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Surface plasmon – polaritons with negative and zero group velocities propagating in thin metal films

D.Yu. Fedyanin, A.V. Arsenin, V.G. Leiman, A.D. Gladun

***Abstract*.** **The conditions of existence of surface electromagnetic waves with negative and zero group velocities propagating in a thin metal film bounded by dielectric media with different refractive indices are found. Analytic expressions are derived to determine the group velocities of such waves, which are of interest for calculating and optimising optical systems of insulator – metal – insulator type.**

Keywords: *surface plasmon – polaritons, plasmonics, zero group velocity, backward waves.*

1. Introduction

It is well known that the dispersion relation for surface plasmon – polaritons (SPPs) at the interface of two semiinfinite media has one branch, i.e. the only value of the frequency ω corresponds to each value of the wave vector k_x [1 – 3]. However, if we consider the films of finite thickness, which are bounded from two sides by the media with other values of the refractive index, the shape of the dependence $\omega(k_x)$ for SPPs substantially changes. The dispersion relation for SPPs propagating in thin metal films is decomposed into two branches [1 – 4]. Below, we will denote these branches by ω^+ and ω^-. The high-frequency branch ω^+ corresponds to the antisymmetric mode, while the low-frequency branch ω^- – to the symmetric mode. By the antisymmetric mode is meant the mode for which the component of the electric field, parallel to the film surface, changes the sign as a function of the transverse coordinate. If the film is thin enough, i.e. the product $k_{\rm p}d$ is rather small (d is the film thickness; $k_{\rm p} = \omega_{\rm p}/c$; $\omega_{\rm p} = \sqrt{4\pi n_{\rm e}e^2/m_{\rm e}}$ is the plasma frequency; $n_{\rm e}$ is the electron density; e is the electron charge; $m_{\rm e}$ is the electron mass; c is the velocity of light in vacuum), the group velocity for the antisymmetric mode can be positive, zero or negative [5, 6] at the corresponding values of k_x. Note that the presence of such modes for a number of plasma waveguides [7, 8] and composite waveguides with the negative refractive index is typical [9].

In this paper, we analyse in detail the dispersion relation for SPPs in insulator – thin metal film – insulator (IMI) structures with the following exposure of conditions under which the SPP group velocity for such a thin-film waveguide is positive, negative or zero. We consider the general case, when the dielectric media bounding the thin metal film can have different refractive indices. Based on the derived theoretical relations, we perform calculations for a silver thin-film waveguide. We restrict our considerations to the lossless model, which is done to reveal the typical peculiarities of the dispersion relation and expressions for the SPP group velocity. In addition, if the path length of SPPs is not taken into account, this model well describes other metals used in plasmonics, for example gold, and allows one to achieve rather accurate numerical results.

2. Dispersion equation for SPPs in IMI structures

The dispersion relation for SPPs propagating along the smooth interface of two semiinfinite media with the dielectric constants ε_1 and ε_2 has the form $\kappa_1\varepsilon_2 = -\kappa_2\varepsilon_1$ [2], where

$$\kappa_i = \sqrt{k_x^2 - (\omega/c)^2\varepsilon_i} \tag{1}$$

is the decay constant ($i = 1, 2$). Within the Drude model $\varepsilon_1(\omega) = \varepsilon_{\rm r} - \omega_{\rm p}^2/\omega^2$ (here, $\varepsilon_{\rm r}$ is the constant taking into account the interband transition in a metal), and at large k_x we obtain that ω tends to $\omega_{\rm sp} = \omega_{\rm p}(\varepsilon_{\rm r} + \varepsilon_2)^{-1/2}$. In this case, the group velocity $v_{\rm g} = {\rm d}\omega/{\rm d}k_x$ is positive for all values of k_x.

Consider the dispersion relation for a film of finite thickness bounded from two sides by media with different, in the general case, refractive indices. We will apply the geometry of the problem shown in Fig. 1, where $a = d/2$ is half the film thickness. The dielectric constant of the metal is $\varepsilon_1 < 0$. The dielectric constants ε_2 and ε_3 of nonconducting media bounding the film will be assumed positive and weakly dependent on the frequency. In the Cartesian coordinate system (Fig. 1) all the fields will be sought for in the form of generalised plane waves of type $V = V\exp(-{\rm i}\omega t + {\rm i}k_x x + {\rm i}k_z z)$ ([10], Ch. 1). The dispersion equation for SPPs propagating in a thin film is written in the form [1 – 4]

D.Yu. Fedyanin, A.V. Arsenin, V.G. Leiman, A.D. Gladun Moscow Institute of Physics and Technology (State University), Institutskii per. 9, 141700 Dolgoprudnyi, Moscow Region, Russia;
e-mail: feddu@mail.ru, a.arsenin@mail.ru

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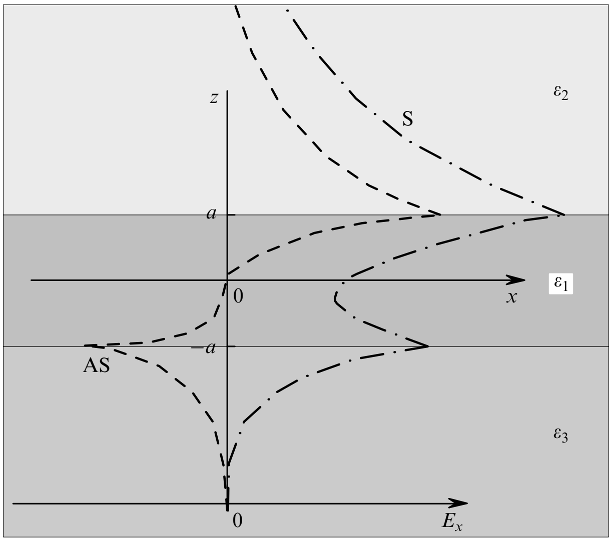

Figure 1. Scheme of a waveguide IMI structure. Dashed and dash-and-dot curves show the spatial distribution of the field E_x (S – for a symmetric mode and AS – for an antisymmetric mode).

$$\exp(-4\kappa_1 a) = \frac{\kappa_1\varepsilon_2 + \kappa_2\varepsilon_1}{\kappa_1\varepsilon_2 - \kappa_2\varepsilon_1}\frac{\kappa_1\varepsilon_3 + \kappa_3\varepsilon_1}{\kappa_1\varepsilon_3 - \kappa_3\varepsilon_1}, \quad (2)$$

where the decay constants k_i are found from (1).

For the case $\varepsilon_2 = \varepsilon_3$ expression (2) can be simplified and represented in the form of two equations for two branches. The first equation corresponds to the antisymmetric mode ω^+,

$$\tanh(\kappa_1 a) = -\frac{\kappa_2\varepsilon_1}{\kappa_1\varepsilon_2}, \quad (3)$$

and the second equation describes the symmetric mode ω^-,

$$\tanh(\kappa_1 a) = -\frac{\kappa_1\varepsilon_2}{\kappa_2\varepsilon_1}. \quad (4)$$

In this case, two values of the frequency ω correspond at once to each value of k_x.

It is obvious that $\exp(-4\kappa_1 a) \simeq 0$ for a rather thick film and equation (2) is equivalent to the expression $(\kappa_1\varepsilon_2 + \kappa_2\varepsilon_1)(\kappa_1\varepsilon_3 + \kappa_3\varepsilon_1) = 0$. In this case, the dispersion equation describes two independent branches: the first one corresponds to the SPPs at the interface of two semiinfinite media with the dielectric constants ε_1 and ε_2, while the second one – to the interface of media with ε_1 and ε_3, i.e. SPPs excited at the lower and upper boundaries of the film are not coupled with each other.

Let us analyse dispersion relation (2) at large k_x. For the case $\varepsilon_2 = \varepsilon_3$ at $k_x \to \infty$ we obtain that $\omega \to \omega_{sp}$, and at $\varepsilon_2 \neq \varepsilon_3$ we have $\omega \to \omega_{sp}^+ = \omega_p[\varepsilon_r + \max(\varepsilon_2, \varepsilon_3)]^{-1/2}$ for the branch ω^+ and $\omega \to \omega_{sp}^- = \omega_p[\varepsilon_r + \min(\varepsilon_2, \varepsilon_3)]^{-1/2}$ for the branch ω^- (Fig. 2). Note that the case $\varepsilon_r = 1$ is well described, for example, in review [3]. For rather thick films the limiting values of ω at $k_x \to \infty$ are the same, which is obviously equivalent to the following assumption: at large k_x the fields of surface waves at both interfaces are not overlapped, i.e. the penetration depth of the electromagnetic field into the film is much smaller than its width.

Consider the coordinate plane k_x, ω (Fig. 2). Of interest is only the region, where $\omega < \omega_p/\sqrt{\varepsilon_r}$ (i.e. $\varepsilon_1 < 0$). For convenience we assume below that $\varepsilon_2 < \varepsilon_3$. For all points in region I, the decay constants κ_2 and κ_3 [see (1)] take only the imaginary values, which corresponds to waves propagating from the film, and, hence, only radiation modes, which are of no interest to us, lie in this region. In region II, κ_2 is a real quantity, while κ_3 is imaginary, i.e. in this case, radiation into the half-space with the dielectric constant ε_3 takes place. Guided (nonradiative) modes, which present interest, lie in region III, where the decay constants κ_2 and κ_3 are real quantities.

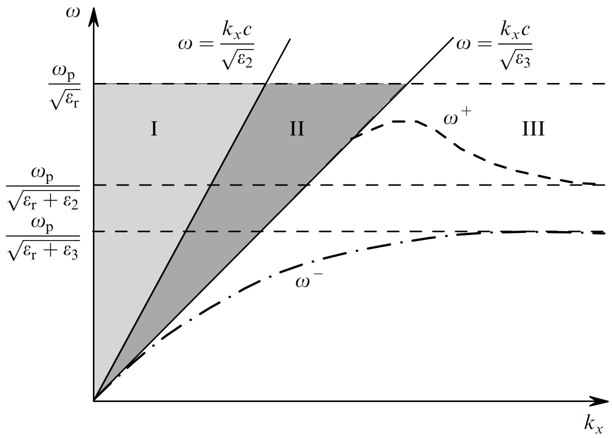

Figure 2. Dispersion dependences of SPPs for a thin metal film (see the description of regions I, II, III in the text).

Therefore, guided modes can be excited by using the scheme, similar to the Otto scheme (or the three-layer Kretchmann scheme) (Fig. 3); in this case, the dielectric constant of the prism should satisfy the condition $\varepsilon_{pr} > \max(\varepsilon_2, \varepsilon_3)$, because the fulfilment of this condition makes it possible to enter region III of the plane k_x, ω with the guided modes. The thickness s of the layer in such experiments should be of the order of the wavelength in a medium with the dielectric constant ε_2. Note at the same time that at a rather large thickness of the films it is possible to assume that guided modes will already lie in region II. This is

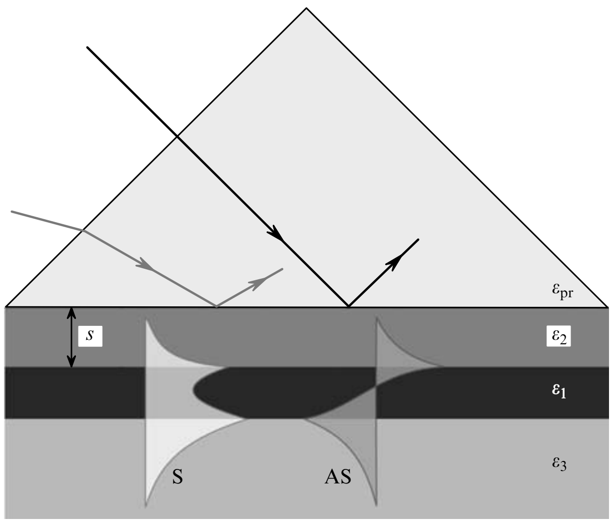

Figure 3. Scheme of excitation of SPPs on a thin metal film.

explained by a small penetration depth of SPP fields, excited at each interface, into the film compared to its thickness.

Solving Maxwell's equations for the configuration depicted in Fig. 1, we find the spatial field distribution of SPPs propagating in thin metal films. We merely consider TM waves (p polarisation) because surface waves exist only for them. The complex amplitude of the magnetic field is written in the form

$$H_y = A\exp(-\kappa_3 z)\exp(\mathrm{i}k_x x) \quad (5)$$

for $z \geqslant a$,

$$H_y = \frac{\kappa_1\varepsilon_3 - \kappa_3\varepsilon_1}{2\kappa_1\varepsilon_3} A\exp[-(\kappa_3+\kappa_1)a]\exp(\kappa_1 z)\exp(\mathrm{i}k_x x)$$

$$+ \frac{\kappa_1\varepsilon_3 + \kappa_3\varepsilon_1}{2\kappa_1\varepsilon_3} A\exp[(\kappa_1-\kappa_3)a]\exp(-\kappa_1 z)\exp(\mathrm{i}k_x x) \quad (6)$$

for $-a \leqslant z < a$,

$$H_y = \frac{\kappa_1\varepsilon_3 + \kappa_3\varepsilon_1}{\kappa_1\varepsilon_2 - \kappa_2\varepsilon_3}\frac{\varepsilon_2}{\varepsilon_3} A\exp[(\kappa_2 - \kappa_3 - 2\kappa_1)a]$$

$$\times\exp(\kappa_2 z)\exp(\mathrm{i}k_x x) \quad (7)$$

for $z < -a$.

The fields E_x and E_z can be easily found from Maxwell's equations. In these equations and everywhere below κ_i are determined from (1) and $k = \omega/c$ is used to reduce the notation. One can see from (5) that the quantity A means the magnetic field amplitude H_y at $x = a$ multiplied by $\exp(\kappa_3 a)$.

3. Conditions of existence of surface electromagnetic waves with negative and zero group velocities

As was shown in [5, 6, 8], backward surface waves, whose energy transfer rate is directed opposite to the phase one, are excited in IMI structures. Let us establish the conditions of existence of backward waves and waves with the zero group velocity in the system presented in Fig. 1. The expression for the x component of the Umov–Poynting vector in the complex form is written as

$$S_x = -\frac{c}{8\pi}E_z H_y^*. \quad (8)$$

The value of the ordinary Umov–Poynting vector averaged per oscillation period has the form

$$\bar{S}_x = \mathrm{Re}(S_x). \quad (9)$$

The group velocity is negative if the general energy flux is directed opposite to the phase velocity, which can be written in the form:

$$\int_{-\infty}^{+\infty}\bar{S}_x \mathrm{d}z < 0, \quad (10)$$

and equal to zero, if the general energy flux is equal to zero, i.e.

$$\int_{-\infty}^{+\infty}\bar{S}_x dz = 0. \quad (11)$$

Recall that only guided modes are of interest (and it means that the component of the energy flux along the z axis is equal to zero) and that the influence of the losses is negligibly small. Conditions (10) and (11) can be fulfilled because $\bar{S}_x \propto \kappa_i/(\varepsilon k)$ in insulators is a positive quantity and in a metal, at $\omega < \omega_\mathrm{p}/\sqrt{\varepsilon_\mathrm{r}}$, – negative. Thus, to satisfy relation (10), it is required that the component of the energy flux in the film along the x axis should exceed that in insulators in the absolute value. Using relations (5)–(7), we obtain that this condition is equivalent to the inequality:

$$\frac{1}{2|\varepsilon_1|}\left\{\frac{\sinh(2\kappa_1 a)}{\kappa_1}\left[\left(\frac{\kappa_1\varepsilon_3-\kappa_3\varepsilon_1}{\kappa_1\varepsilon_3}\right)^2\exp(-2\kappa_1 a)\right.\right.$$

$$\left.\left.+\left(\frac{\kappa_1\varepsilon_3+\kappa_3\varepsilon_1}{\kappa_1\varepsilon_3}\right)^2\exp(2\kappa_1 a)\right]+4a\frac{\kappa_1^2\varepsilon_3^2-\kappa_3^2\varepsilon_1^2}{\kappa_1^2\varepsilon_3^2}\right\}$$

$$>\frac{1}{\kappa_3\varepsilon_3}+\frac{\varepsilon_2}{\kappa_2\varepsilon_3^2}\left(\frac{\kappa_1\varepsilon_3+\kappa_3\varepsilon_1}{\kappa_1\varepsilon_2-\kappa_2\varepsilon_1}\right)^2\exp(4\kappa_1 a). \quad (12)$$

This cumbersome expression can be significantly simplified for the case $\varepsilon_2 = \varepsilon_1$. Indeed, in this case dispersion equation (3) for the branch ω^+ allows one to establish a one-value relation between $\kappa_1\varepsilon_2$ and $\kappa_2\varepsilon_1$ via the function of only $\kappa_1 a$. The amplitudes of the magnetic filed $H_y(z)$ and the electric field $E_z(z)$, $E_x(z)$ at $z < -a$ stop depending on the parameters κ_1, ε_1, related to the film material. In addition $H_y(z) = H_y(-z)$, $E_z(z) = E_z(-z)$, $E_x(z) = -E_x(-z)$ at $|z| \geqslant a$. Expressions for the fields in the film in accordance with (3) become the functions only of $\kappa_1 a$, multiplied by $\exp(-\kappa_2 a)$, and $H_y(z) = H_y(-z)$, $E_z(z) = E_z(-z)$, $E_x(z) = -E_x(-z)$ at $|z| < a$. Thus, the symmetry with respect to the plane $z = 0$ appears. Finally, we obtain

$$\tanh(\kappa_1 a)\left[\tanh(\kappa_1 a) + \frac{\kappa_1 a}{\cosh^2(\kappa_1 a)}\right] > \frac{\varepsilon_1^2}{\varepsilon_2^2}. \quad (13)$$

The particular expression for the case $\varepsilon_2 = \varepsilon_3 = 1$ can be found in [8] (Ch. 10).

The convenience of relation (13) consists in the fact that each part of this inequality is a function of only one variable: on the left – only $\kappa_1 a$, while on the right – only ω. By performing similar transformations for the branch ω^- to which relation (4) corresponds, one can easily see that the low-frequency branch does not allow the existence of zero and negative group velocities – for any k_x the group velocity is positive and the dispersion curve, corresponding to (4) lies below ω_sp. This result can be qualitatively explained as follows: the field amplitude $E_x(z)$ for the symmetric mode in the film is significantly higher, while $E_z(z)$ is significantly lower than the same quantities for the antisymmetric mode and the component of the energy flux along the x axis cannot already exceed the energy flux in insulators with respect to the absolute value.

While deriving relation (12), (13), we restricted ourselves to consideration of guided modes and made no suggestions about the type of the dependence $\varepsilon_1(\omega)$ except for the fact that we neglected losses and assumed ε_1 to be negative. Therefore, relations (12), (13) remain also valid, for example, for surface phonon–polaritons in the case of an isotropic crystal with one dispersion oscillator $\{\varepsilon_1(\omega) = \varepsilon_\infty - (\varepsilon_0 - \varepsilon_\infty)\omega_{\rm to}^2(\omega^2 - \omega_{\rm to}^2)^{-1}$, where ε_∞ and ε_0 are the high-frequency and static dielectric constants; $w_{\rm to}$ is the frequency of long wavelength optical phonons [2]$\}$.

Similar results can be obtained for the condition of the zero group velocity. It is enough to change the signs '$>$' into '$=$' in expressions (12), (13) and we obtain the resonance condition. Then, a detailed analysis requires the consideration of complex k_x in the general case of allowance for the losses; however, this problem will be considered elsewhere. Note that if we manage to create conditions under which the losses will not be too large, the system under study will represent a high-Q resonator in which fields are strongly amplified [11, 12].

4. Expressions for determining the group velocity

From the conditions of existence (12), (13) we can pass to the quantitative description of the group velocity. According to the variation theorem proposed by Bers ([8], Ch. 7), at real k_x and ω we have

$$\upsilon_g = \frac{\partial\omega}{\partial k_x} = \int_{-\infty}^{+\infty} \bar{S}_x \mathrm{d}z \Big/ \int_{-\infty}^{+\infty} w \mathrm{d}z, \tag{14}$$

where w is the time-averaged bulk energy density. The numerator in (14) is a time-averaged energy flux over the entire cross section per unit width of the film (unity y) and the denominator – a time-averaged energy corresponding to the unit length (unity x) per unit width of the film. The integrals in the numerator and denominator in the right-hand side of equality (14) can be calculated by using expressions (5)–(9):

$$\int_{-\infty}^{+\infty} \bar{S}_x \mathrm{d}z = \frac{c}{16\pi}\frac{k_x}{k}|A|^2 \exp(-2\kappa_3 a)$$
$$\times\left(\frac{1}{\kappa_3\varepsilon_3} + \frac{\varepsilon_2}{\kappa_2\varepsilon_3^2}\left(\frac{\kappa_1\varepsilon_3 + \kappa_3\varepsilon_1}{\kappa_1\varepsilon_2 - \kappa_2\varepsilon_1}\right)^2 \exp(4\kappa_1 a)\right.$$
$$+\frac{1}{2\varepsilon_1}\left\{\frac{\sinh(2\kappa_1 a)}{\kappa_1}\left[\left(\frac{\kappa_1\varepsilon_3 - \kappa_3\varepsilon_1}{\kappa_1\varepsilon_3}\right)^2 \exp(-2\kappa_1 a)\right.\right.$$
$$\left.\left.\left.+\left(\frac{\kappa_1\varepsilon_3 + \kappa_3\varepsilon_1}{\kappa_1\varepsilon_3}\right)^2 \exp(2\kappa_1 a)\right] + 4a\frac{\kappa_1^2\varepsilon_3^2 - \kappa_3^2\varepsilon_1^2}{\kappa_1^2\varepsilon_3^2}\right\}\right), \tag{15}$$

$$\int_{-\infty}^{+\infty} w\, \mathrm{d}z = \frac{1}{16\pi}\frac{1}{k^2}|A|^2 \exp(-2\kappa_3 a)$$
$$\times\left\{\frac{k_x^2}{\varepsilon_3\kappa_3} + \frac{k_x^2\varepsilon_2}{\varepsilon_3\kappa_2}\left(\frac{\kappa_1\varepsilon_3 + \kappa_3\varepsilon_1}{\kappa_1\varepsilon_2 - \kappa_2\varepsilon_1}\right)^2 \exp(4\kappa_1 a)\right.$$
$$+\frac{2\varepsilon_{\rm r}\varepsilon_1 k^2 + 2(\varepsilon_{\rm r} - \varepsilon_1)\kappa_1^2}{2\varepsilon_1^2}\frac{\sinh(2\kappa_1 a)}{\kappa_1}$$
$$\times\left[\left(\frac{\kappa_1\varepsilon_3 - \kappa_3\varepsilon_1}{\kappa_1\varepsilon_3}\right)2\exp(-2\kappa_1 a) + \left(\frac{\kappa_1\varepsilon_3 + \kappa_3\varepsilon_1}{\kappa_1\varepsilon_3}\right)^2\right.$$
$$\left.\left.\times\exp(2\kappa_1 a)\right] + k^2\frac{\kappa_1^2\varepsilon_3^2 - \kappa_3^2\varepsilon_1^2}{\kappa_1^2\varepsilon_3^2}\left(\frac{1+\varepsilon_1}{\varepsilon_1}\right)\right\}. \tag{16}$$

By simplifying these expression for the branch ω^+ in the case, when $\varepsilon_2 = \varepsilon_3$, we obtain

$$\int_{-\infty}^{+\infty} \bar{S}_x \mathrm{d}z = \frac{c}{8\pi}\frac{k_x}{k}|A|^2 \exp(-2\kappa_2 a)$$
$$\times\left\{\frac{1}{\kappa_2\varepsilon_2} + \frac{1}{\kappa_1\varepsilon_1}\left[\tanh(\kappa_1 a) + \frac{\kappa_1 a}{\cosh^2(\kappa_1 a)}\right]\right\}, \tag{17}$$

$$\int_{-\infty}^{+\infty} w \mathrm{d}z = \frac{1}{8\pi}\frac{1}{k^2}|A|^2 \exp(-2\kappa_2 a)$$
$$\times\left[\frac{k_x^2}{\kappa_2\varepsilon_2} + \frac{\varepsilon_{\rm r}\varepsilon_1 k^2 + (\varepsilon_{\rm r} - \varepsilon_1)\kappa_1^2}{\kappa_1\varepsilon_1^2}\tanh(\kappa_1 a)\right.$$
$$\left.+\frac{\varepsilon_{\rm r} k^2}{\kappa_1\varepsilon_1}\frac{\kappa_1 a}{\cosh^2(\kappa_1 a)}\right]. \tag{18}$$

Note that we can derive similarly expressions for the group velocity of the branch ω^-, but we will not consider this problem because, as was mentioned above, only in the case of ω^+, the group velocity can be positive, zero, and negative. The group velocity can be also calculated by using the method of numerical differentiation of dispersion relations (2)–(4). However, this method is rather cumbersome, for example: the dispersion relations are given in the implicit form, which complicates differentiation; it is needed to find, with a high degree of accuracy, the points of the plane k_x, ω satisfying the dispersion relation, the points lying in the small vicinity of the point under study.

Expression (14) supplemented with relations (15), (16) allows one to calculate the group velocities at the point satisfying the dispersion relation only if its coordinates on the plane k_x, ω are known. Note that expressions (16)–(18) are derived for a particular model. However, necessary assumptions and additional computations having been made, one can obtain analogous expressions for other models, for example, surface phonon–polaritons in the case of an isotropic crystal with one dispersion oscillator.

5. Discussion of the results

Let us demonstrate the possible use of the derived relations for the analysis of backward waves and waves with the zero group velocity in IMI waveguides by the example of specific materials employed in plasmonics. We will mainly discuss the possibilities of practical applications of the results obtained in the paper.

Consider a thin silver film in the configuration illustrated in Fig. 1. We will restrict our consideration to the frequency range corresponding to the wavelengths of light in vacuum, i.e. 300–600 nm. The frequency dependence of the dielectric constant of silver in this particular frequency range can be determined by the relation $\varepsilon_1(\omega) = 6.0 - (1.43 \times 10^{16})^2/\omega^2$, which was obtained within the Drude model taking into account the correction for interband transitions in metal that was made by using the experimental data [13]. This relation makes it possible to describe approximately the dielectric constant of silver at room temperature (the collision frequency of electrons is $\gamma_{\rm e} \approx 7.7 \times 10^{13}$ s^{-1} and, hence, $\omega, \omega_p \gg \gamma_e$) and to describe, virtually without any errors, the dielectric constants of high-purity silver samples at liquid helium temperature ($\gamma_e < 10^{12}$ c^{-1}) [14].

Figure 4 presents the dispersion dependences for the waveguide structures Al_2O_3-Ag-vacuum, $Al_2O_3-Ag-Al_2O_3$, $Al_2O_3-Ag-SiO_2$ and $Al_2O_3-Ag-ZrO_2$, which demonstrate that the slope of the high-frequency branch is positive at small k_x and, when k_x increases, it smoothly changes into negative. The point that the dispersion dependence of SPPs for the high-frequency branch originates not from zero at $\varepsilon_2 \neq \varepsilon_3$ is explained by the fact than only guided modes are being considered. The obtained dispersion dependences make it possible to estimate the SPP group velocity; however, they do not allow one to calculate its exact value and to determine the shape of the dependence on different parameters of the IMI structure under study. This problem is solved with the help of relation (14) supplemented with expressions (15), (16).

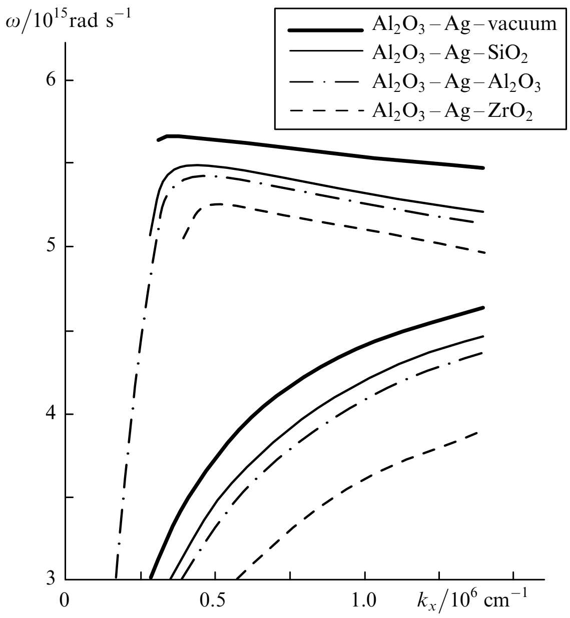

Figure 4. Dispersion dependences of SPPs for waveguide structures Al_2O_3-Ag-vacuum, $Al_2O_3-Ag-SiO_2$, $Al_2O_3-Ag-Al_2O_3$ and $Al_2O_3-Ag-ZrO_2$. The thickness of the silver film is $d = 10$ nm, the dielectric constants are $\varepsilon = 2.84$ (Al_2O_3), 2.2 (SiO_2) и 5.5 (ZrO_2).

The example of this calculation is the dependence of the group velocity on the waveguide vector of SPPs shown in Fig. 5 for four IMI structures. This dependence is characterised by the presence of a minimum for the group velocity. In addition, Fig. 5 clearly demonstrates that the SPP group velocity depends on the dielectric constants of the media surrounding the metal film. Relations (14)–(16) allow one to establish this dependence.

Figure 6 presents the dependence of the SPP group velocity on the dielectric constant ε_3 of one of the media surrounding a silver film under the assumption that the quantities ε_2, d and k_x are fixed. The group velocity has a minimum for the high-frequency branch at $\varepsilon_3 \approx 3.2$ and a weak dependence on ε_3 for the low-frequency branch. Obtained relations (12), (14)–(16) can be used to solve the computation problems and to optimise the waveguide structures of the IMI type.

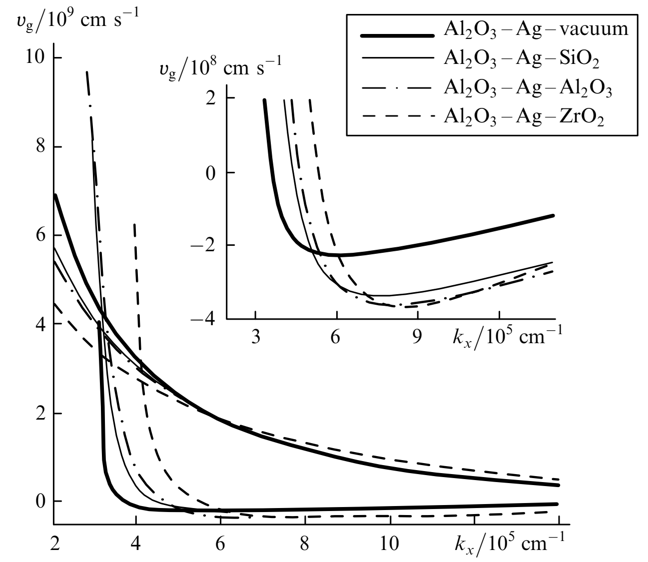

Figure 5. Dependences of the group velocity on the wave vector for the structures shown in Fig. 4.

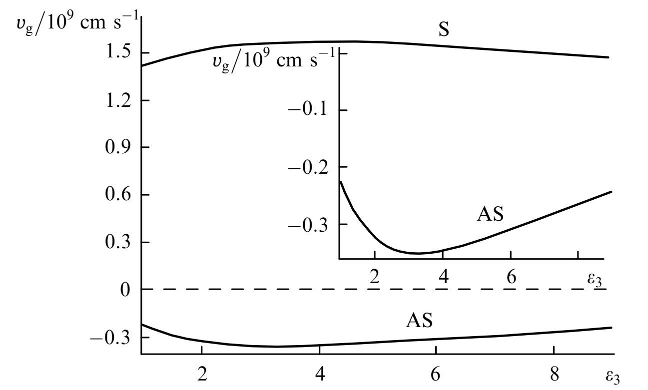

Figure 6. Dependences of the group velocity on ε_3; the dielectric constant is $\varepsilon_2 = 2.84$ (Al_2O_3), the film thickness is $d = 10$ nm, the wave vector is $k_x = 7 \times 10^5$ cm^{-1}. The inset shows the high-frequency branch.

6. Conclusions

In this paper, we have derived relations for determining certain conditions of existence of backward waves and waves with the zero group velocity as well as the SPP group velocity in IMI structures in the general case, when the media surrounding a metal film can have different dielectric constants. We have presented the calculations and analysis of the dependence of the SPP group velocity on different parameters of particular structures.

Note that as was shown in [6], due to the existence of backward waves in IMI structures one should expect negative refraction of SPPs at the interface of two layered structures [15]. The relations obtained in this paper make it possible to predict negative refraction in the given structure and to optimise the parameters of the device, for example, to solve the problem of search for a layered structure with a minimum group velocity or with a specified group velocity. They can also be used as a theoretical basis for designing a number of plasmonic devices based on the backward waves. The condition of existence of a wave with zero group velocities can be employed for designing high-Q plasmon resonators.

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