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Application of the stochastic parallel gradient descent algorithm for numerical simulation and analysis of the coherent summation of radiation from fibre amplifiers*

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***Abstract*. Coherent summation of fibre laser beams, which can be scaled to a relatively large number of elements, is simulated by using the stochastic parallel gradient descent (SPGD) algorithm. The applicability of this algorithm for coherent summation is analysed and its optimisaton parameters and bandwidth limitations are studied.**

Keywords: *coherent summation, fibre laser, stochastic parallel gradient descent algorithm.*

1. Introduction

Coherent summation of radiation from lasers/amplifiers in a phased array configuration can increase the overall output laser power while maintaining a good beam quality, which is an important research area for laser communication, laser radar and energy delivering system [1]. In coherent summation, all the laser elements operate with the same spectrum and the relative phases are controlled to maintain the constructive interference increasing the intensity. Fibre lasers/amplifiers are especially well-suited for beam summation because of their inherent compact size [2]. Several approaches have been demonstrated to scale up the output power by summing radiations of multiple lasers. Stable coherent summation of beams from several fibre lasers has been demonstrated in [3 – 5]. Multi-core phase-locked fibre lasers [3] employing the evanescent coupling between multiple cores to achieve coherent summation significantly reduce nonlinear processes within the fibre core; however, the maximum power is still limited by the available pump power and brightness of pump laser diodes as in the single-core fibre system.

Coupling schemes with a single fibre output [4] are probably not suitable for high power scaling. The problem is that the nonlinear effects within the fibre cannot be eliminated. Coherent summation of beams from fibre lasers with a master oscillator – power amplifier (MOPA) configuration solves the problem of power limitation. To obtain stable constructive interference, phase control is required to compensate for the phase noise based on the heterodyne-detecting method in the present MOPA configuration. To the best of our knowledge, the highest power in the MOPA configuration is achieved by using the active phase control [5]. Besides, in high-power beam systems, heterodyne-detecting cannot be applied not only because of the presence of the sophisticated phase control system but also due to the partial coherence of such beams [6, 7].

To solve these challenging problems and to scale the MOPA powers to a higher-power level, coherent beam summation with the help of the stochastic parallel gradient descent (SPGD) algorithm without phase-detecting components was proposed and demonstrated by Liu [8, 9] and Kansky [2]. The SPGD algorithm was first used in the adaptive optics [10, 11]. If coherent summation of the beams from fibre lasers is considered as the compensation for piston-type phase aberrations for a single large beam in a monolithic optical system, it is possible to apply straightforward the SPGD algorithm to solve this problem. By now, 48 collimated micro-beams have been coherently summed with the phase control by using the SPGD algorithm [2].

In this paper, we present an extensive study of coherent summation of a relatively large number of laser beams using the SPGD algorithm.

2. Formulation of the problem

2.1 System setup

Consider a coherent beam summation system with N channels of elementary beams (see Fig. 1 showing only two beams as an example). The laser beam from the master oscillator is split into N channels and is coupled to the phase modulators. The laser beams from the phase modulators are then delivered to the fibre amplifiers and optical isolators and then to fibre collimators. Note that for high-power applications, multi-stage fibre amplifiers are usually required. The beam array is coupled out to free space via the collimators. The collimated output beams are split by a beamsplitter. After the beamsplitter, a part of the beams is incident on a focusing lens which images the far-field pattern onto the detector. The cost function $J = J(\boldsymbol{u})$ obtained or calculated from the signal collected by the detector, is a function of the control parameters $\boldsymbol{u} = \{u_1, \ldots, u_N\}$, which are typically voltages applied to the phase modulators. At each iteration cycle of the SPDG algorithm, the phase control signal is applied to the phase modulators via the analogue-to-digital converter and the

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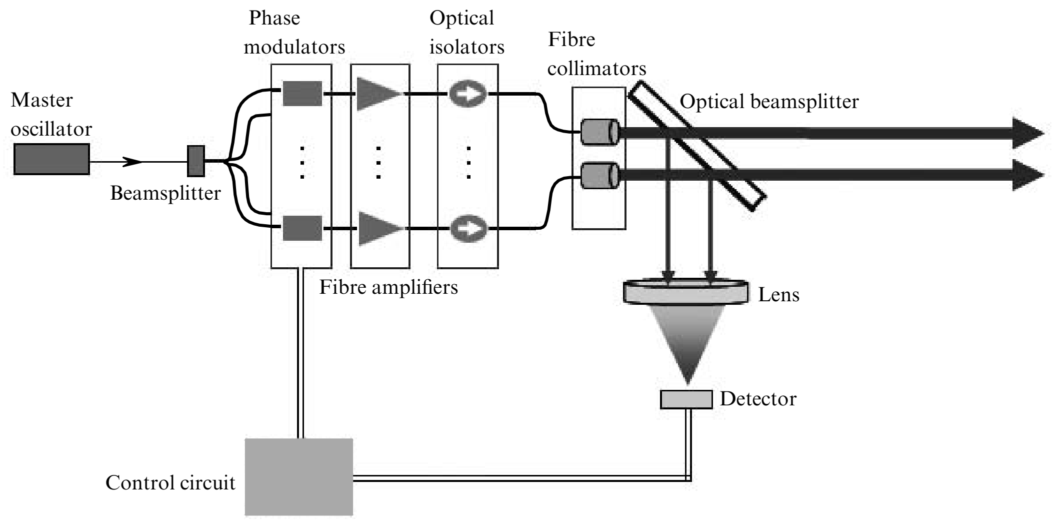

Figure 1. System setup for coherent beam summation with the help of the SPGD algorithm.

control circuit. The other part of the beam after the beamsplitter is the desired coherent summed beam array. The object of coherent beam summation is to maximise the energy in the central lobe, i.e. maximise the cost function J by using the SPGD algorithm .

2.2 Theory of the SPGD Algorithm

The theory of the SPGD algorithm can be briefly described as follows [8]. The cost function is expressed as $J = J\{u_1, u_2, \ldots, u_n\}$, where u_i are the control voltages ($i = 1, \ldots, n$) generated by the computer. Each iteration cycle has the following stages:

(i) the statistically independent random perturbations $\delta u_1, \delta u_2, \ldots, \delta u_n$ are generated, where all $|\delta u_n|$ are small values which are typically chosen as statistically independent variables with the zero mean and equal dispersions: $\langle \delta u_k \rangle = 0$, $\langle \delta u_k \delta u_l \rangle = \sigma^2 \delta_{kl}$, where δ_{kl} is the Kronecker symbol;

(ii) the control voltages with the positive perturbations are applied and the cost function $J^+ = J(u_1 + \delta u_1, u_2 + \delta u_2, \ldots, u_n + \delta u_n)$ is evaluated, then the control voltages with the negative perturbations are applied and the cost function, $J^- = J(u_1 - \delta u_1, u_2 - \delta u_2, \ldots, u_n - \delta u_n)$ is again evaluated;

(iii) the difference between two evaluations of the cost function is calculated: $\delta J = J^+ - J^-$;

(iv) the control voltages $u_i = u_i + \gamma \delta u_i \delta J$ are updated, where γ is the correction coefficient, $\gamma > 0$ corresponding to the minimisation procedure and $\gamma < 0$ – to the maximisation procedure.

2.3 Cost function

A set of image quality metrics referred as sharpness functions or sharpness metrics is used as cost functions in adaptive optics with the SPGD algorithm [10]. These cost functions require the time-consuming calculations of matrices computation and are hardly suitable for real-time applications. The image quality parameter proposed in [11, 12] can be measured in real time using an analogue coherent optoelectronic processor. The radiation power propagated through the target pinhole is used as the cost function in [8]. Both these parameters can be measured in real time. For the simplicity of the whole system, the radiation power propagated through the target pinhole is used in this paper as the cost function for coherent summation of radiation from fibre lasers.

3. Simulation and analysis

3.1 Feasibility validation

Because the perturbation signal in practice varies with time, the performance of the whole system will depend on the perturbations. The laser array has the form of two rings with 6 lasers and 12 lasers in the inner and outer rings, respectively. We used the following parameters for the whole laser array: $w_0 = 1$ cm is the beam waist of a single laser and $d = 3$ cm is the distance between the neighbouring lasers. The rms value of the phase error for the laser array is 3π. The correction coefficient was chosen equal to 4 after analysing many results of numerical calculations. Figure 2 shows the average result of 100 numerical simulations. One can see from Fig. 2 that the SPGD algorithm can converge to the extremum of the metric. The concept of coherent summation using the SPGD algorithm is feasible.

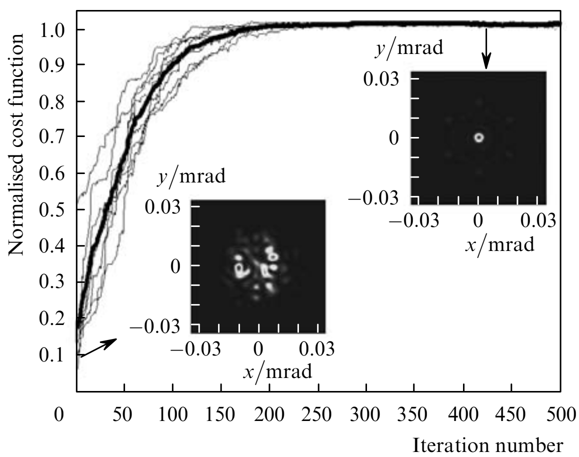

Figure 2. Dependences of the normalised cost functions on the iteration number for different initial perturbations. The insets show the examples of the far-field intensity distribution before (on the left) and after (on the right) application of the SPGD algorithm.

3.2 Optimisation of parameters

3.2.1 Perturbations

One of the key components in implementing the SPGD algorithm is the perturbations. Basically, there are four kinds of probability density distributions for generating perturbations with zero mean and equal dispersions: the Bernoulli distribution, segmented-uniform, uniform and normal distributions.

The convergence curve for different kinds of probability density distributions is shown in Fig. 3a, all the parameters used in calculation being the same as those in Fig. 2. One can see that the Bernoulli perturbations result in the fastest convergence rate. Taking into account that these Bernoulli perturbations can be generated easily by the hardware, they should be applied in the coherent beam summation. Another parameter of the perturbations is their amplitudes. The evolution curves for different perturbation amplitudes are presented in Fig. 3b. One can see from Fig. 3b that perturbations with a smaller amplitude will decelerate the convergence rate of the evolution curve while perturbations with a larger amplitude will bring instability to the coherent summation effect. Therefore, we can conclude that there exists an optimal perturbation amplitude for coherent beam summation.

3.2.2 Correction coefficient

One of main drawbacks limiting the potential applications of this or that optimisation method is their convergence rate [13]. It was shown that when applied to the adaptive optics problems, the convergence rate of the SPGD algorithm is proportional to $N^{1/2}$ [14]. To accelerate the convergence rate of the evolution curves, the adaptive SPGD algorithm with a modified correction coefficient can be used. In the adaptive SPGD algorithm, the parameter γ can take adaptive values according to the expression $\gamma = g_0 J_0 / J$, where J_0 is the ideal value of the power propagated through the target pinhole, which can be calculated once after the coherent beam summation system is specified. The evolution curves of the cost functions calculated for constant and adaptive correction coefficients are plotted in Fig. 4. One can see that the adaptive correction coefficients can provide a faster convergence rate. Further statistical calculations with a large number of simulations reveal that using the adaptive correction coefficient can increase the convergence rate, at least, by 15 %.

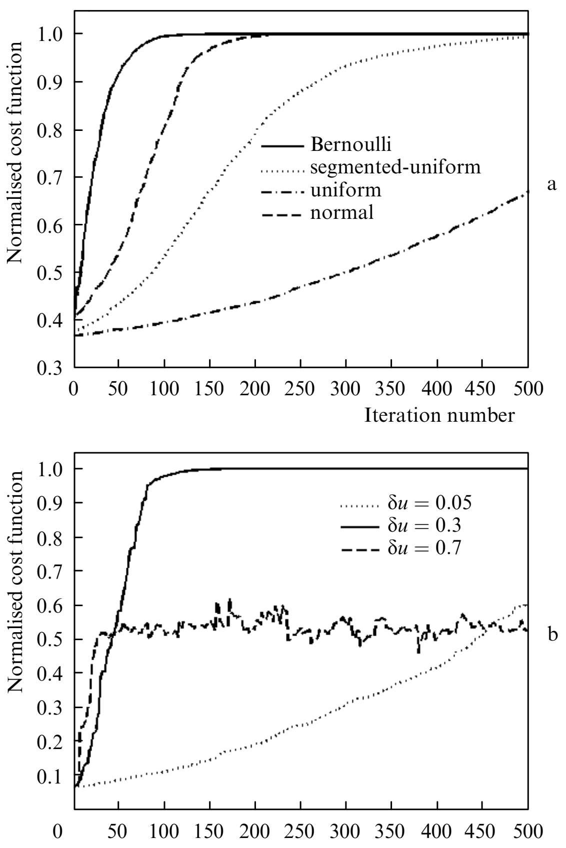

Figure 3. Dependences of the normalised cost functions on the iteration number for different distributions of the probability density (a) and perturbation amplitudes δu (b).

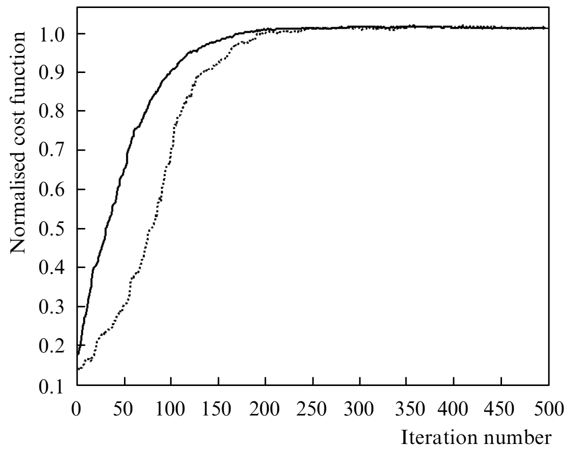

Figure 4. Dependences of the normalised cost functions on the iteration number for adaptive (solid curve) and constant (doted curve) correction coefficients.

3.2.3 Bandwidth analysis

The formal analysis indicates that the convergence rate typically increases at least as N in adaptive optics, where N is the number of elementary beams [14]. We studied coherent summation of the beams from fibre amplifiers using the SPGD algorithm for different laser beam arrays (i.e. laser arrays containing 37 or 61 lasers). The dependence of the convergence rate on the number of elementary beams is presented in Fig. 5. One can see that in coherent summation of N laser beam, the convergence rate depends linearly on N. This circumstance significantly complicates the application of the SPGD algorithm for coherent summation of a relatively large number of beams from high power fibre amplifiers. Besides, the question as to how rapidly the phase noise worsens with increasing the power at the fibre output remains open [15].

Thus, when scaling the fibre laser to a kilo-watt power level, the bandwidth of the phase control system should be improved. This means that the effective control bandwidth of the coherent beam summation system based on the SPGD algorithm decreases with increasing the laser beam power and the number of elementary beams. We believe that this problem can be solved in three stages. Firstly, a more advanced SPGD controller with a faster update rate should be used. Secondly, the corresponding countermeasures should be taken to suppress the influence of phase dis-

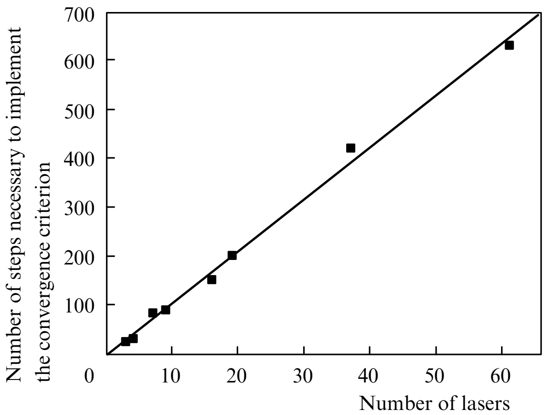

Figure 5. Calculated dependence of the number of steps necessary to implement the convergence criterion on the number of lasers (points) and its linear approximation (straight line).

tortions. Thirdly, the SPGD algorithm should be modified to accelerate the convergence rate when increasing the number of lasers.

4. Conclusions

We have shown the possible application of the SPGD algorithm for coherent summation of the beams from fibre amplifiers. We have found that the quality of the coherent beam summation system can be improved by optimising the parameters of the algorithm. In this case, the Bernoulli perturbations with a proper amplitude and adaptive correction coefficient should be used. Application of this method for a large number of lasers is limited by the bandwidth of the algorithm. Nevertheless, the advantage of beam summation with the help of the SPGD algorithm makes this algorithm so attractive that there are already several projects in this direction [16, 17]. We believe that the algorithm has a considerable potential for the development of a new architecture of high energy laser systems.

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