspond to the equality $\Delta L_{x,y}^{\rm deg} = \delta L_{x,y}$ at $w_{\rm p} = 100$ μm and $D_x - D_y = 0.2$ (*1*), 0.1 (*2*), 0.05 (*3*) and 0.02 m^{-1} (*4*). The region of realisation of the circular symmetry for $D_x - D_y = 0.1$ m^{-1} is shown in grey. Points □, ◇, and ○ correspond to combinations of L_0 and T for which Fig. 3 shows the intensity distributions.

$$t_{mn,m'n'} = \int \mathrm{HG}^*_{mn}(x,y)K(x,y)\mathrm{HG}_{m'n'}(x,y)\mathrm{d}x\mathrm{d}y, \qquad (5)$$

where $\mathrm{HG}_{mn}(x,y)$ and $\mathrm{HG}_{m'n'}(x,y)$ are the complex amplitudes of Hermite–Gaussian beams in the plane of the active medium. In addition, it is necessary to replace in (4) $2\Delta p$ by Δm (Δn) because the phase incursions of Laguerre–Gaussian and Hermite–Gaussian beams are proportional to $2p+l+1$ and $m+n+1$, respectively (see, for example, [13]). We will assume the transverse dimension of the mirrors rather large to neglect the difference in diffraction losses of the beams under study. Thus, we will obtain an estimate of the mode-locking region widths of Hermite–Gaussian modes of the families HG_{m0} and HG_{0n}:

$$\Delta(g_1g_2)^{\mathrm{HG}_{m0}} = 2\sin\left(2\pi\frac{r}{s}\right)\left[\Delta m\left(\frac{t_{\Delta m0,\Delta m0}}{t_{00,00}}\right)^{1/2}\right]^{-1} \times\left(1-\frac{t_{\Delta m0,\Delta m0}}{t_{00,00}}\right), \qquad (6a)$$

$$\Delta(g_1g_2)^{\mathrm{HG}_{0n}} = 2\sin\left(2\pi\frac{r}{s}\right)\left[\Delta n\left(\frac{t_{0\Delta n,0\Delta n}}{t_{00,00}}\right)^{1/2}\right]^{-1} \times\left(1-\frac{t_{0\Delta n,0\Delta n}}{t_{00,00}}\right). \qquad (6b)$$

This yields the width of the mode-locking region in the coordinates of the resonator length

$$\delta L_{x,y} = \Delta(g_1g_2)^{\mathrm{HG}_{m0,0n}}\left|\frac{\partial(g_1g_2)}{\partial L}\right|^{-1} = \Delta(g_1g_2)^{\mathrm{HG}_{m0,0n}} \times L_0\left[\sin^2\left(\pi\,\frac{r}{s}\right)\right]^{-1}, \qquad (7)$$

and then we have

$$\delta L_x = 4L_0\cot\left(\pi\,\frac{r}{s}\right)\left[\Delta m\left(\frac{t_{\Delta m0,\Delta m0}}{t_{00,00}}\right)^{1/2}\right]^{-1}\left(1-\frac{t_{\Delta m0,\Delta m0}}{t_{00,00}}\right), \qquad (8a)$$

$$\delta L_y = 4L_0\cot\left(\pi\,\frac{r}{s}\right)\left[\Delta n\left(\frac{t_{0\Delta n,0\Delta n}}{t_{00,00}}\right)^{1/2}\right]-1\left(1-\frac{t_{0\Delta n,0\Delta n}}{t_{00,00}}\right). \qquad (8b)$$

Obviously, the degeneracy splitting is manifested if $\Delta L^{\mathrm{deg}}_{x,y}$ substantially exceeds the effective widths δL_x and δL_y of the mode-locking regions of transverse modes of the families HG_{m0} and HG_{0n} with the centres at points corresponding to L^{deg}_x and L^{deg}_y:

$$\Delta L^{\mathrm{deg}}_{x,y} > \delta L_{x,y}. \qquad (9)$$

When (9) is realised, we can assume that the fundamental mode at $L \approx L^{\mathrm{deg}}_x$ is produced mainly by the modes of the HG_{m0} family, while the contribution of the HG_{0n} family is negligibly small. And vice verse, at $L \approx L^{\mathrm{deg}}_y$ the contribution of the modes of the HG_{m0} family is negligibly small.

Thus, under the condition

$$\Delta L^{\mathrm{deg}}_{x,y} \ll \delta L_{x,y} \qquad (10)$$

the distance between L^{deg}_x and L^{deg}_y with respect to $\delta L_{x,y}$ can be neglected, which should lead to the absence of splitting.

Figure 2, for the semiconfocal resonator configuration ($r/s = 1/4$, $L_0 = R/2$), shows in the coordinates L_0 and T the interfaces on which the equality $\Delta L^{\mathrm{deg}}_{x,y} = \delta L_{x,y}$ is fulfilled at different values of $D_x - D_y$. These interfaces were calculated by expressions (3), (8) and represent the characteristic regions (9) and (10). In calculations we used the Gaussian gain profile {see expression (2) in [12]}. One can see that to pass to the case of the circular symmetry, it is necessary to decrease the resonator length and/or to increase losses in it.

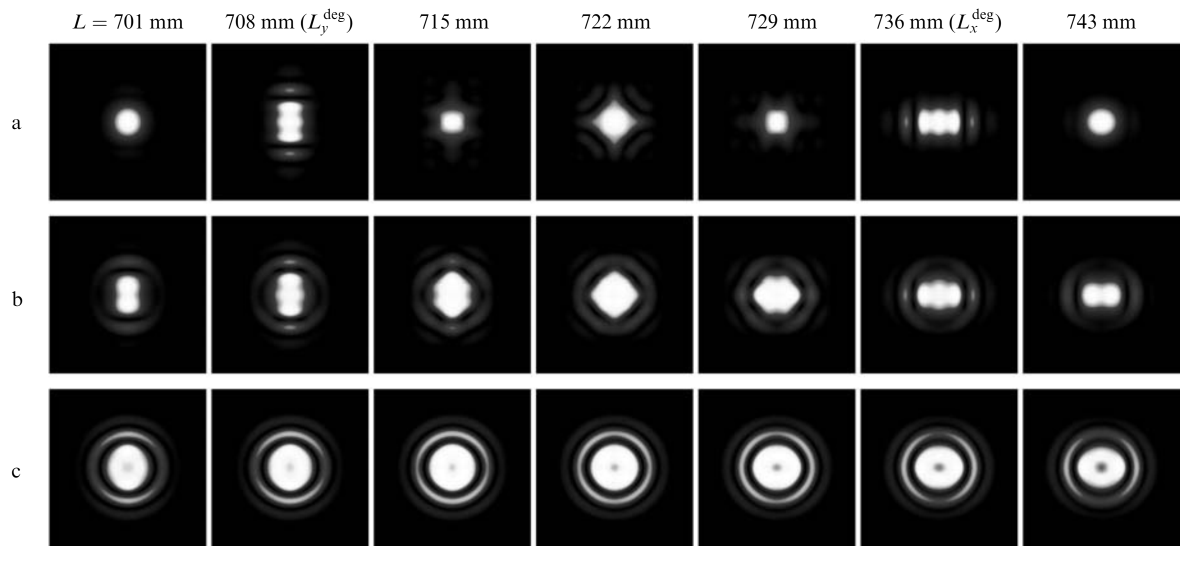

Figure 3. Calculated intensity distributions of the fundamental mode for $R = 150$ cm, $D_x = 0.05\ \mathrm{m}^{-1}$, $D_y = 0.15\ \mathrm{m}^{-1}$, $w_\mathrm{p} = 100$ μm and $T = 2.5\,\%$ (a), 7.5 % (b) and 20 % (c) at different L.

Figure 3 presents the intensity distributions of the fundamental mode at the output spherical mirror, calculated for the semiconfocal configuration at fixed $L_0 = 750$ mm and characteristic values of T. The combination of L_0 and T, for which intensity distributions are presented in Fig. 3, are shown in Fig. 2 by points □, ◇ and ○. The points □ and ○ are located in the characteristic region, while the point ◇ lies near their interface. The results of the calculation show tha passage from the case of degeneracy splitting accompanied by the violation of the circular symmetry ($T = 2.5$ %, Fig. 3a) to the case of distributions with the ring structure of the symmetry close to the circular one ($T = 20$ %, Fig. 3c).

Calculations were performed with the help of the model based on the re-distribution of the amplitudes in the system of Hermite – Gaussian modes of the astigmatic resonator due to the spatially inhomogeneous amplification. This model is described in the Appendix.

3. Experiment

We studied experimentally the intensity distribution of radiation behind the output mirror for diode end-pumped Nd : YAG and Nd : YLF lasers with the resonators of different lengths L and different transmissions of output mirrors T. The distribution was detected with a CCD camera.

A crystal with $D_x - D_y = 0.124$ m^{-1} was used in the Nd : YAG laser with $R = 150$ cm [9]. The effective diameter of the pump beam in the active medium was 100 μm. To eliminate the appearance of a thermal lens, the pump beam was modulated with a mechanic chopper, which allowed the average pump power to be reduced down to 10 mW. For resonators whose lengths differing significantly from those at which degeneracy appears, the intensity distributions are close to Gaussian. The degeneracy splitting is most strongly pronounced when a highly reflecting mirrors is used ($T < 1$ %), the lengths L_x^{deg} and L_y^{deg} being measured with the highest accuracy. Figure 4 presents the corresponding intensity distributions with the violated circular symmetry for a semiconfocal resonator configuration ($L_0 = R/2 =$ 750 mm). These distributions are close to those obtained numerically (Fig. 3a) in the entire range of changes in L. When T was increased to 18 %, we observed the intensity distributions with a ring structure demonstrating the circular symmetry (Fig. 4d). In this case, the characteristic lengths L_x^{deg} and L_y^{deg} were not manifested and the distributions with a ring structure were observed in the entire vicinity of the degeneracy, which corresponds to the calculations (Fig. 3c).

For the Nd : YLF laser [14] with a semiconfocal resonator L_0 is 7 cm, which is close to $L_0 = 10$ cm used in [2, 3], where distributions with a circular structure were observed at $T = 10$ %. The pump beam in the active medium had the effective cross section 100 × 100 μm. In the Nd : YLF laser employing mirrors with $T = 7.8$ %, 15 % and 25 %, the intensity distributions also had the circular structure. One can see from Fig. 2 that for resonators of length up to 10 cm, it is difficult to realise the degeneracy splitting at $D_x - D_y < 0.1$ m^{-1}. However, when a highly reflecting mirror with $T < 1$ % is used, we managed to observe experimentally the violation of the circular symmetry of the intensity distribution. Figures 5a – c show the corresponding distributions at $L = L_x^{\text{deg}} = 6.91$ cm, $L = 6.96$ cm and $L = L_y^{\text{deg}} = 6.99$ cm ($D_x - D_y = 0.096$ m^{-1} corresponds to these values of L_x^{deg} and L_y^{deg}). The case with $L_0 = 7$ cm and $T < 1$ % is an intermediate one between the cases satisfying (9) and (10). The calculated distributions for a similar intermediate case are shown in Fig. 3b. The characteristic intensity distribution with the circular symmetry obtained at $T = 25$ % is presented in Fig. 5d.

4. Conclusions

We have shown theoretically and experimentally that the

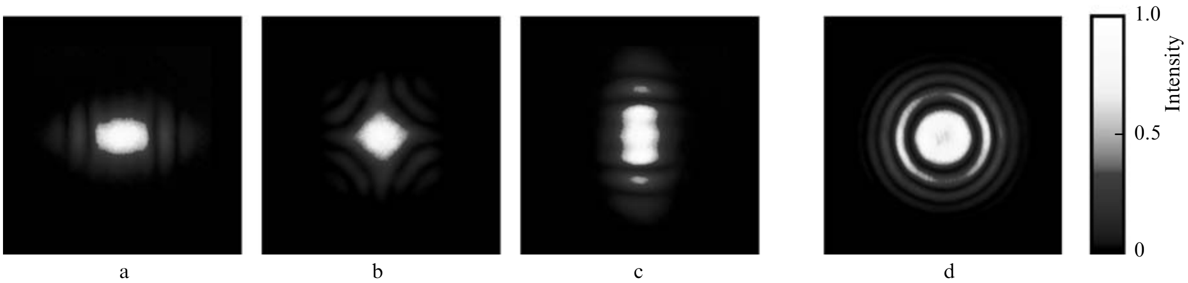

Figure 4. Experimental intensity distributions of radiation of the Nd : YAG laser behind the output spherical mirror at $R = 150$ cm and $T < 1$ % for $L = L_x^{\text{deg}} = 700$ mm (a), $L = (L_x^{\text{deg}} + L_y^{\text{deg}})/2 = 717$ mm (b), and $L = L_y^{\text{deg}} = 734$ mm (c), and at $T = 18$ % for $L = 715$ mm (d).

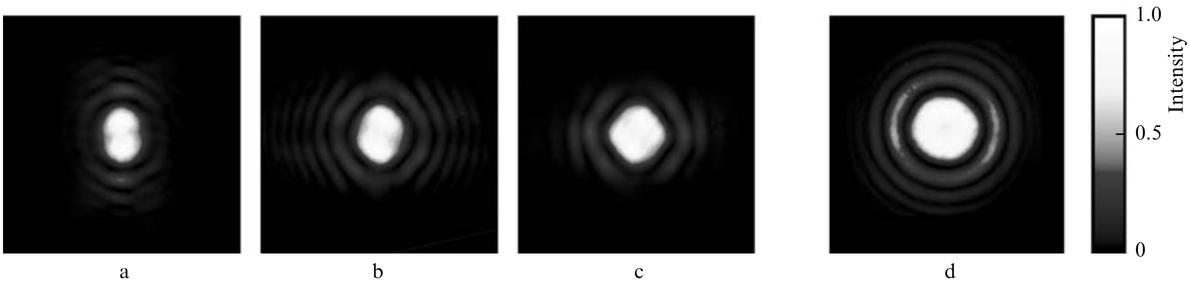

Figure 5. Experimental intensity distributions of radiation of the Nd : YLF laser with a semiconfocal resonator at $T < 1$ % for $L = L_y^{\text{deg}} = 6.91$ cm (a), $L = 6.96$ cm (b), and $L = L_x^{\text{deg}} = 6.99$ cm (c), and at $T = 25$ % for $L = 6.9$ cm (d).

astigmatism of the resonator in an end-pumped laser being one and the same, the fundamental mode both with the ring spatial structure having the circular symmetry and with the structure demonstrating the degeneracy splitting can be produced under the conditions of transverse mode locking. We have shown that the astigmatism-induced degeneracy splitting accompanied by the violation of the circular symmetry is not manifested when the resonator length is decreased and/or the losses increase. For this reason, it is expedient to use a highly reflecting long resonator to diagnose weak astigmatism with the help of the degeneracy splitting in the case of transverse mode locking. The theoretical conclusions have been confirmed by the experiments with diode end-pimped Nd:YAG and Nd:YLF lasers.

Acknowledgements. This work was supported by the Russian Foundation for Basic Research (Grant Nos 08-08-00108-a, 08-02-12143-ofi and 09-02-01190-a) and the program of the fundamental research of the Department of Physical Sciences, RAS 'Fundamental Problems of Photonics and Physics of New Optical Materials'.

Appendix

Numerical model for calculating the radiation intensity distributions of a laser with an astigmatic resonator and spatially nonuniform pumping

Consider a laser, which consists of a thin active medium of radius a with optical forces D_x and D_y, and of two mirrors. One of the mirrors is deposited on the face through which pumping is performed and the other is a spherical output mirror with the radius of curvature R. The eigenmodes of the cold resonator represent astigmatic Hermite–Gaussian beams [13]:

$$u_{mn}(q^x, q^y; x, y) = \sqrt{\frac{2}{\pi}}\frac{1}{\sqrt{2^{m+n}m!n!}}\frac{1}{\sqrt{w_x w_y}}H_m\left(\frac{\sqrt{2}\,x}{w_x}\right)$$

$$\times H_n\left(\frac{\sqrt{2}\,x}{w_y}\right)\exp\left(-\,\mathrm{i}\,\frac{kx^2}{2R_x}-\frac{x^2}{w_x^2}\right)\exp\left(-\,\mathrm{i}\,\frac{ky^2}{2R_y}-\frac{y^2}{w_y^2}\right),\quad\text{(A1)}$$

where $k = 2\pi/\lambda$. The horizontal and vertical radii of the beam, w_x and w_y, and the radii of curvature of the wave front, R_x and R_y, can be expressed via the complex parameters of curvature, q^x and q^y:

$$w_{x,y} = \left[-\frac{2}{k\mathrm{Im}(1/q^{x,y})}\right]^{1/2},\quad R_{x,y} = \frac{1}{\mathrm{Re}(1/q^{x,y})}.\quad\text{(A2)}$$

In the resonator under study we select three planes: 1 and 2 are the input and output planes of the active medium, respectively; 3 is the plane of the spherical mirror. The complex parameters of curvature are defined by the expressions:

$$\frac{1}{q_1^{x,y}} = \frac{1-g_1^{x,y}}{L} - \frac{\mathrm{i}}{L}\left[\frac{g_1^{x,y}\left(1-g_1^{x,y}g_2^{x,y}\right)}{g_2^{x,y}}\right]^{1/2},$$

$$\frac{1}{q_2^{x,y}} = -\frac{1-g_1^{x,y}}{L} - \frac{\mathrm{i}}{L}\left[\frac{g_1^{x,y}\left(1-g_1^{x,y}g_2^{x,y}\right)}{g_2^{x,y}}\right]^{1/2},\quad\text{(A3)}$$

$$\frac{1}{q_3^{x,y}} = \frac{1-g_2^{x,y}}{L} - \frac{\mathrm{i}}{L}\left[\frac{g_2^{x,y}\left(1-g_1^{x,y}g_2^{x,y}\right)}{g_1^{x,y}}\right]^{1/2}.$$

The complex amplitude distribution of the laser field $u(x,y)$ can be described by the matrix β_{mn}, whose elements represent amplitudes of the Hermite–Gaussian beams in the expansion

$$u(x,y) = \sum_{m,n}\beta_{mn}u_{mn}(q^x, q^y; x, y),\quad\text{(A4)}$$

where $u_{mn}(q^x, q^y; x, y)$ are the complex amplitudes of the Hermite–Gaussian beams (A1). Expansion (A4) is valid for arbitrary q^x and q^y but in the case of the resonator it is expedient to use the values of q^x and q^y determined in (A3). In the case of natural normalisation $\sum_{m,n}|\beta_{mn}|^2 = 1$, the quantity $|\beta_{mn}|^2$ is a fraction of laser radiation energy concentrated in the Hermite–Gaussian beam with the subscripts m and n.

When $u(x,y)$ is known, the values of β_{mn} can be found from relations

$$\beta_{mn} = \int\!\!\int u_{mn}^*(q^x, q^y; x, y)u(x,y)\mathrm{d}x\mathrm{d}y.\quad\text{(A5)}$$

The transformation of the elements of the matrix β_{mn} during the radiation propagation from plane 2 to plane 3 and back to plane 1 has the from [15]

$$\beta'_{mn} = \beta_{mn}\exp\Big[2\mathrm{i}kL - \mathrm{i}(2m+1)\arccos\pm\sqrt{g_1^x g_2^x}$$

$$-\,\mathrm{i}(2n+1)\arccos\pm\sqrt{g_1^y g_2^y}\,\Big],\quad\text{(A6)}$$

where the signs '+' and '−' coincide with those of g_1^x and g_1^y.

The re-distribution of the beam amplitudes β_{mn} after the double transit in the active medium with the gain profile $K(x,y)$ is given by the expression

$$\beta'_{mn} = \sum_{j,l} t_{mnjl}\beta_{jl},\quad\text{(A7)}$$

where

$$t_{mnjl} = \int\!\!\int u_{mn}^*(x,y)K(x,y)u_{jl}(x,y)\mathrm{d}x\mathrm{d}y.\quad\text{(A8)}$$

Note that $t_{mnjl} = \delta_{mj}\delta_{nl}$ only when $K(x,y)\equiv 1$.

The transformation of the beam amplitudes upon the round trip of the resonator with allowance for (A6)–(A8) can be written in the form:

$$\beta'_{mn} = \sum_{j,l} f_{mnjl}\beta_{jl},\quad\text{(A9)}$$

where

$$f_{mnjl} = t_{mnjl}\exp\Big[2\mathrm{i}kL - \mathrm{i}(2j+1)\arccos\pm\sqrt{g_1^x g_2^x}$$

$$-\mathrm{i}(2l+1)\arccos\pm\sqrt{g_1^y g_2^y}\,\Big].\quad\text{(A10)}$$

The values of β_{mn}, corresponding to the fundamental mode, are found as the components of the eigenvector of the equation

$$\beta_{mn} = \lambda \sum_{j,l} f_{mnjl} \beta_{jl} \tag{A11}$$

with the eigenvalue λ, which is maximal in modulus. Equation (A11) was solved by the power iteration method. Its solution represents a set of beam amplitudes in plane 2. The beam amplitudes in plane 3 are found from the solution of (A11) as follows:

$$\beta'_{mn} = \beta_{mn} \exp\big[-\mathrm{i}(m+1/2)\big(\psi_x - \psi_x^0\big) - \mathrm{i}(n+1/2)\big(\psi_y - \psi_y^0\big)\big], \tag{A12}$$

where

$$\psi_{x,y} - \psi_{x,y}^0 = \arctan\left[-\frac{\mathrm{Re}(1/q_3^{x,y})}{\mathrm{Im}(1/q_3^{x,y})}\right] - \arctan\left[-\frac{\mathrm{Re}(1/q_2^{x,y})}{\mathrm{Im}(1/q_2^{x,y})}\right].$$

The obtained values allow one, by using (A1) – (A4), to obtain the intensity distribution of the fundamental mode $|u(x,y)|^2$ at the output mirror.

References

1. Laporta P., Brussard M. *IEEE J. Quantum Electron.*, **27**, 2319 (1991).
2. Wu H.-H., Sheu C.-C., Chen T.-W., Wei M.-D., Hsieh W.-F. *Opt. Commun.*, **165**, 225 (1999).
3. Chen C.-H., Tai P.-T., Hsieh W.-F. *J. Opt. Soc. Am. B*, **20**, 1220 (2003).
4. Frauchiger J., Albers P., Weber H.P. *IEEE J. Quantum Electron.*, **28**, 1046 (1992).
5. Martel G., Labbe C., Sanches F., Frimager M., Ait-Ameur K. *Opt. Commun.*, **201**, 117 (2002).
6. Ramsay I.A., Degnan J.J. *Appl. Opt.*, **9**, 385 (1970).
7. Chen C.-H., Tai P.-T., Chiu W.-H., Hsieh W.-F. *Opt. Commun.*, **245**, 301 (2005).
8. Gorbunkov M.V., Kostryukov P.V., Telegin L.S., Tunkin V.G., Yakovlev D.V. *Kvantovaya Elektron.*, **37**, 173 (2007) [*Quantum Electron.*, **37**, 173 (2007)].
9. Bezotosnyi V.V., Cheshev E.A., Gorbunkov M.V., Kostryukov P.V., Tunkin V.G. *Appl. Opt.*, **47**, 3651 (2008).
10. Anan'ev Yu.A. *Opt. Spektrosk.*, **64**, 650 (1988).
11. Ozygus B., Zhang Q. *Appl. Phys. Lett.*, **71**, 2590 (1997).
12. Gorbunkov M.V., Kostryukov P.V., Tunkin V.G. *Kvantovaya Elektron.*, **38**, 689 (2008) [*Quantum Electron.*, **38**, 689 (2008)].
13. Bykov V.P., Silichev O.O. *Lazernye resonatory* (Laser Resonators) (Moscow: Fizmatlit, 2004) pp 70, 137.
14. Bezotosnyi V.V., Gorbunkov M.V., Kostryukov P.V., Petukhov V.A., Popov Yu.M., Tunkin V.G., Cheshev E.A. *Trudy 6 belorusko-rossiiskogo seminara 'Poluprovodnikovye lazery i sistemy na ikh osnove'* (Proceedings of the VI Belorussian – Russian Seminar 'Semiconductor Lasers and Systems') (Minsk, 2007) pp 73 – 76.
15. Siegman A.E. *Lasers* (Mill Valley, CA: Univ. Sci. Books, 1986) p. 761.