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Reflection and absorption of a p-polarised wave by a metal with a nonuniform electron temperature

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***Abstract*. The complex reflection coefficient of a p-polarised wave incident on a metal with electrons nonuniformly heated over the depth is found under the conditions of a high-frequency skin effect. The derived expressions for the absorption coefficient and the phase shift of a reflected wave are applicable in the case of an arbitrary ratio of the penetration depth of radiation to the spatial scale of the inhomogeneity of small contributions to the dielectric constant, which are caused by electron collisions. It is shown that the use of Fresnel formulae is not always possible and can lead to significant errors if the temperature of hot electrons changes at the skin-layer scale.**

Keywords: *p-polarised wave, absorption coefficient, phase shift, Fresnel formulae, electron – electron collisions.*

1. Introduction

When high-power femtosecond laser pulses interact with metals, electrons are efficiently heated in the skin layer (see, for example, [1 – 11]). In the case of a rather fast heating, a nonequilibrium state appears in which the electron temperature significantly exceeds that of the lattice and drastically changes at the skin-layer scale [5, 8]. Because of this, the electron – electron collision frequency, that is proportional to the square of the temperature, proves to be relatively large and nonuniform over the skin-layer thickness, which leads to the corresponding nonuniformity of the dielectric constant of a metal. Under these conditions, it is necessary to describe the optical properties of a substantially inhomogeneous metal.

The theory of the optical properties of inhomogeneous media is now being actively developed (see, for example, [12 – 15]). A productive approach to the description of optical properties of strongly inhomogeneous media applicable in the case of a small ratio of the scale of the medium inhomogeneity to the radiation wavelength was used in [16 – 19]. Because the skin-layer thickness is usually small compared to the wavelength, the approach of papers [16 – 19] can be also employed to describe the optical properties of metals with nonuniformly heated electrons. At the same time, in the case of a high-frequency skin effect, when the emission frequency exceeds the electron collision frequency and the ratio of the Fermi velocity to the radiation frequency is small compared to the penetration depth of radiation into the medium, another description model proposed in [20] is possible. It is based on the consideration, with the help of the perturbation theory, of small nonuniform terms in the expression for the dielectric constant of a metal, which are proportional to the electron collision frequency. In this case, the ratio of the inhomogeneity scale of small terms to the radiation wavelength, unlike that used in [16 – 19], is not assumed small, which makes it possible to construct the theory of optical properties permitting the passage to the known results of the optics of homogeneous metals. Paper [20] describes the consequences of the new approach for the case of reflection of an s-polarised wave from a nonuniformly heated metal. On the other hand, much attention in the experiments is paid to studying the reflection of p-polarised waves (see, for example, [3, 7, 9 – 11]). This interest is related to the possibility of investigating a number of physical phenomena appearing only during the interaction of p-polarised waves with metals. Taking into account the existing situation, in this paper we generalise the approach [20] to the case of interaction of a p-polarised wave with a metal whose electrons are nonuniformly heated by a femtosecond laser pulse. We study below the reflection of a probe p-polarised wave from a nonuniformly heated metal and its absorption in the metal. The expressions for the absorption coefficient and the phase shift of the reflected wave are derived with an accuracy to corrections quadratic in the small ratio of the electron collision frequency to the probe wave frequency. These expressions are applicable in the case of arbitrary spatial profiles of the electron temperature. When the temperature changes at a distance exceeding the skin-layer thickness, these expressions yield known Fresnel formulae for metals with relatively small electron collision frequencies. If the temperatures changes drastically, which takes place in the case of an insignificant heat removal, both the absorption coefficient and the relative change in the phase shift upon reflection prove to be significantly lower than the values yielded by the Fresnel formulae.

2. Model description of a metal

The specific features of penetration of an electromagnetic field into a metal significantly depend on the type of the

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constitutive equation determining the relation of the current density with the electric field strength. Under the conditions of a high-frequency skin effect discussed below, the distance propagated by the electron during the period of a change in the field is small compared to the skin-layer thickness. This allows one to neglect the spatial dispersion in the constitutive equation. In addition, in the high-frequency skin effect the characteristic frequency ν of electron collisions is small compared to the field frequency ω, which makes it possible to use the small parameter $\nu/\omega \ll 1$. At the same time, we assume that the frequency ω is smaller than the frequency corresponding to the band-gap width and the influence of interband transitions can be neglected. Under these conditions, we describe the response of the metal by using the known expression for a dielectric constant $\varepsilon(z)$ nonuniform along the z axis

$$\varepsilon(z) = \varepsilon_0 - \frac{\omega_{\rm p}^2}{\omega[\omega + {\rm i}\nu(z)]} \simeq \varepsilon_1 + {\rm i}\varepsilon_2(z) + \delta\varepsilon_1(z), \qquad (1)$$

where ε_0 is the contribution of coupled electrons and lattice ions to the dielectric constant; $\omega_{\rm p}$ is the plasma frequency;

$$\varepsilon_1 = \varepsilon_0 - \frac{\omega_{\rm p}^2}{\omega^2}; \quad \varepsilon_2(z) = \frac{\nu(z)}{\omega^3}\omega_{\rm p}^2; \quad \delta\varepsilon_1(z) = \frac{\nu^2(z)}{\omega^4}\omega_{\rm p}^2. \qquad (2)$$

The imaginary part ε_0 is assumed small compared to $\varepsilon_2(z)$. The electron collision frequency $\nu = \nu(z)$ is made up from the frequency of electron–phonon collisions $\nu_{\rm ep}$ and the frequency of electron–electron collisions $\nu_{\rm ee}(z)$ occurring with the umklappe processes of a quasi-momentum:

$$\nu(z) = \nu_{\rm ep} + \nu_{\rm ee}(z). \qquad (3)$$

Under the action of rather high-power femtosecond pulses, electrons are efficiently heated, while the lattice remains cold during the time smaller than the time of the energy transfer from electrons to the lattice, which is several picoseconds for typical metals. To describe this nonequilibrium state, we will use the two-temperature model of a metal. Due to the electron heating, the frequency of electron–electron collisions $\nu_{\rm ee}(z)$ increases with their temperature $T_{\rm e} = T_{\rm e}(z)$, and the frequency of electron–phonon collisions $\nu_{\rm ep}$, which is proportional to the lattice temperature T_l , remains virtually constant. At $\kappa T_{\rm e} \ll \varepsilon_{\rm F}$ (κ is the Boltzmann constant, $\varepsilon_{\rm F}$ is the Fermi energy), the relation of the frequency $\nu_{\rm ee}(z)$ with the temperature has the from [21]:

$$\nu_{\rm ee}(z) = a[\kappa T_{\rm e}(z)]^2/\hbar\varepsilon_{\rm F}, \qquad (4)$$

where $\hbar$ is Plank's constant; a is the numerical coefficient, whose quantity depends on the type of the metal band structure. For typical metals whose electron temperature already exceeds several thousand Kelvin degrees, the conditions are fulfilled, when $\nu_{\rm ee}(z) > \nu_{\rm ep}$. According to (4), the inhomogeneity scale $\nu_{\rm ee}(z)$ depends on the distribution of the electron temperature $T_{\rm e}(z)$ and can be of the order of the skin-layer thickness. In particular, under the conditions of a high-frequency skin effect, the evolution of the electron temperature in the metal is described by the expression [8]

$$C\frac{\partial T_{\rm e}}{\partial t} = \frac{4}{c}I(t)\nu(z)\exp\left(-\frac{2z}{\delta}\right) + \frac{\partial q}{\partial z}, \qquad (5)$$

where $C = \pi^2 N\kappa^2 T_{\rm e}/2\varepsilon_{\rm F}$ is the heat capacity of electrons with the density N; $I(t)$ is the density, slowly varying during the time $2\pi/\omega$, of the radiation flux heating the electrons; δ is the skin-layer thickness at the frequency of the heating laser beam; c is the speed of light; q is the density of the thermal flux of electrons. Expression (5) does not take into account the energy transfer from electrons to the lattice and is applicable at times shorter than several picoseconds. Relations (1)–(5) form the basis for the description of the optical properties of a metal heated by a femtosecond laser pulse.

3. Basic relations

Consider the interaction of the probe p-polarised wave with the metal, which occupies the half-space $z \geqslant 0$ and is heated by the femtosecond pulse (Fig. 1). The magnetic field of the incident wave is represented in the form

$$\tfrac{1}{2}\boldsymbol{B}_{\rm L}\exp(-{\rm i}\omega t + {\rm i}\boldsymbol{kr}) + {\rm c.c.}, \quad z \leqslant 0, \qquad (6)$$

where $\boldsymbol{B}_{\rm L} = (0, B_{\rm L}, 0)$; $\omega = kc$; ω is the frequency; k is the wave number; $\boldsymbol{k} = k(\sin\theta, 0, \cos\theta)$; θ is the angle between the vector $\boldsymbol{k}$ and the direction of the z axis. The magnetic field $\boldsymbol{B}_{\rm L}$ of the incident wave is orthogonal to the plane of incidence. The electric field of the p-polarised wave lies in the plane of incidence and has the components along the x and z axes: $\boldsymbol{E}_{\rm L} = E_{\rm L}(\cos\theta, 0, -\sin\theta)$, where $E_{\rm L} = B_{\rm L}$. The magnetic field of the reflected wave is described by the relation

$$\tfrac{1}{2}\boldsymbol{B}_{\rm r}\exp(-{\rm i}\omega t + {\rm i}kx\sin\theta - {\rm i}kz\cos\theta) + {\rm c.c.}, \quad z \leqslant 0. \qquad (7)$$

where $\boldsymbol{B}_{\rm r} = r_{\rm p}\boldsymbol{B}_{\rm L}$; $r_{\rm p}$ is the complex reflection coefficient. In this case, we have $\boldsymbol{E}_{\rm r} = -r_{\rm p}E_{\rm L}(\cos\theta, 0, \sin\theta)$ for the electric field of the reflected wave. We will seek for the field in the metal in the form

$$\boldsymbol{E}(\boldsymbol{r}, t) = \tfrac{1}{2}\boldsymbol{E}(z)\exp(-{\rm i}\omega t + {\rm i}kx\sin\theta) + {\rm c.c.}, \quad z \geqslant 0, \qquad (8)$$

$$\boldsymbol{B}(\boldsymbol{r}, t) = \tfrac{1}{2}\boldsymbol{B}(z)\exp(-{\rm i}\omega t + {\rm i}kx\sin\theta) + {\rm c.c.}, \quad z \geqslant 0, \qquad (9)$$

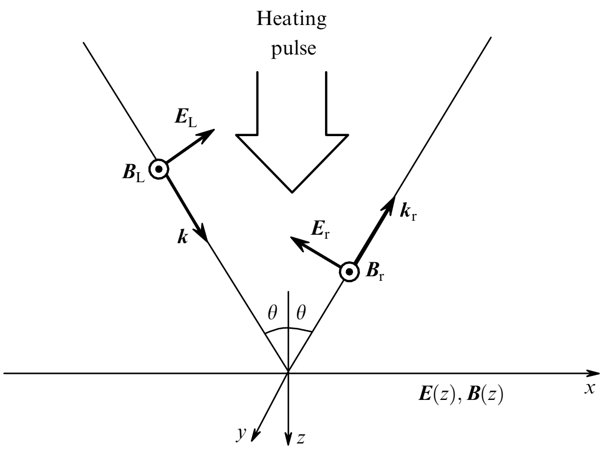

Figure 1. Reflection of a p-polarised wave from the metal surface heated by a femtosecond pulse.

where

$\boldsymbol{E}(z) = (B'(z)/\mathrm{i}k\varepsilon(z), 0, -B(z)\sin\theta/\varepsilon(z))$; $\boldsymbol{B}(z) = (0, B(z), 0)$.

The electric and magnetic fields are continuous on the metal surface ($z = 0$). Taking into account relations (6)–(9), we will write the conditions for the field continuity in the form

$$E_{\mathrm{L}}\cos\theta(1-r_{\mathrm{p}}) = \frac{1}{\mathrm{i}k\varepsilon(z)}B'(z)|_{z=0} \equiv E_x(z)|_{z=0}, \tag{10}$$

$$E_{\mathrm{L}}(1+r_{\mathrm{p}}) = B(z)|_{z=0} \equiv B_y(z)|_{z=0}. \tag{11}$$

Then, using the surface impedance of the p-polarised wave

$$Z_{\mathrm{p}} = [E_x(z)/B_y(z)]|_{z=0} \equiv [B'(z)/\mathrm{i}k\varepsilon(z)B(z)]|_{z=0}, \tag{12}$$

we obtain

$$r_{\mathrm{p}} = \frac{\cos\theta - Z_{\mathrm{p}}}{\cos\theta + Z_{\mathrm{p}}} \equiv |r_{\mathrm{p}}|\mathrm{e}^{\mathrm{i}\phi_{\mathrm{p}}}, \tag{13}$$

where $|r_{\mathrm{p}}|$ is the absolute quantity of the reflection coefficient; ϕ_{p} is the phase shift of the reflected wave. The absorption coefficient of the p-polarised wave is

$$A_{\mathrm{p}} = 1 - |r_{\mathrm{p}}|^2. \tag{14}$$

In accordance with Maxwell's equations, the magnetic field $B(z)$ in the metal satisfies the expression

$$\frac{\mathrm{d}}{\mathrm{d}z}\left[\frac{1}{\varepsilon(z)}\frac{\mathrm{d}B(z)}{\mathrm{d}z}\right] + k^2\left[1 - \frac{\sin^2\theta}{\varepsilon(z)}\right]B(z) = 0, \quad z \geqslant 0. \tag{15}$$

The boundary condition for the field $B(z)$ on the metal surface follows from (10), (11) and has the form

$$\left.\left[\frac{1}{\mathrm{i}k\varepsilon(z)}B'(z) + B(z)\cos\theta\right]\right|_{z=0} = 2E_{\mathrm{L}}\cos\theta. \tag{16}$$

Another boundary condition follows from the requirement that the magnetic field vanishes at $z \to \infty$, $B(z\to\infty) = 0$. The solution of equation (15) satisfying boundary condition (16) on the metal surface and the condition of the field absence at $z \to \infty$ unambiguously describes the field distribution in the metal.

4. Field in a metal

Taking into account the approximate form of the dielectric constant (1), (2), we have the expression from (15)

$$B''(z) - \frac{\varepsilon'(z)}{\varepsilon(z)}B'(z) + \left[-\frac{1}{d^2} + \mathrm{i}k^2\varepsilon_2(z) + k^2\delta\varepsilon_1(z)\right]B(z) = 0, \quad z \geqslant 0, \tag{17}$$

where $d = \left(k\sqrt{-\varepsilon_1 + \sin^2\theta}\right)^{-1}$ is the characteristic penetration depth of radiation. The field $B(z)$ described by expression (17) satisfies boundary condition (16) and the requirement to the absence of the field in the metal depth. We will seek for the solution of (17) in the form

$$B(z) = B_{\mathrm{a}}(z)\exp[\mathrm{i}\psi(z)], \tag{18}$$

where $B_{\mathrm{a}}(z)$ and $\psi(z)$ are real functions determining the amplitude and the phase of the magnetic field. To construct the approximate solution of expression (17), we will use the small parameter $\nu(z)/\omega \ll 1$. We will seek for the functions $B_{\mathrm{a}}(z)$ and $\psi(z)$ in the form of a series in powers $[\nu(z)/\omega]^n \ll 1$ ($n = 1, 2, \ldots$):

$$B_{\mathrm{a}}(z) = \sum_{n=0}^{\infty} B_n(z), \quad \psi(z) = \sum_{n=0}^{\infty}\psi_n(z), \tag{19}$$

where $B_n = B_n(z)$ and $\psi_n = \psi_n(z)$ have the order of smallness $[\nu(z)/\omega]^n$. In the zero order over the small parameter, taking into account relations $\varepsilon_2(z) \sim |\varepsilon_1|\nu(z)/\omega$ and $\delta\varepsilon_1(z) \sim \varepsilon_2(z)\nu(z)/\omega$, (15) and (16) yield the expressions

$$B_0'' - \frac{1}{d^2}B_0 - B_0(\psi_0')^2 = 0, \tag{20}$$

$$2B_0'\psi_0' + B_0\psi_0'' = 0 \tag{21}$$

and the boundary conditions:

$$B_0'\cos\psi_0 - (B_0\psi_0' + \varepsilon_1 kB_0\cos\theta)\sin\psi_0 = 0, \tag{22}$$

$$B_0'\sin\psi_0 + (B_0\psi_0' + \varepsilon_1 kB_0\cos\theta)\cos\psi_0 = 2E_{\mathrm{L}}k\varepsilon_1\cos\theta, \tag{23}$$

$$B_0(z\to\infty) = 0, \quad B_0'(z\to\infty) = 0, \quad B_0(z\to\infty)\psi_0'(z\to\infty) = 0. \tag{24}$$

In expression (24) two last relations follow from the condition that the electric field vanishes. By multiplying equation (21) by $B_0 \neq 0$, we have $B_0^2\psi_0' = \mathrm{const}$. In accordance with the boundary conditions at $z \to \infty$ (24), $B_0^2\psi_0' = 0$. Because at finite z the magnetic field differs from zero ($B_0 \neq 0$), we have

$$\psi_0' = 0 \tag{25}$$

for all $z \geqslant 0$. Using relation (25) from expression (20) and boundary condition at $z = 0$ (22), (23), we find

$$B_0(z) = B_0(0)\exp\left(-\frac{z}{d}\right) \equiv -\frac{2E_{\mathrm{L}}kd\varepsilon_1\cos\theta}{[1+(\varepsilon_1 kd\cos\theta)^2]^{1/2}}\exp\left(-\frac{z}{d}\right), \tag{26}$$

$$\tan\psi_0 = -\frac{1}{\varepsilon_1 kd\cos\theta} > 0. \tag{27}$$

In the linear approximation in $\nu(z)/\omega$ from (17), we have expressions for B_1 and ψ_1:

$$B_1'' - \frac{1}{d^2}B_1 = 0, \tag{28}$$

$$\psi_1'' - \frac{2}{d}\psi_1' = -g(z) \equiv -k^2\varepsilon_2(z) - \frac{\varepsilon_2'(z)}{d\varepsilon_1}. \tag{29}$$

The boundary conditions for the functions B_1 and ψ_1 directly follow from relations $B(z\to\infty) = 0$, $E(z\to\infty) = 0$

and (16). In analogy with the derivation of expressions for B_0 and ψ_0, we will write in the explicit form the solution of the equation for the functions B_1 and ψ_1 (see [22]). However, in considering the reflection of the probe wave, we will need only explicit expressions for derivatives of the functions ψ_n $(n = 0, 1, 2, ...)$, the explicit form of the dependence of the functions B_0 and B_1 on the coordinate, and expression (35) presented below. Taking this remark into account, we will present below only the relations required to calculate the complex reflection coefficient. Taking into account the boundary condition $B_0(z \to \infty) \times \psi'_1(z \to \infty) = 0$, we find from (29)

$$\psi'_1(z) = \exp\left(\frac{2z}{d}\right)\int_z^\infty g(z')\exp\left(-\frac{2z'}{d}\right)dz. \tag{30}$$

The solution of equation (28) satisfying the boundary conditions $B_1(z \to \infty) = 0$ and $B'_1(z \to \infty) = 0$, has the form

$$B_1(z) = B_1(0)\exp\left(-\frac{z}{d}\right). \tag{31}$$

The quantities $B_1(0)$ and the function $\psi_1(0)$ not used below are presented in paper [22]. In the second order of the perturbation theory, from (17) for functions B_2 and ψ_2 we have the expressions

$$B''_2 - \frac{1}{d^2}B_2 = B_0 b(z), \tag{32}$$

$$2B'_0\psi'_2 + B_0\psi''_2 = 0, \tag{33}$$

where

$$b(z) \equiv (\psi'_1)^2 - k^2\delta\varepsilon_1(z) + \frac{1}{\varepsilon_1^2 d}[\varepsilon_1\delta\varepsilon'_1(z) + \varepsilon'_2(z)\varepsilon_2(z)] - \psi'_1\frac{\varepsilon'_2(z)}{\varepsilon_1}. \tag{34}$$

Taking into account the conditions $B_2(z \to \infty) = 0$ and $B'_2(z \to \infty) = 0$, we obtain from (32)

$$\frac{d}{dz}\left[\frac{B_2(z)}{B_0(z)}\right] = -\exp\left(\frac{2z}{d}\right)\int_z^\infty b(z')\exp\left(-\frac{2z'}{d}\right)dz', \tag{35}$$

where the function $b(z)$ is described by expression (34). Equation (33) is similar to equation (21) studied above. Taking into account this similarity and using the boundary condition $B_0(z \to \infty)\psi'_2(z \to \infty) = 0$, we obtain

$$\psi'_2(z) = 0, \quad \psi_2(z) = \psi_2(0). \tag{36}$$

The function $B_2(z)$ and constants $B_2(0)$, $\psi_2(0)$ are presented in paper [22].

5. Absorption coefficient and phase shift

According to (12), (13), and (18), the complex reflection coefficient r_p is expressed by the derivatives of the functions $\psi(z)$ and $\ln B_a(z)$ at $z = 0$:

$$r_p = \left.\frac{k(\varepsilon_1 + \delta\varepsilon_1)\cos\theta - \psi'(z) + i\{k\varepsilon_2\cos\theta + [\ln B_a(z)]'\}}{k(\varepsilon_1 + \delta\varepsilon_1)\cos\theta + \psi'(z) + i\{k\varepsilon_2\cos\theta - [\ln B_a(z)]'\}}\right|_{z=0}, \tag{37}$$

where the notations $\varepsilon_2 = \varepsilon_2(0)$, $\delta\varepsilon_1 = \delta\varepsilon_1(0)$ are used. In accordance with definitions (13), (14), we find the absorption coefficient A_p and the phase-shift tangent ϕ_p:

$$A_p = \left.\frac{4k\cos\theta[(\varepsilon_1 + \delta\varepsilon_1)\psi'(z) - \varepsilon_2 B'_a(z)/B'_a(z)]}{[k(\varepsilon_1 + \delta\varepsilon_1)\cos\theta + \psi'(z)]^2 + [k\varepsilon_2\cos\theta - B'_a(z)/B_a(z)]^2}\right|_{z=0}, \tag{38}$$

$$\tan\phi_p = \left.\frac{2k\cos\theta[(\varepsilon_1 + \delta\varepsilon_1)B'_a(z)/B_a(z) + \psi'(z)\varepsilon_2]}{k^2(\varepsilon_1 + \delta\varepsilon_1)^2\cos^2\theta + k^2\varepsilon_2^2\cos^2\theta - [\psi'(z)]^2 - [B'_a(z)/B_a(z)]^2}\right|_{z=0}. \tag{39}$$

To calculate the absorption coefficient of the p-polarised wave and the phase of the reflected wave, we will use the approximate relation for the derivative of the logarithm of the magnetic field

$$\frac{d}{dz}\ln B_a(z) \simeq -\frac{1}{d} + \frac{d}{dz}\left[\frac{B_2(z)}{B_0(z)}\right] \tag{40}$$

and the above expressions for the corrections to the field amplitude and phase in the metal (25), (26), (30), (31), (35), (36). Then, we find from (38), (39)

$$A_p = \frac{4kd\cos\theta}{1 + (kd\varepsilon_1\cos\theta)^2}\left[\varepsilon_1 d\int_0^\infty g(z)\exp\left(-\frac{2z}{d}\right)dz + \varepsilon_2\right], \tag{41}$$

$$\tan\phi_p = \frac{2kd\varepsilon_1\cos\theta}{1 - (kd\varepsilon_1\cos\theta)^2}\left\{1 - \frac{1 + (kd\varepsilon_1\cos\theta)^2}{1 - (kd\varepsilon_1\cos\theta)^2} \times d\int_0^\infty b(z)\exp\left(-\frac{2z}{d}\right)dz - d\frac{\varepsilon_2}{\varepsilon_1}\int_0^\infty g(z)\exp\left(-\frac{2z}{d}\right)dz + \frac{1 + (kd\varepsilon_1\cos\theta)^2}{1 - (kd\varepsilon_1\cos\theta)^2}\frac{\delta\varepsilon_1}{\varepsilon_1} + \frac{(kd\varepsilon_2\cos\theta)^2}{1 - (kd\varepsilon_1\cos\theta)^2} - \frac{d^2}{1 - (kd\varepsilon_1\cos\theta)^2}\left[\int_0^\infty g(z)\exp\left(-\frac{2z}{d}\right)dz\right]^2\right\}. \tag{42}$$

Expression (41) is written with an accuracy to the terms linear in $v(z)/\omega$. Expressions (41), (42) generalise the Fresnel formulae to the case when the dielectric constant of the metal contains a small inhomogeneous part [see relations (1), (2)]. If the change in $\varepsilon_2(z)$ and $\delta\varepsilon_1(z)$ can be neglected, expressions (41), (42) yield Fresnel relations written with an accuracy to terms quadratic in $v/\omega \ll 1$.

6. Effect of electron heating on absorption and reflection

Let us use above relations (41), (42) to describe the effect of the electron heating on the properties of the reflection of the p-polarised wave from the metal. We assume that before the action of the heating pulse the electrons have a uniform temperature T_0. Small collision terms $\varepsilon_2(T_0)$ and $\delta\varepsilon_1(T_0)$, in the dielectric constant corresponding to this temperature are also independent of the coordinate. When $\varepsilon_2(T_0)$ and $\delta\varepsilon_1(T_0)$ are constant, we obtain Fresnel relations from (41), (42)

$$A_{\rm pF} = \frac{4kd\cos\theta}{1+(kd\varepsilon_1\cos\theta)^2}\left(1+\frac{1}{2}k^2d^2\varepsilon_1\right)\varepsilon_2(T_0), \tag{43}$$

$$\tan\phi_{\rm pF} = \frac{2kd\varepsilon_1\cos\theta}{1-(kd\varepsilon_1\cos\theta)^2}\left\{1-\frac{1}{2\varepsilon_1}k^2d^2[\varepsilon_2(T_0)]^2\right.$$

$$-\frac{1}{8}\frac{3+(kd\varepsilon_1\cos\theta)^2}{1-(kd\varepsilon_1\cos\theta)^2}k^4d^4[\varepsilon_2(T_0)]^2+\frac{(kd\cos\theta)^2}{1-(kd\varepsilon_1\cos\theta)^2}$$

$$\times[\varepsilon_2(T_0)]^2+\frac{1+(kd\varepsilon_1\cos\theta)^2}{1-(kd\varepsilon_1\cos\theta)^2}$$

$$\left.\times\left(1+\frac{1}{2}k^2d^2\varepsilon_1\right)\frac{\delta\varepsilon_1(T_0)}{\varepsilon_1}\right\}. \tag{44}$$

As usual, the absorption coefficient achieves a maximum at $(kd\varepsilon_1\cos\theta)^2=1$. At $\theta=\pi/2$, we have $A_{\rm pF}=0$ and at $\theta=0$, $A_{\rm pF}$ has a local minimum. The phase shift $\phi_{\rm pF}$ is a monotonic function of θ, increasing from the minimal value at $\theta=0$ to the maximal value $\phi_{\rm pF}=\pi$ at $\theta=\pi/2$ (see details in [23]).

Consider the influence of the electron heating on $A_{\rm p}$ and $\tan\phi_{\rm p}$. In the case of fast electron heating in the skin layer, the derivative of the thermal flux in (5) can be neglected. In this case, we find from (3)–(5)

$$\frac{v(z,t)}{v(T_0)} = \exp\left[\alpha\exp\left(-\frac{2z}{\delta}\right)\right], \tag{45}$$

where the parameter α is proportional to the integral of the radiation flux density heating the electrons:

$$\alpha=\alpha(t)=\frac{16a}{\pi^2N\hbar c}\int_{t_0}^{t}{\rm d}t'I(t'); \tag{46}$$

t_0 is the onset time of the electron heating at which their temperature is T_0. Relation (45) allows one to write small terms $\varepsilon_2(z)$ and $\delta\varepsilon_1(z)$ (2) in the from

$$\varepsilon_2(z)=\varepsilon_2(T_0)\exp\left[\alpha\exp\left(-\frac{2z}{\delta}\right)\right], \tag{47}$$

$$\delta\varepsilon_1(z)=\delta\varepsilon_1(T_0)\exp\left[2\alpha\exp\left(-\frac{2z}{\delta}\right)\right]. \tag{48}$$

By using dependences (47), (48) for the absorption coefficient (41) and the phase-shift tangent of the reflected wave (42), we find

$$A_{\rm p}=\frac{4kd\cos\theta}{1+(kd\varepsilon_1\cos\theta)^2}\varepsilon_2(T_0)$$

$$\times\left(1+\frac{1}{2}k^2d^2\varepsilon_1\right)\frac{1}{\alpha}({\rm e}^{\alpha}-1)\geqslant A_{\rm pF}, \tag{49}$$

$$\tan\phi_{\rm p}=\tan\phi_{\rm pF}+2kd\delta\varepsilon_1(T_0)\cos\theta\frac{1+(kd\varepsilon_1\cos\theta)^2}{1-(kd\varepsilon_1\cos\theta)^2}$$

$$\times\left[G(2\alpha)+G(0)-G(0)\frac{1}{\alpha}({\rm e}^{2\alpha}-1)\right]+\frac{(\varepsilon_2(T_0))^2}{\varepsilon_1}$$

$$\times\frac{2kd\cos\theta}{1-(kd\varepsilon_1\cos\theta)^2}\left\{G(\alpha)-G(0)-\frac{G^2(\alpha)-G^2(0)}{1-(kd\varepsilon_1\cos\theta)^2}\right.$$

$$+\frac{1+(kd\varepsilon_1\cos\theta)^2}{1-(kd\varepsilon_1\cos\theta)^2}\left[\frac{1}{2}G^2(0)-\frac{1}{2\alpha}\int_0^{\alpha}{\rm d}\xi(G^2(\xi)\right.$$

$$\left.\left.-2\xi G(\xi){\rm e}^{\xi}-2\xi{\rm e}^{2\xi})\right]\right\}, \tag{50}$$

where

$$G(\alpha)=\frac{1}{\alpha}\left[{\rm e}^{\alpha}(\alpha-1)+1-\frac{1}{2}k^2d^2\varepsilon_1({\rm e}^{\alpha}-1)\right]. \tag{51}$$

Relations (49)–(51) are written under assumption that the difference in the penetration depths of heating (δ) and probe (d) waves can be neglected. The latter is possible under conditions of a high-frequency skin effect at $-\varepsilon_1\gg1$.

According to (49), the absorption coefficient rapidly and almost exponentially increases with increasing the parameter α. By measuring $A_{\rm p}$ we can find α. Because the electron density N is known and the form of the dependence $I(t)$ is given by the shape of the heating pulse, knowing α (46) we can determine the unknown parameter a or the effective frequency of electron–electron collisions. The quantity α can be also found by measuring the phase difference $\phi_{\rm p}-\phi_{\rm pF}$ [see (50)] upon reflection of the probe pulse from the metals with hot and cold electrons. Figure 2 shows the scheme of the corresponding experiment. One can see that the probe pulse is divided into two, each of them passing equal distances. One of the pulses is reflected from the hot metal, while the other – from the cold. As a result, during the interference the pulses have the phases differing by the quantity $\phi_{\rm p}-\phi_{\rm pF}$. Thus, having determined $\phi_{\rm p}-\phi_{\rm pF}$ from the interference pattern, we can find α and then a. The dependence of the function $\phi_{\rm p}-\phi_{\rm pF}$ on α is presented in Fig. 3. The solid curve is plotted at $\theta=\pi/3$ for the metal with $v(T_0)=10^{14}$ s^{-1} and $\omega_{\rm p}=1.4\times10^{16}$ s^{-1}. The fundamental frequency ω of the probe pulse is taken equal to 1.8×10^{15} s^{-1}, which gives $kd=(-\varepsilon_1+\sin^2\theta)^{-1/2}\sim0.1$. For comparison Fig. 3 shows a dashed curve plotted by using Fresnel formula in which the values of small parameters in the dielectric constant on the metal surface $\varepsilon_2(z=0,t)$ and $\delta\varepsilon_1(z=0,t)$ increase with increasing α in accordance with relations (47), (48). One can see from the comparison of the two curves in Fig. 3 that the Fresnel

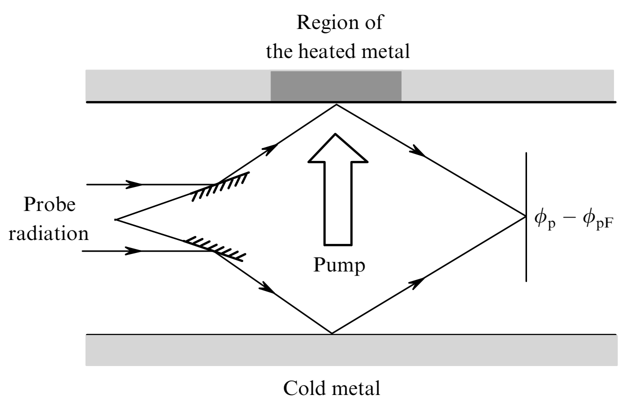

Figure 2. Possible scheme of the experiment on measuring the phase difference $\phi_{\rm p}-\phi_{\rm pF}$ appearing due to the heating of electrons by the pump.

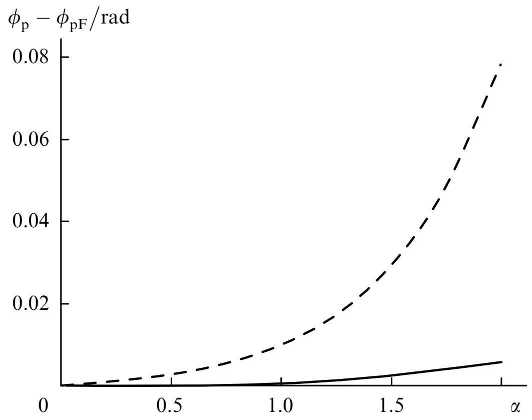

Figure 3. Dependence of the phase difference $\phi_p - \phi_{pF}$ on the degree of electron heating α upon reflection of the p-polarised wave from cold and hot metals. The solid curve is plotted at $\theta = \pi/3$ and the dashed curve is plotted by using Fresnel formulae for ϕ_p depending on $\varepsilon_2(z = 0)$ (47) and $\delta\varepsilon_1(z = 0)$ (48).

formulae overestimate the phase difference the more, the more nonuniform the electron heating. Fresnel formulae also overestimate the value of the absorption coefficient. Indeed, if we replace in (43) $\varepsilon_2(T_0)$ by the term $\varepsilon_2(z = 0, t)$ described by relation (47), the absorption coefficient will be larger than that yielded by expression (49), which takes into account the nonuniform heating of electrons in the skin layer.

The dependences of type (47), (48) are valid until the heat removal from the skin layer is neglected. At large times, the heat removal results in smoothing the temperature profile near the metal surface (see details in [8]). The electron temperature changing with time at the skin-layer scales changes weakly. At large times we can assume that at $z = 0$, dielectric constant (1), (2) depends on the electron temperature $T_e(z = 0, t)$ and radiation reflection and absorption are described by Fresnel formulae (43), (44) in which $\varepsilon_2[T_e(z = 0, t)]$ and $\delta\varepsilon_1[T_e(z = 0, t)]$ are used instead of $\varepsilon_2(T_0)$ and $\delta\varepsilon_1(T_0)$, respectively.

7. Conclusions

It follows from the above that the investigation of the reflection of the probe p-polarised wave from the surface of the metal with hot electrons can be an effective tool for determining the frequencies of electron–electron collisions. In this case, the analysis of the experimental dependences of the absorption coefficient and the phase shift of the reflected wave should be based on relations (41), (42), which take into account the possible significant change in the collision terms in the dielectric constant at the skin-layer scale.

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