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Stochastic effects during the action of the pump noise on bistable self-modulation oscillations in a solid-state ring laser

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***Abstract*.** **Nonlinear radiation dynamics of a solid-state ring laser is studied in the region of laser parameters corresponding to the parametric resonance between the self-modulation and relaxation oscillations. Bistable regions are found in which, apart from the self-modulation regime of the first kind, a stable quasi-periodic self-modulation regime exists. Temporal and spectral emission parameters of counterpropagating waves are considered in the bistable self-modulation generation regimes. The effect of noise on the bistable self-modulation oscillations is studied. It is shown that during the interaction of noise, spectral peaks split at relaxation and self-modulation frequencies.**

Keywords: *solid-state ring laser, self-modulation regime of the first kind, quasi-periodic self-modulation regime, bistability, stochastic effects.*

1. Introduction

In the majority of studies on nonlinear dynamics of solid-state ring lasers (SRLs), stochastic processes related to intrinsic and external noises in lasers have not been considered so far. Only recently, there appeared papers which discovered interesting nonlinear stochastic effects appearing in two-directional SRLs in the presence of noise effects. Among these effects it is necessary to mention the change in the frequency of relaxation oscillations, which depends on the noise intensity, a considerable broadening of the relaxation and self-modulation peaks in the emission power spectrum, and the nonmonotonic dependence of the relaxation peak height on the noise intensity (the peak height achieves a maximum at a given noise intensity) [1]. Authors of paper [2] studied noise precursors of period-doubling bifurcations. Nevertheless, a number of nonlinear stochastic effects in SRLs have not been studied, including nonlinear stochastic effects under conditions of bistability.

Experiments performed in this paper have revealed some new features in the behaviour of SRLs with the noise pump modulation, which can be explained only by the presence of bistability of self-modulation oscillations in the laser under study.

In dynamic systems of different nature, much attention is being paid to the study of the effect of bistability on stochastic phenomena. It is shown in a number of papers that under bistability conditions, noise effects can play a constructive role. One of the examples demonstrating the constructive role of noise is the stochastic resonance. As applied to laser physics, it was studied in bistable generation regimes of a ring dye laser [3, 4] and a single-mode semiconductor laser [5, 6].

The aim of this paper is to study the influence of noise pump modulation on the temporal and spectral emission parameters of a SRL operating in the bistability region of self-modulation oscillations. The experimental investigations of the intensity spectra of counterpropagating waves in the SRL with the noise pump modulation have revealed some new features associated with the appearance of bistable self-modulation oscillations in the laser under study. To study bistable self-modulation generation regimes and to explain the results observed in the experiments, we performed numerical simulation of the processes under consideration.

2. Experimental

2.1 Experimental setup

In our experiments we used a 1.06-μm monolithic chip Nd : YAG laser with a nonplanar resonator. The geometrical perimeter of the ring resonator was 28 mm, and the nonplanarity angle was 85°. The laser was pumped by a 0.810-μm diode laser with the power supply incorporating a noise generator. The noise spectral width was 200 kHz. During the experiments, the noise intensity of the pump was varied by changing the generator output voltage from zero to the maximal value, which was determined by the spectral density of the electric noise at the generator output and achieved 10^{-6} W Hz^{-1}. When processing experimental data, the pump noise was measured in relative units (each relative unit of the pump noise corresponds to the spectral density of the electric noise at the generator output, 10^{-7} W Hz^{-1}).

In the experiments, we simultaneously measured temporal and spectral characteristics of the radiation intensity of counterpropagating waves as functions of the noise power and the excess of the pump power above the laser threshold. A 20-12-PCI analogue-to-digital converter (ADC) and Tektronix TDS 2014 digital broadband oscillo-

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scope were used in the measurements of signals. While processing experimental results, we used statistic averaging over a large number (about 100) of realisations of random processes. We paid special attention to the study of the influence of noise on the spectral emission characteristics of the ring laser in spectral regions close to the fundamental relaxation frequency and to frequency of self-modulation oscillations.

2.2 Experimental results

In the absence of noise pump modulation in the laser under study, self-modulation regime of the first kind takes place at a relatively small excess of the pump power above the threshold ($\eta < \eta_c = 0.25$). At $\eta > \eta_c = 0.25$, period-doubling bifurcations of self-modulation oscillations appear in the laser and the self-modulation regime of the first kind undergoes a transition to another periodic period-doubled self-modulation regime. The stability region $\eta < \eta_c$ of the self-modulation regime of the first kind was studied (up to the period-doubling bifurcation point).

In the self-modulation regime of the first kind, the emission power spectrum exhibits the following peaks: the main peak at the self-modulation frequency ω_m (Fig. 1b) and two additional peaks at the fundamental relaxation frequency ω_r (Fig. 1a) and at the combination frequency $\omega_m - \omega_r$ (the latter is absent in Fig. 1, only spectral regions in the vicinity of the fundamental relaxation frequency and self-modulation frequency are shown). At $\eta = 0.1$, the frequency of self-modulations oscillations in the laser under study is $\omega_m/2\pi = 215$ kHz and the relaxation frequency is $\omega_r/2\pi = 62.5$ kHz. Note that in the self-modulation regime of the first kind, apart from the fundamental relaxation frequency, there exists also an additional relaxation frequency ω_{r1}, which is observed in the emission spectra only in the presence of the frequency nonreciprocity of the ring resonator. In this paper, studies were performed in the absence of this nonreciprocity.

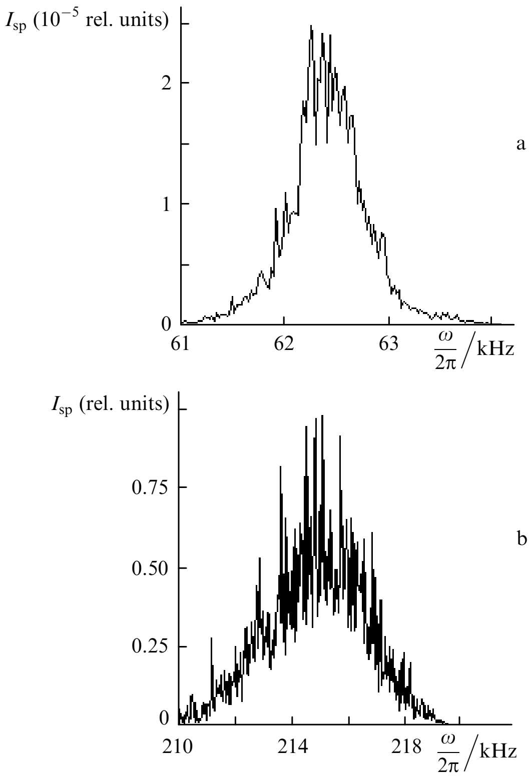

Figure 1. Intensity spectrum of one of counterpropagating waves in the self-modulation regime of the first kind in the absence of noise pump modulation at $\eta = 0.1$ in the vicinity of the fundamental relaxation frequency (a) and self-modulation frequency (b).

Figure 2 shows the spectrum of self-modulation oscillations in the absence of noise pump modulation when the lasing threshold ($\eta = 0.125$) is exceeded. One can see that the peak in the intensity spectrum at the frequency of self-modulation oscillations consists of two spectral components with almost equal intensities and maxima at the frequencies $\omega_{m1}/2\pi = 221$ kHz and $\omega_{m2}/2\pi = 227.5$ kHz, while at the fundamental relaxation frequency the emission spectrum exhibits one peak at the frequency $\omega_r/2\pi = 71.5$ kHz.

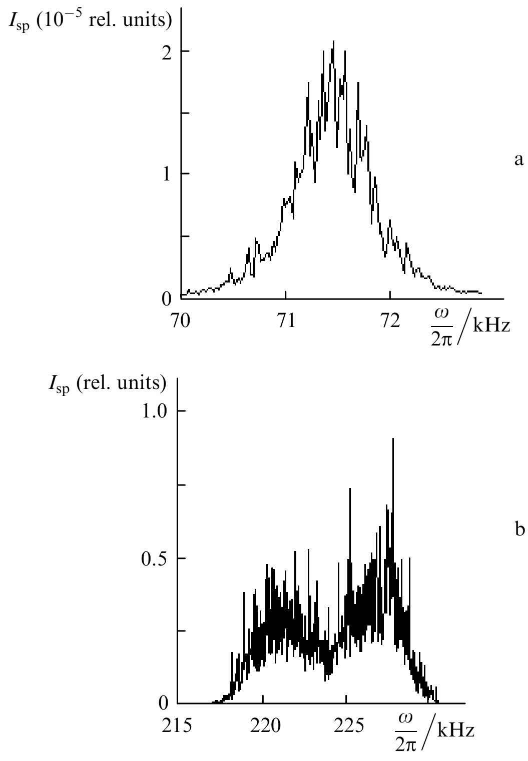

Figure 2. Intensity spectrum of one of counterpropagating waves in the absence of noise pump modulation and in the presence of bistability of self-modulation oscillations at $\eta = 0.125$ in the vicinity of the fundamental relaxation frequency (a) and self-modulation frequency (b).

The effect of the noise pump modulation was studied experimentally in this paper at several values of the pump excess over the threshold. Figure 3 presents the structure of radiation peaks at the frequency of self-modulation oscillations at different noise intensities and a constant excess over the threshold, $\eta = 0.195$. One can see that with increasing the noise intensity, the dip between two spectral components decreases and at a relatively large intensity both components virtually merge with each other (Fig. 3c).

The structure of spectral peaks is studied experimentally at the fundamental frequency of relaxation oscillations. Typical structures of relaxation peaks are presented in Fig. 4 at $\eta = 0.145$ and some values of the external noise intensity. One can see from these figures that at some region of noise intensities, the spectrum of relaxation oscillations

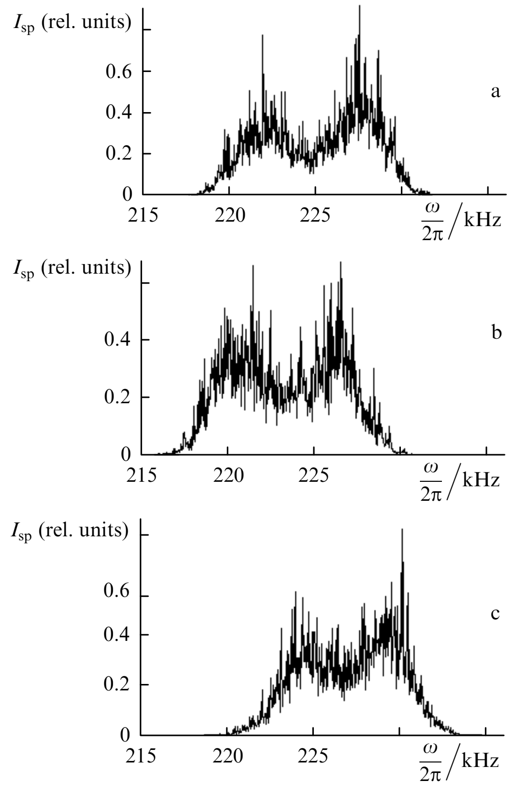

Figure 3. Structures of spectral peaks at the self-modulation frequency in the absence of external noise modulation (a) and at the intensity of the output signal of the noise generator 3.6×10^{-8} W Hz^{-1} (b) and 2×10^{-7} W Hz^{-1} (c); $\eta = 0.195$.

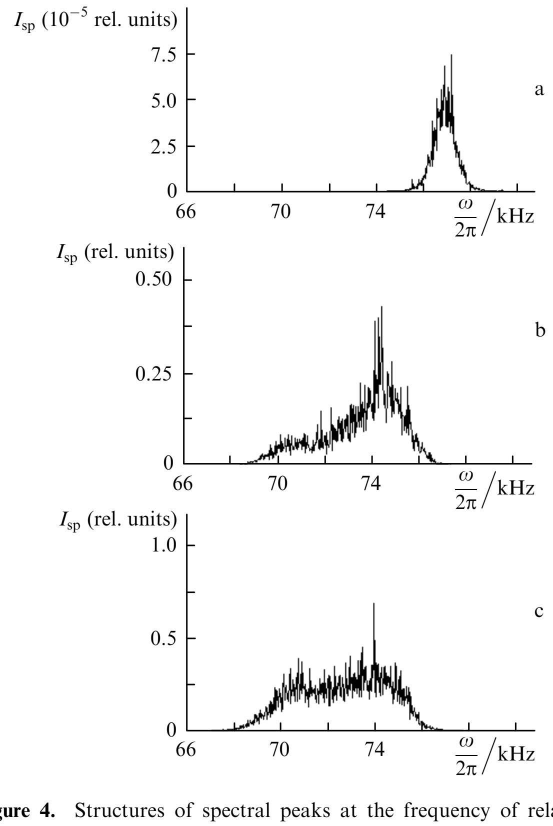

Figure 4. Structures of spectral peaks at the frequency of relaxation oscillations in the absence of external noise modulation (a) and at the intensity of the output signal of the noise generator 3.6×10^{-8} W Hz^{-1} (b) and 6.4×10^{-8} W Hz^{-1} (c); $\eta = 0.145$.

exhibits two components and the ratio of the intensities of these components depends on the noise intensity.

Spectra presented in Figs 1–4 were obtained by averaging over a large number of realisations (~ 100). Figure 5 shows the peak structure at the self-modulation frequency for three realisations of duration 41 ms. These spectra exhibit two spectral components, whose intensities can significantly differ from one realisation to another (Fig. 5).

Taking into account the results of numerical simulation presented below, we can assert that one of the spectral components (with the frequency $\omega_{m1}/2\pi$) corresponds to the periodic self-modulation regime, while the other (with the frequency $\omega_{m2}/2\pi$) – to the quasi-periodic regime. Thus, in accordance with the results presented, the laser under study operates within some time intervals either in the periodic or quasi-periodic regime by undergoing random noise-induced transitions from one regime to the other.

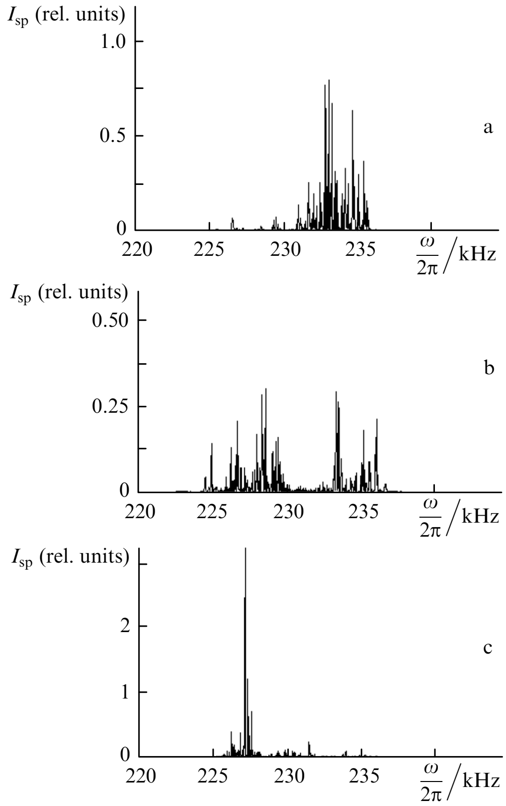

Figure 5. Peak structures at the self-modulation frequency corresponding to separate realisations within 41 ms at the intensity of the output signal of the noise generator 1.6×10^{-8} W Hz^{-1} and $\eta = 0.225$.

3. Theoretical study

3.1 Theoretical model

Numerical simulations of the phenomena under study were performed by using the SRL vector model [7], in which the effect of the pump noise on the laser dynamics was taken into account by introducing the term describing the noise pump modulation into the inverse population equation. In this case, the initial system of equations of the vector model has the form:

$$\frac{\mathrm{d}}{\mathrm{d}t}\tilde{E}_{1,2} = -\frac{\omega_c}{2Q_{1,2}}\tilde{E}_{1,2} \pm \mathrm{i}\frac{\Omega}{2}\tilde{E}_{1,2} + \frac{\mathrm{i}}{2}\tilde{m}_{1,2}\tilde{E}_{2,1}$$

$$+\frac{\sigma l}{2T}(N_0\tilde{E}_{1,2} + N_{\mp}\tilde{E}_{2,1}),$$

$$\frac{\mathrm{d}N_0}{\mathrm{d}t} = \frac{1}{T_1}[N_{\mathrm{th}}(1+\eta) - N_0 - N_0 a(|E_1|^2 + |E_2|^2) \qquad (1)$$

$$-N_+ aE_1E_2^* - N_- aE_1^*E_2] + g_{\mathrm{w}},$$

$$\frac{\mathrm{d}N_+}{\mathrm{d}t} = -\frac{1}{T_1}[N_+ + N_+ a(|E_1|^2 + |E_2|^2) + \beta N_0 aE_1^*E_2],$$

$$N_- = N_+^*.$$

Here, the complex field amplitudes of counterpropagating waves $\tilde{E}_{1,2}(t) = E_{1,2}\exp(\mathrm{i}\varphi_{1,2})$ and spatial harmonics of the inverse population N_0, $N_\pm$, determined by the expressions

$$N_0 = \frac{1}{L}\int_0^L N\mathrm{d}z, \quad N_\pm = \frac{1}{L}\int_0^L \boldsymbol{e}_1^*\boldsymbol{e}_2 N\exp(\pm \mathrm{i}2kz)\mathrm{d}z \qquad (2)$$

are dynamics variables; $\omega_c/Q_{1,2}$ are the resonator bandwidths; $Q_{1,2}$ are the resonator Q factors for counterpropagating waves; $T = L/c$ is the is the light round-trip time in the resonator of length L; T_1 is the time of the longitudinal relaxation; l is the active element length; $a = T_1 c\sigma/(8\hbar\omega\pi)$ is the saturation parameter; σ is the laser transition cross section; $\Omega = \omega_1 - \omega_2$ is the frequency nonreciprocity of the resonator; ω_1, ω_2 are the resonator eigenfrequencies for counterpropagating waves; $N_{\mathrm{th}}(1+\eta)/T_1$ is the pump rate; N_{th} is the threshold inverse population; $\eta = P/P_{\mathrm{th}} - 1$ is the pump power excess over the threshold. The linear coupling of counterpropagating waves is determined by phenomenologically introduced complex coupling coefficients

$$\tilde{m}_1 = m_1\exp(\mathrm{i}\theta_1), \quad \tilde{m}_2 = m_2\exp(-\mathrm{i}\theta_2), \qquad (3)$$

where $m_{1,2}$ are the moduli of coupling coefficients and $\theta_{1,2}$ are their phases. The field polarisations of counterpropagating waves are characterised by arbitrary unit vectors $\boldsymbol{e}_{1,2}$. The polarisation factor β is determined by the expression $\beta = (\boldsymbol{e}_1\boldsymbol{e}_2)^2 = \cos^2\gamma$, where γ is the angle between the unit vectors $\boldsymbol{e}_{1,2}$. The field polarisations of counterpropagating waves inside the resonator and the angle between the vectors $\boldsymbol{e}_{1,2}$ depend on the coordinate of the considered point inside the resonator. The quantity β involves the value γ averaged over the resonator length. Note that equations (1) are written for the case of lasing at the gain line centre.

The noise pump modulation is described with the help of the source of the white Gaussian noise g_{w} having the following statistical characteristics:

$$\langle g_{\mathrm{w}}(t)\rangle = 0, \qquad (4)$$

$$\langle g_{\mathrm{w}}(t)g_{\mathrm{w}}(s)\rangle = D\delta(t-s), \qquad (5)$$

where D is the noise intensity; $\delta(t)$ is the Dirac delta function.

Stochastic differential equations (1) were numerically solved by using the Euler method with the help of the Box–Muller transform for white noise generation [8]. In the numerical simulation of equations (1) a part of the parameters was assumed equal to the experimentally measured parameters of the laser under study. The resonator bandwidth ω_c/Q was determined by the relaxation frequency $\omega_r = \sqrt{\eta\omega_c/QT_1}$ and was equal to 4.4×10^8 s^{-1}. When the pump is exceeded over the threshold $\eta = 0.218$ in the laser under study, the main relaxation frequency is $\omega_r/2\pi = 101$ kHz. We assumed that the frequency and amplitude nonreciprocities of the ring resonator are absent ($\Omega = 0$, $\Delta = \omega_c/2Q_2 - \omega_c/2Q_1 = 0$). The polarisation parameter $\beta = 0.75$ was determined by the experimentally measured dependence of the additional relaxation frequency ω_{r1} on the frequency nonreciprocity Ω of the resonator (see [2]).

We failed to measure in experiments such parameters as moduli and phases of complex coupling coefficients $\tilde{m}_{1,2}$. For simplicity, the coupling coefficients are assumed below complex conjugated ($\theta_1 - \theta_2 = 0$). The values of moduli $m_{1,2}$ were varied to elucidate the effect of their inequality on the instability boundaries of the self-modulation regime of the first kind (points of self-modulation period-doubling bifurcations) and on the conditions for the appearance of self-modulation oscillation bistability. The values of the moduli of coupling coefficients were specified so that the self-modulation frequency calculated by using expression $\omega_m/(2\pi) = \sqrt{m_1 m_2}$, remained constant and equal to 202 kHz, which is close to the frequency measured in the experiment. In calculations we also varied the pump excess η over the threshold.

3.2 Bistable self-modulation regimes in the absence of noise

One of the main lasing regimes in an autonomous SRL is the self-modulation regime of the first kind, which is characterised by the out-of-phase sinusoidal modulation of intensities of counterpropagating waves. The theoretical and experimental investigations performed above showed that this regime exists and proves stable in a broad range of laser parameters. We have established that the main mechanism leading to the instability of the self-modulation regime in the region of its existence is a parametric interaction of self-modulation and relaxation oscillations [9–12]. This regime is characterised by two relaxation frequencies – the fundamental (ω_r) and additional (ω_{r1}) frequencies.

At the parameters typical of monolithic ring chip lasers, the broadest of all the regions of the parametric interaction is the region in which the frequency ω_m of self-modulation oscillations is close to the doubled value of the fundamental relaxation frequency. Period-doubling bifurcation of self-modulation oscillations appears in it. The theoretical analysis [12] showed that the width of this region depends on the symmetry of coupling coefficients. In the case of equal moduli of coupling coefficients of counterpropagating waves it vanishes and increases with increasing the relative difference in the moduli of coupling coefficients. This paper shows that when m_1/m_2 increases in the SRL, bistable self-modulation generation regimes can appear. It follows from the results presented below that the asymmetry (inequality of moduli) of coupling coefficients of counterpropagating waves and the pump power excess η over the threshold significantly affect the conditions for the appearance of the quasi-periodic generation regime and the bistability regions of self-modulation oscillations. In bistability regions, apart from periodic self-modulation regimes (self-modulation regime of the first kind and double-period self-modulation regime), quasi-periodic self-modulation regimes can exist.

Figure 6 presents typical time dependences and intensity spectra of one of counterpropagating waves in the self-modulation regimes of the first kind and in the quasi-periodic regime obtained by solving numerically equations (1) in the absence of noise at $m_1/m_2 = 0.41$ and $\eta = 0.2$. Under these conditions, both regimes shown in Fig. 6 are bistable. One can see from Fig. 6b that in the quasi-periodic regime, unlike the periodic regime of the first kind (Fig. 6a), the periodic modulation of the envelope of self-modulation oscillations takes place.

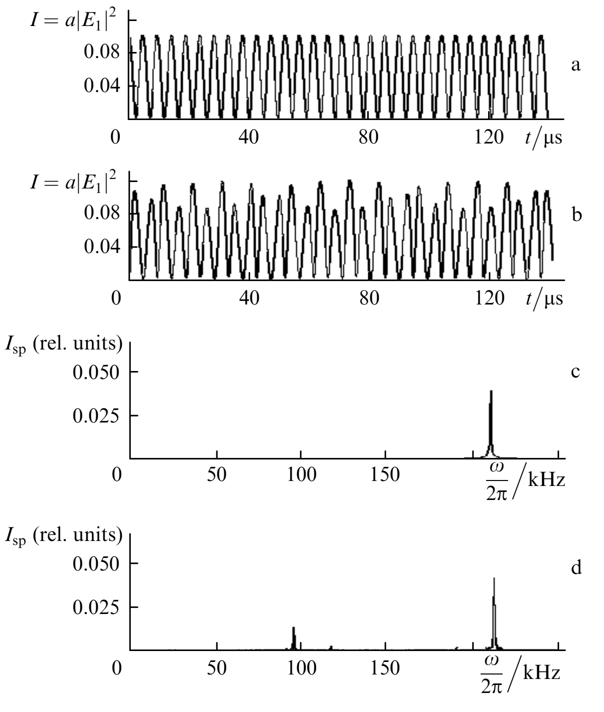

Figure 6. Time dependences (a, b) and intensity spectra (c, d) of one of counterpropagating waves in the self-modulation regime of the first kind (a, c) and in the quasi-periodic regime (b, d), obtained by solving numerically equations (1) in the absence of noise at $m_1/m_2 = 0.41$ and $\eta = 0.2$.

One can also see that both bistable self-modulation regimes have almost equal (differing by several kilohertz) self-modulation frequencies. In the quasi-periodic regime, unlike the self-modulation regime of the first kind, the intensity spectrum exhibits two spectral components: $\omega_{mq}/2\pi = 212.7$ kHz and $\omega_{rq}/2\pi = 95.5$ kHz. The spectral component $\omega_{rq}/2\pi$ differs by approximately 1 kHz from the fundamental relaxation frequency in the self-modulation regime of the first kind for which, under the same conditions, the self-modulation frequency is $\omega_m/2\pi = 210.6$ kHz.

Figure 7 presents the stability regions of the self-modulation regime of the first kind and the quasi-periodic self-modulation regime the plane of parameters m_1/m_2 and η. One can see that there exists the bistability region in which both these regimes are stable; its width increases with increasing the inequality of moduli of coupling coefficients.

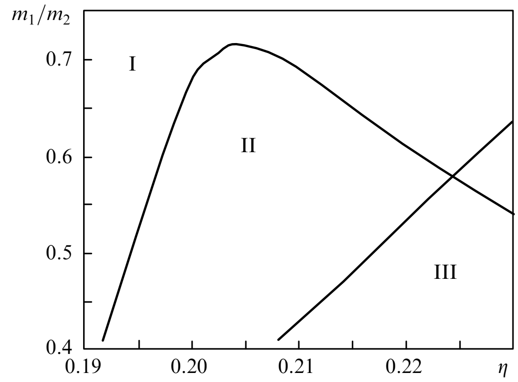

Figure 7. Stability regions of the self-modulation regime of the first kind (I) and quasi-periodic self-modulation regime (II) in the parameter plane $m_1/m_2, \eta$. Region III corresponds to the periodic period-doubled self-modulation regime, region II is a bistability region of the self-modulation regime of the first kind and the quasi-periodic self-modulation regime.

3.3 Effect of noise pump modulation on the characteristics of self-modulation oscillations

The influence of the white noise intensity on the bistable generation regimes was numerically simulated. We found that bistability can be observed only at rather small intensities of the pump noise.

Figure 8 shows the frequency dependences of the relaxation peak maxima on the pump noise intensity for two bistable regimes. Curve (*1*) corresponds to the case when in the absence of noise the laser operates in the self-modulation regime of the first kind. In this case, the central frequency of the relaxation peak decreases with increasing the noise intensity resulting in the appearance of two maxima which correspond to the periodic and quasi-periodic self-modulation regimes and exist in a narrow region of pump noise intensities. If the noise intensity is further increased, only one maximum at the frequency

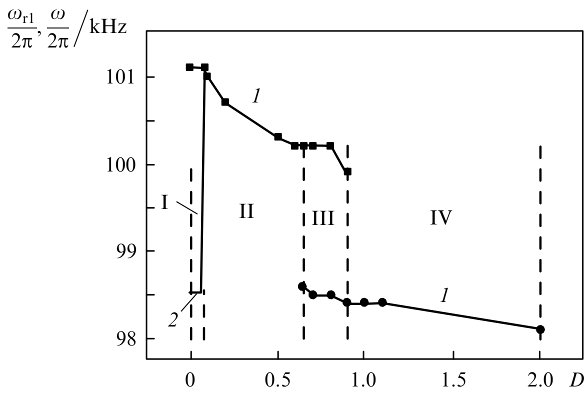

Figure 8. Dependences of central frequencies of relaxation peaks on the noise pump intensity for two bistable regimes [curve (*1*) corresponds to the case when in the absence of noise the SRL operates in the self-modulation regime of the first kind, curve (*2*) – to its operation in the quasi-periodic self-modulation regime]. In region I bistability of self-modulation oscillations takes place, in region II the emission spectrum exhibits only one peak in the vicinity of the relaxation frequency, which corresponds to the self-modulation regime of the first kind, in region III, independently of initial conditions, two maxima are observed in the relaxation peak and in region IV only one maximum is present in the relaxation peak, which corresponds to the quasi-periodic self-modulation regime.

$\omega_{rq}/2\pi$ is observed, which corresponds to the quasi-periodic self-modulation regime.

Curve (*2*) in Fig. 8 corresponds to the case, when in the absence of noise the laser operates in another bistable self-modulation (quasi-periodic) regime. One can see that bistability exists only at rather low noise intensities (region I). In region III bistability is absent and, independently of the initial conditions we can observe two maxima in the relaxation peak. In region IV only one maximum is observed in the relaxation peak, which corresponds to the quasi-periodic self-modulation regime. With increasing the noise intensity from region I to region II, there occurs a jump-like transition from the quasi-periodic self-modulation regime to the self-modulation regime of the first kind [passage from curve (*2*) to curve (*1*)].

Thus, the noise pump modulation leads to transitions between the regimes under study. Depending on the noise intensity, the laser can predominantly operate either in the periodic or quasi-periodic regimes. In these cases, the intensity spectra near the fundamental relaxation frequency exhibit only one peak (at the frequency $\omega_r/2\pi$ or $\omega_{rq}/2\pi$). In some narrow range of noise intensities, both bistable regimes can appear with the equal probability; in this case both these spectral components are present in the intensity spectrum.

It follows from the results of numerical simulations that in the intensity spectra of counterpropagating waves, the peak structure at the frequency of self-modulation oscillations both in the absence of external noise modulation and in its presence is identical to that observed in experiments. Note also that we failed to observe experimentally region I (Fig. 8) where bistability appears. This is probably explained by the fact that in the absence of noise pump modulation, the laser is affected by intrinsic technical noises, whose intensities are higher than the noise intensity corresponding to the upper boundary of region I.

4. Conclusions

Thus during theoretical and experimental investigations we have found the following peculiarities in the radiation dynamics of SRLs.

(i) In some region of laser parameters, bistability of self-modulation oscillations takes place (along with the self-modulation regime of the first kind, quasi-periodic self-modulation regime exists and proves stable).

(ii) Bistability appears only in the presence of the asymmetry (inequality of moduli) of coupling coefficients of counterpropagating waves.

(iii) Because of bistability, the intensity spectra of counterpropagating waves exhibit splitting of peaks at the relaxation and self-modulation frequencies in the presence of the pump noise, splitting depending on the noise intensity.

(iv) At low noise intensities, laser, depending on the initial conditions, can operate in one of bistable regimes. With increasing the noise intensity, there appear random noise-induced transitions from one bistable regime to another.

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