

# Calculation of the optimal polarisation anisotropy of interference mirrors of a ring laser resonator

O.D. Vol'p'yan, V.N. Kuryatov, A.L. Sokolov

**Abstract.** The influence of the amplitude–phase polarisation anisotropy of the interference mirrors on the polarisation characteristics of a ring laser (ellipticity, frequency shifts, and losses) is analysed. The combination of the mirror parameters, at which the maximum sensitivity of the polarisation characteristics of radiation to the nonplanar deformation of the axial contour is observed, is determined. It is shown that there exists a range of optimal phase anisotropies of the mirrors.

**Keywords:** ring laser, interference mirror, nonplanar resonator, polarisation eigenstates.

## 1. Introduction

Ring lasers used in laser gyroscopes have a number of specific features such as low amplification in the active medium, a high degree of radiation monochromaticity, single-mode and single-frequency lasing regimes [1–5]. Because of this, stringent requirements (namely, superlow losses, stability of the parameters in a broad temperature range) are imposed on the ring laser resonators and on the influence of different external factors. It is also necessary to control and reduce the errors in manufacturing and assembly of optical resonators.

One of the errors is the nonplanar deformation of the axial contour, which appears if the resonator is formed by more than three vertices of the axial contour (the vertex of the axial contour is a point where the axial beam changes its direction [6]). This deformation is sometimes used specially, for example, to produce circular polarisation eigenstates [3–5].

The polarisation characteristics of ring lasers significantly depend on the polarisation anisotropy of the reflectors forming the nonplanar optical resonator: interference mirrors or totally reflecting prisms [5–11]. The aim of this paper consists in determining the optimal anisotropy

of the interference mirrors, which provides minimal radiation losses and small sensitivity of the parameters of a four-mirror ring laser to the magnetic field in the case of nonplanar deformation of the axial contour.

## 2. Calculation technique

In nonplanar resonators, the incidence–reflection planes of the axial beam for adjacent vertices of the axial contour do not coincide. The polarisation state of the waves is determined in the rotating coordinate system  $\xi_i\eta_i\zeta_i$ , whose axis  $\xi_i$  lies in the plane of radiation incidence on the  $i$ th optical surface. On passing to the next resonator arm, the axis  $\eta_i$  is rotated through the angle  $\alpha_i$  around the longitudinal axis. For a counterpropagating wave, the direction of the axis  $\zeta_i$  changes to the opposite one, the coordinate system remaining right-handed.

The method of the analysis of nonplanar resonators consists in the following. It is necessary to specify the type of the nonplanar deformation of the axial contour, to calculate the coordinates of the axial contour vertices, and to determine the rotation angles between the planes of incidence of the axial beam on each vertex. Then, the cyclic Jones operator should be constructed taking into account the rotators describing the turn of the subsequent planes of incidence, and matrices of the amplitude–phase anisotropy, characterising each vertex and each arm of the axial contour, and, finally, to determine the polarisation eigenstates, losses, and frequency shifts of counterpropagating waves.

Consider a four-mirror travelling wave resonator with a deformed square axial KLMN contour (Fig. 1) whose arms have the same length  $a$ . The resonator mirrors will be assumed identical. Let the initial axial KLMN' contour be deformed along the diagonal MOK so that the point  $N$  be displaced at a distance  $\Delta$  outside the plane of the initial axial contour. The break angle  $\gamma$  of the diagonal LON is equal to  $\arcsin(\sqrt{2}\Delta/a)$ .

We will select the calculated section at the point P near the vertex K and will pass clockwise (PLMNK) around the resonator. On passing the resonator, we rotate, in front of each mirror, the coordinate basis  $\xi_i\eta_i\zeta_i$  around the axial beam through a rotation angle  $\alpha$ . This means that an unpolarised (natural) rotator with the parameter  $\alpha$  is located in each resonator arm. If the angle  $\gamma$  is small, the angle is  $\alpha \approx \pm\sqrt{2}\gamma/2$ . For a counterpropagating wave, the orientation of the coordinate basis axes is different (see Fig. 1, where the superscript ‘-’ means that this basis refers to the wave passing the resonator counter-clockwise).

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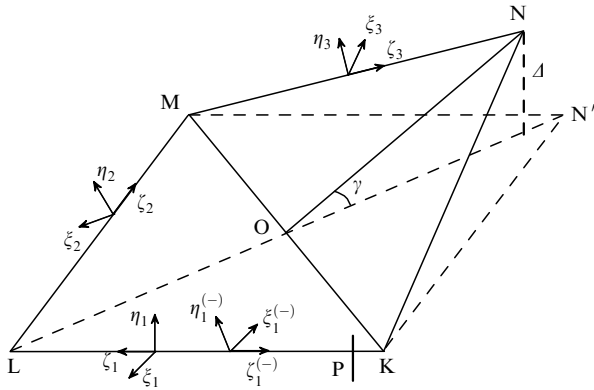


Figure 1. Scheme of a nonplanar resonator.

The complete Jones operator is found as a product of Jones matrices of rotators and reflectors:

$$\hat{T}_0 = \hat{T}(\alpha)\hat{T}_m\hat{T}(-\alpha)\hat{T}_m\hat{T}(\alpha)\hat{T}_m\hat{T}(-\alpha)\hat{T}_m, \quad (1)$$

where

$$\hat{T}_m = \begin{pmatrix} -R & 0 \\ 0 & 1 \end{pmatrix}; \quad \hat{T}(\alpha) = \begin{pmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{pmatrix}.$$

Here,  $R = R_p/R_s = (1 - t)\exp(-i\phi)$  is the amplitude-phase anisotropy coefficient of the mirror;  $\phi = \phi_p - \phi_s$  is the phase shift difference of orthogonal components of the electromagnetic wave strength vector  $\mathbf{E}$  during reflection;  $t \approx t_p - t_s \approx (\tau_p - \tau_s)/2$  is the parameter of the relative amplitude anisotropy;  $t_p$  and  $t_s$  are the amplitude transmission coefficients; and  $\tau_p$  and  $\tau_s$  are energy transmission coefficients. For the interference mirrors, the parameter  $t$  and the phase shift  $\phi$  additional to  $180^\circ$  are usually small quantities.

### 3. Results of calculations

For a resonator formed by mirrors, the s-polarised mode, whose vector  $\mathbf{E}$  is perpendicular to the plane of radiation incidence on each mirror, has a higher  $Q$  factor (lower losses). As follows from the calculations based on the above described method, additional polarisation losses  $A_s$ , caused by the nonplanarity, and the polarisation variable  $\Gamma_s = E_x/E_y$  are described in this case by expressions

$$A_s \approx 8\text{Re}\left(\frac{1 - R}{1 + R}\right)\alpha^2, \quad \Gamma_s \approx \pm \frac{\alpha}{1 + R}. \quad (2)$$

In a four-mirror nonplanar resonator (Fig. 1), the radiation polarisation, as a rule, becomes elliptic and changes after reflection from each mirror. In all the arms of the resonator, the rotation direction of the vector  $\mathbf{E}$  for counterpropagating waves are opposite (this means that in their coordinate bases with the opposite directions of the longitudinal axis, the waves have identical signs of the ellipticity angles). If a magnetic field is applied to the active medium of such a ring laser, a nonreciprocal frequency shift of counterpropagating waves appears.

Consider the influence of mirror parameters on the polarisation characteristics of a ring laser for which the

ratio of the frequencies and losses of counterpropagating high- $Q$  modes is important [1, 2].

The anisotropy parameters of interference mirrors depend on the number of mirror layers  $N$ , the ratio  $\lambda/\lambda_0$  ( $\lambda$  is the wavelength of incident radiation,  $\lambda_0$  is the nominal value of the wavelength), on the angle of incidence, etc. When  $\lambda/\lambda_0$  differs from unity, the phase anisotropy  $\phi$  increases linearly. In this case, if  $\lambda/\lambda_0 \approx 1.02$ ,  $\phi \approx 5^\circ$  (Fig. 2). The amplitude anisotropy mainly depends on the number of mirror layers: the larger is  $N$ , the lower is  $t$ . For example, at  $N = 21$ , we have  $t \approx 10^{-3}$ . At  $\lambda/\lambda_0 \approx 1.05$ , the parameter  $t$  increases approximately by 1.5 times (Fig. 3).

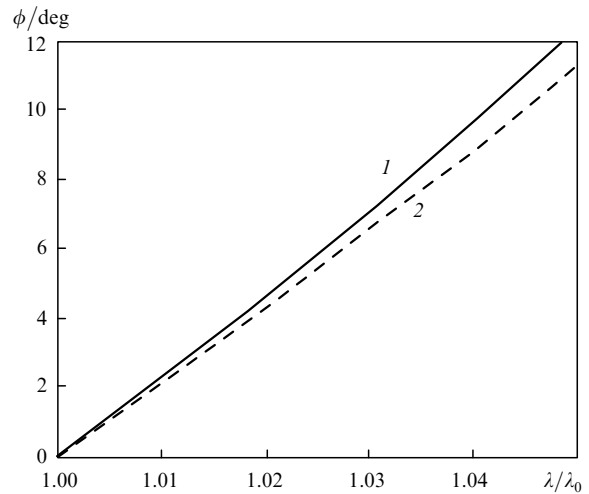


Figure 2. Dependences of the phase shift  $\phi$  between orthogonal components of the strength vector on the ratio  $\lambda/\lambda_0$  for the number of the mirror layers  $N = 23$  (1) and 13 (2).

The analysis performed shows that there exists a combination of the mirror parameters at which the sensitivity of the polarisation characteristics to the nonplanar deformation is maximal (at small rotation angles):

$$\phi_0 = t. \quad (3)$$

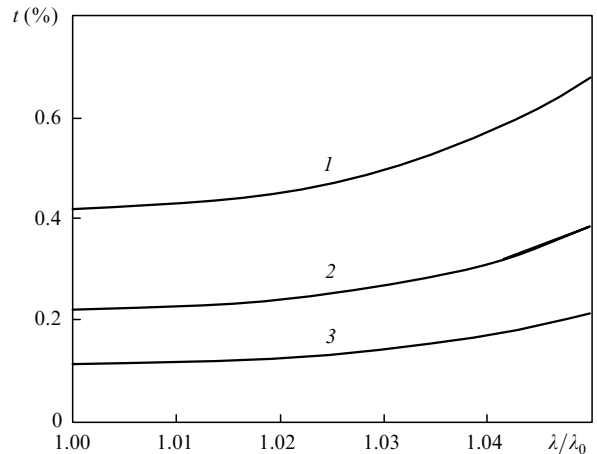
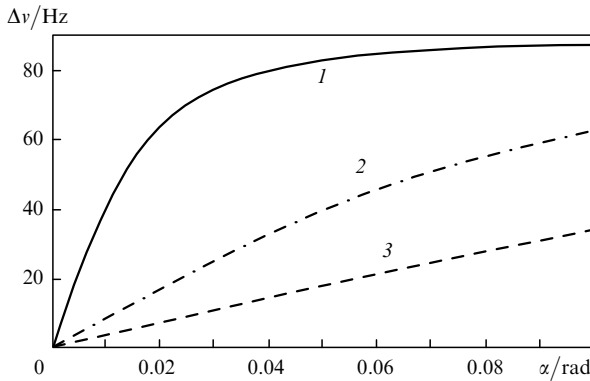


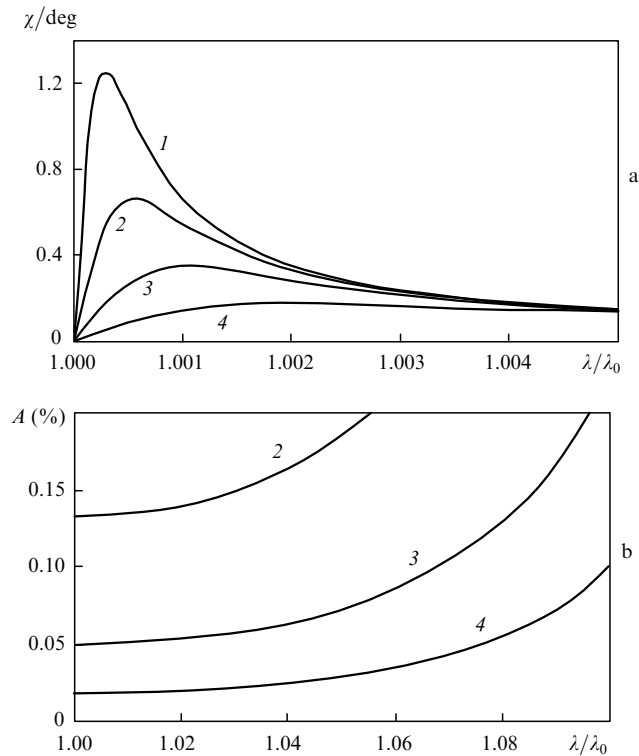
Figure 3. Dependences of the parameter of the relative amplitude anisotropy  $t$  on the ratio  $\lambda/\lambda_0$  at  $N = 17$  (1), 19 (2), and 21 (3).

This combination of the parameters of the amplitude–phase anisotropy of the mirrors should be avoided during their fabrication. When relation (3) is fulfilled, additional losses are  $A_{\max} \approx 4\alpha^2/\phi_0$  and the ellipticity angle  $\chi_{\max} \approx \alpha/(2\phi_0)$  achieves maximal values.

Figure 4 shows the calculated dependence of the frequency shift of counterpropagating waves in a magnetic field on the rotation angle  $\alpha$  calculated for the parameter of the Faraday rotator  $0.26 \times 10^{-6}$  [10], which approximately corresponds to the gain in the active medium of a ring laser, the magnetic field strength 1 Oe, and the axial contour perimeter 0.28 m.



**Figure 4.** Dependences of the frequency shift  $\Delta\nu$  of counterpropagating waves of ring laser in a magnetic field on the rotation angle  $\alpha$  at  $\lambda/\lambda_0 = 1.01$  (1), 1.05 (2), and 1.1 (3).



**Figure 5.** Dependences of the ellipticity angle  $\chi$  (a) and polarisation losses  $A$  (b) on the ratio  $\lambda/\lambda_0$  at the angle  $\alpha = 5 \times 10^{-5}$  and  $N = 15$  (1), 17 (2), 19 (3), and 21 (4).

Figure 5 presents the dependences of the ellipticity angle  $\chi$  and losses  $A$  on  $N$  and  $\lambda/\lambda_0$  determining different ratios of the parameters of the amplitude–phase anisotropy of the mirrors. It follows from the analysis of Figs 4 and 5 that the sensitivity of the polarisation characteristics to the nonplanar deformation of the axial contour can be decreased by increasing the phase anisotropy of the mirrors up to  $10^\circ$ ; in this case,  $\lambda/\lambda_0 \approx 1.04$ .

### 4. Conclusions

(i) The amplitude–phase polarisation anisotropy of interference mirrors significantly affects the losses, ellipticity angle, and frequency shifts of counterpropagating waves of a nonplanar resonator ring laser.

(ii) For a certain ratio of the parameters of the amplitude and phase anisotropy of the mirrors, the sensitivity of the polarisation characteristics of a ring laser to the nonplanar deformation of the axial contour of the resonator increases.

(iii) To reduce the influence of the nonplanar deformation of the axial contour on the polarisation characteristics, it is necessary to provide an optimal range of the phase anisotropies of the mirrors ( $5^\circ < \phi < 10^\circ$ ) by changing the parameters of the interference coating (the number, thickness, and the refractive index of the mirror layers).

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